2.141 Modeling and Simulation of Dynamic Systems

Assignment #1

Out: Monday September 11, 2006
Due: Wednesday September 20, 2006

Problem 1

Typical examples of common linear drives are described in the attached pages taken from the Warner Electric catalog. You are to model and analyze the in-line configuration shown at the bottom of the page.

A permanent-magnet DC motor is used to drive a recirculating-ball screw (a nut and screw with ball-bearings interposed between the nut and screw so that frictional energy losses are minimized). The motor shaft is connected directly to the screw. The nut is prevented from rotating but may slide as the screw rotates within it. The screw has 5 threads per inch with a pitch diameter of 0.5 inches. The screw may be considered to be a steel cylinder 18 inches long with diameter equal to the screw pitch diameter. The nut may be considered to be a steel cylinder 3 inches long, outside diameter 3 inches, with a hole though its center equal to the pitch diameter of the screw. The extension tube is a foot long with outside and inside diameters of 3.0 and 2.9 inches respectively.

Parameters of the motor are given in the table below (excerpted from a direct-current permanent-magnet DC motor specification sheet)

MOTOR CONSTANTS:			
INTRINSIC (AT 25 DEG C)	SYMBOL	UNITS	
TORQUE CONSTANT	KT	OZ IN/AMP	5.03
BACK EMF CONSTANT	KE	VOLTS/KRPM	3.72
TERMINAL RESISTANCE	RT	OHMS	1.400
ARMATURE RESISTANCE	RA	OHMS	1.120
VISCOUS DAMPING CONSTANT	KD	OZ IN/KRPM	0.59
MOMENT OF INERTIA	JM	OZ IN SEC-SEC	0.0028
ARMATURE INDUCTANCE	L	MICRO HENRY	<100.0
TEMPERATURE COEFFICIENT OF KE	C	%/DEG C RISE	-0.02

(a) First assume the DC motor is driven by a current-controlled amplifier (i.e., the current applied to the electrical terminals of the motor may be specified independent of the voltage required[1]). Assuming all model elements have linear constitutive equations, develop a model competent to describe the linear displacement of the rod end in response to motor current input. Express your model as a bond graph, clearly identifying the different energy domains.

(b) What is the order of this model?

(c) Suppose an external force is applied to the rod end. Numerically evaluate the total apparent translational inertia opposing that force (i.e., the equivalent mass "seen at" the rod end due to everything that moves).

[1] This is commonly achieved using high-gain current feedback.

(d) Next assume the DC motor is driven by a voltage-controlled amplifier (i.e., the voltage applied to the electrical terminals of the motor may be specified independent of the current required). Assuming all model elements have linear constitutive equations, develop a model competent to describe the linear displacement of the rod end in response to motor voltage input. Express your model as a bond graph, clearly identifying the different energy domains.

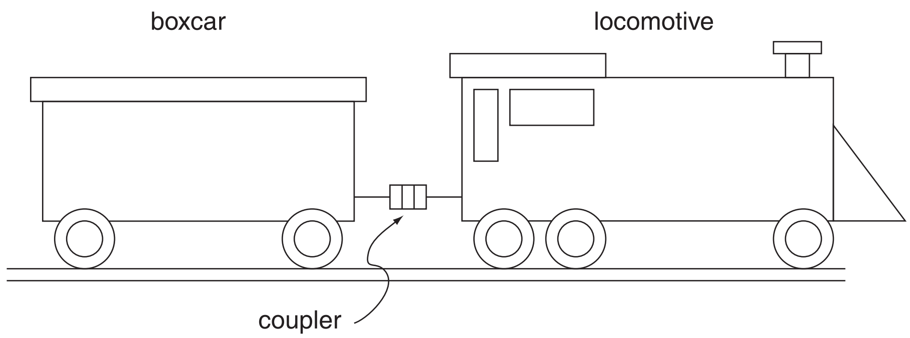
(e) What is the order of this model?

(f) Describe (preferably using a standard "iconic" diagram) an equivalent all-mechanical-translation system with the same dynamic behavior.

(g) In that equivalent model, what kind of mechanical element (mass, spring or damper) corresponds to the electrical inductor? In units consistent with your model above, what is its numerical value?

Problem 2

A locomotive pulling a single boxcar on a straight, flat track is depicted in the sketch. Your task is to develop a simple model of the motion of the system in response to inputs from the locomotive



One singular advantage of compressed gas (i.e., steam) engines is that they generate maximal effort at minimal speed, even at zero speed (i.e., stalled)[2]. For that reason, assume the locomotive generates a prescribed step change in force $F(t)$. Initially assume the coupling between locomotive and boxcar has both stiffness (elastic energy storage) and damping (energy dissipation) properties with linear constitutive equations. The boxcar and locomotive are subject to rolling friction at the wheels.

(a) Propose a model of the system and represent it as a bond graph. Specify whether the coupler stiffness and damping should be in series or in parallel and justify your choice physically (without excessive analysis).

(b) Approximate some realistic values for the parameters of your model[3].

(c) From your model, develop a set of state and output equations sufficient to predict the position of the locomotive and boxcar.

(d) It is probably difficult to justify the assumption of linearity for the stiffness and damping elements of this model. For example, real couplers exhibit pronounced

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[2] Compare that with a typical internal-combustion engine.

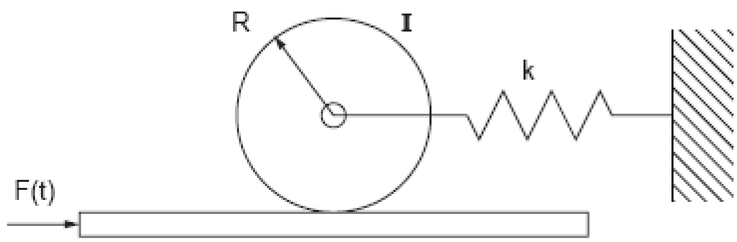
[3] This is often one of the most challenging aspects of model formulation and a strong reason for keeping models as simple as possible.

"backlash" (due to clearance between connecting members) and nonlinear friction, e.g., as described by Coulomb's law, $F_{Coulomb} = F_c \operatorname{sgn}(v)$. Modify your model and propose parameter values to include these effects. Does the model order change?

(e) Develop simulations (e.g., using MATLAB) of the responses of both the linear and nonlinear models to a step input force that would yield a steady speed of 30 mph. Start from rest and simulate until speed is at or near steady state.

Problem 3

The sketch depicts a cylinder that rolls on a horizontal plate which may be driven horizontally by force $F(t)$. The plate has mass m_p. The cylinder has radius R, mass m_c, moment of inertia I about its mass center and is restrained by a spring of stiffness k attached to its axis as shown.



(a) Assuming no slip between cylinder and plate, formulate a model relating input force to the horizontal position of the cylinder mass center. Represent your model as a bond graph.

(b) What's the minimum number of generalized coordinates required to describe the system's configuration?

(c) What's the maximum number of energy-storage elements that may be assigned integral causality?

(d) Formulate a minimal-order state-determined model relating input force $F(t)$ to mass center position.

Now assume (i) that the cylinder may slip relative to the plate and (ii) that the plate is driven by a velocity source that specifies plate velocity $v_p(t)$. Assume the friction between plate and cylinder may be described by the usual dry friction model: zero slip until force reaches a threshold value F_f, constant force at that value opposing motion at all non-zero slip velocities.

(e) Formulate a model relating input force to the horizontal position of the cylinder mass center. Represent your model as a bond graph.

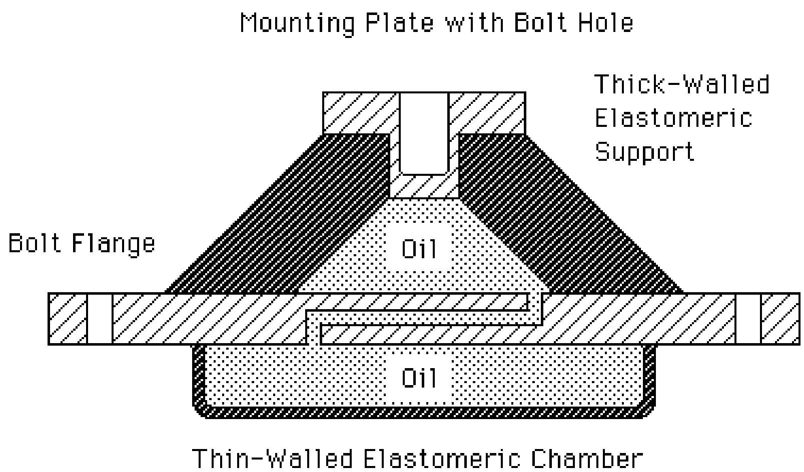
(f) What's the minimum number of generalized coordinates required to describe the system's configuration?

(g) What's the maximum number of energy-storage elements that may be assigned integral causality?

(h) Can a single system order be assigned to the resulting model? If so, what is it? If not, explain.

Problem 4

The diagram shows a simplified cross-sectional view of an engine shock-mount developed by Lord Corporation for use in aircraft and automobiles. In use, an engine is bolted to the top mounting plates of a number of these (usually four). The shock mounts are attached to the vehicle frame via the bolting flange. The thick-walled elastomeric support provides steady-state support for the engine. The inside of the shock-mount is filled with oil in two chambers that communicate via a narrow passage called an "inertia track". This is an ingeniously cunning way to make the shock mount function as a *dynamic vibration absorber*.[4] Your task is to develop a model of this system to explain how it works and describe which geometric and material parameters are critical to its function.



(a) Propose the simplest model competent to describe how the shock-mount functions as a dynamic vibration absorber and represent it as a bond graph, clearly identifying the different energy domains. To keep things simple, assume the bolt flange does not move and that an engine is mounted on the top mounting plate and vibrates about an equilibrium position in response to vertical forces. To gain insight, initially assume that all frictional losses are zero.

(b) Describe (preferably using a standard "iconic" diagram) an equivalent all-mechanical-translation system, and specify the relation between the parameters of the elements in the two models.

(c) What's the minimum number of generalized coordinates required to describe the system's configuration?

(d) What's the maximum number of energy-storage elements that may be assigned integral causality?

(e) What's the maximum number of energy-storage elements that may be assigned *differential* causality?

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[4] The dynamic vibration absorber was invented by Frahm in 1909 and is discussed in detail in Den Hartog, J. P. (1956) Mechanical Vibrations, 4th ed. Section 3.2, pp. 87 et seq.

(f) Derive minimal-order state equations relating input vertical force to output vertical position of the top mounting plate. Assume all elements have linear constitutive equations.

(g) Derive an expression for the admittance of the system (the transfer function relating input vertical force to output vertical position). Make a Bode plot of its magnitude and phase. (If you do this using, e.g., MATLAB you may find it convenient to include small amounts of frictional loss to avoid infinite amplitudes.)

(h) What is the physical meaning of the poles and zeroes of this transfer function? If the engine were subjected to a sinusoidally varying force at some known frequency, Ω, explain how the shock mount could be designed to "absorb vibration" due to this excitation.

(i) In order for the system to function properly as a vibration absorber, its dimensions must be carefully chosen. Write a design equation relating the excitation frequency, Ω, to the physical parameters of the system (e.g. geometric dimensions, density of oil, etc.).

Design Features

An inside look…

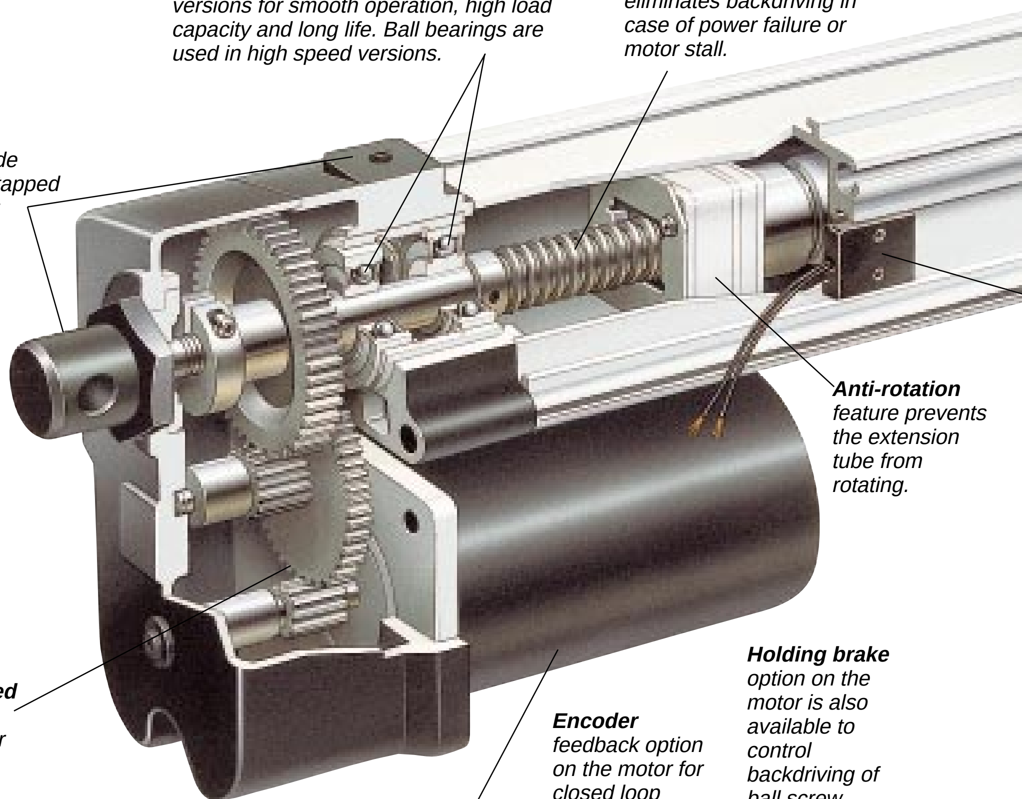
Parallel configuration

(Shown with acme screw and gear drive. Other configurations are available.)

***Roller thrust bearings** on high load versions for smooth operation, high load capacity and long life. Ball bearings are used in high speed versions.*

***Acme screw** with self locking operation eliminates backdriving in case of power failure or motor stall.*

***Mounting options** include clevis, front flange, side tapped holes, and mounting feet*



***Anti-rotation** feature prevents the extension tube from rotating.*

***Belt, gear or direct coupled** depending on ratio and configuration of the actuator*

***Sealed motors** for wash down applications (IP-56 rated)*

***Encoder** feedback option on the motor for closed loop positioning control*

***Holding brake** option on the motor is also available to control backdriving of ball screw version in the event of electrical power failures.*

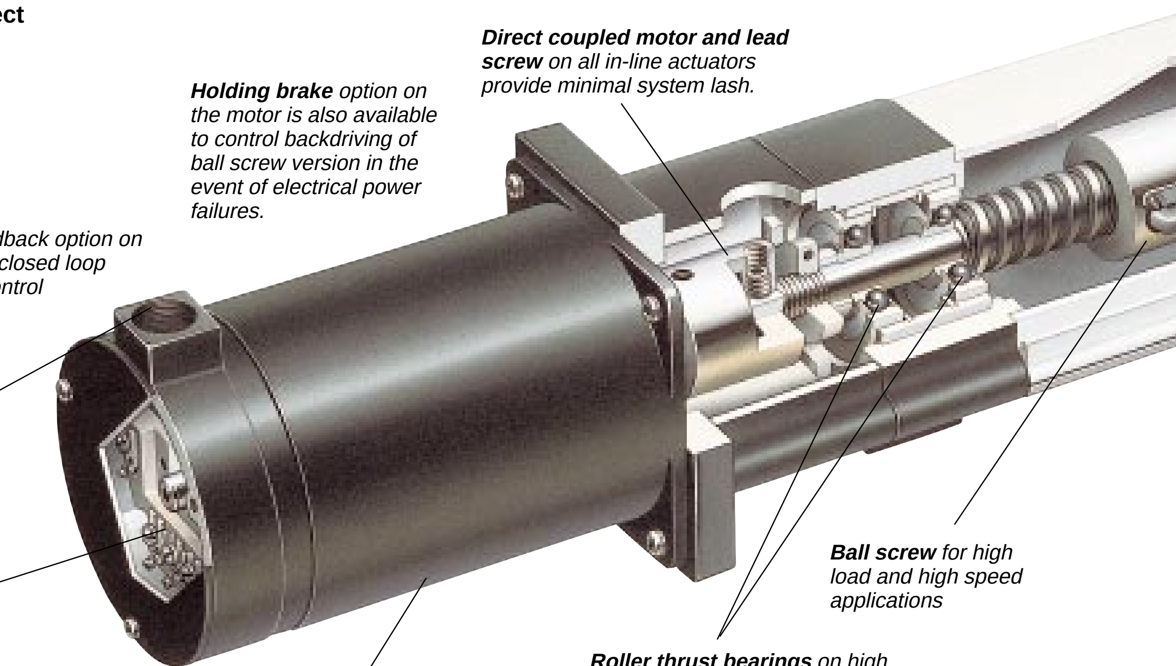
In-line configuration

(Shown with ball screw and direct drive. Acme screw drive is also available.)

***Direct coupled motor and lead screw** on all in-line actuators provide minimal system lash.*

***Holding brake** option on the motor is also available to control backdriving of ball screw version in the event of electrical power failures.*

***Encoder** feedback option on the motor for closed loop positioning control*



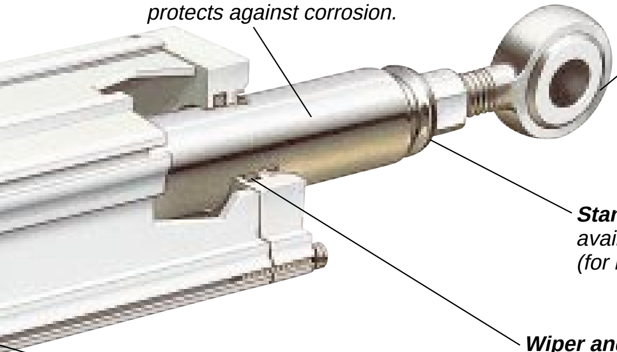
***Threaded cable entry and watertight connector** provides washdown capability.*

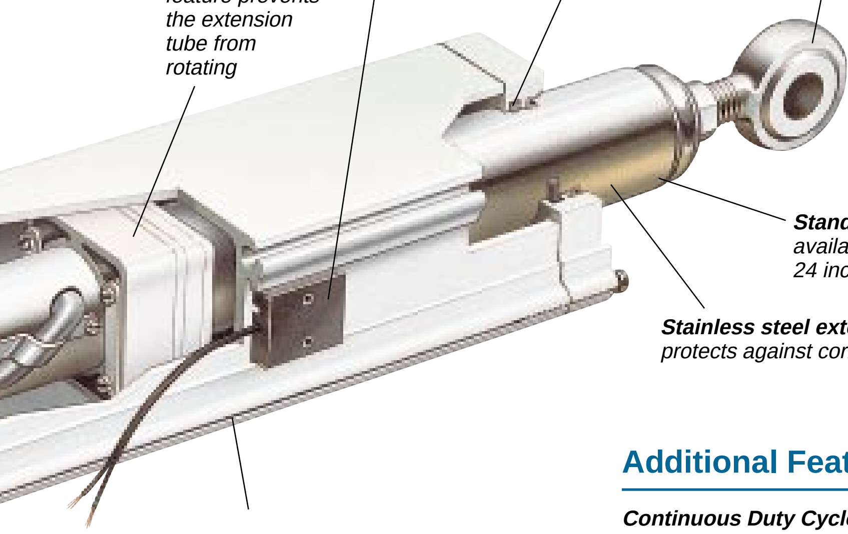
***Screw terminals** on size 34 and size 42 motors provide ease of wiring.*

***Ball screw** for high load and high speed applications*

***Sealed motors** for wash down applications (IP-56 rated)*

***Roller thrust bearings** on high load versions for smooth operation, high load capacity and long life. Ball bearings are used in high speed versions.*





Actuator Components:

Drive types

Acme screw – economical choice for lower load, speed and duty cycle requirements. Acme screws are self-locking by design and can not be back driven by the load. They are recommended for vertical applications or other applications where back driving can occur during loss of power. Acme screw actuators have a 50% maximum duty cycle rating.

Ball bearing screw – choice for higher speed and load applications. Ball bearing screws are highly efficient and consume little power in converting rotary motion into linear motion. Ball screw actuators have a 100% duty cycle rating. Motor power must be maintained to prevent backdriving by the load.

Configurations

Parallel – this style of actuator allows the broadest performance range with various combinations of drive types and ratios, screw types and leads, and motor sizes. Speeds up to 15 inches per second at 20 lb. and loads up to 2000 lb. at 0.3 inches per second define the performance limits. A variety of mounting configurations are available for parallel style actuators to make installation as easy as possible.

In-line – direct drive with combinations of motors, screws and leads provides speeds up to 25 inches per second at 15 lb. and loads up to 1000 lb. at 0.2 inches per second. In-line actuators are available with front flange, trunion, side tapped holes, or foot mounting to suit a variety of applications.

Additional Features

Continuous Duty Cycle *will handle the most rigorous applications. (Ball screw version)*

Life *up to 50 million cycles depending upon screw type and load*

Resolution *is less than 0.0005 inches depending on the control selected.*

Ambient temperature rating *of -40° to +150°F allows a wide range of applications.*

Parallel or in-line versions *for different space requirements*

Load *ratings up to 2,000 lb. at .3 inches per second continuous*

Speed *up to 25 inches per second continuous at 15 lbs., variable up to the maximum speed. A constant speed regardless of load up to load/speed rating of the actuator*

Programmable positioning *available with Superior Electric controls*

Software package *available that is menu driven and units are expressed in linear terms for easier and faster programming. No need to learn special codes*

Hazardous location versions *are available. Contact the factory for explosion proof or dust ignition proof models.*