2.161 Signal Processing: Continuous and Discrete
Fall 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING

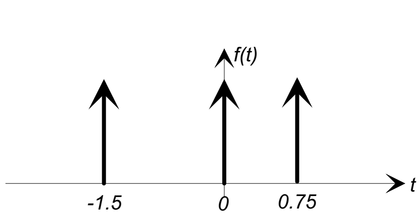
2.161 Signal Processing - Continuous and Discrete
Fall Term 2008

Problem Set 1: Convolution and Fourier Transforms

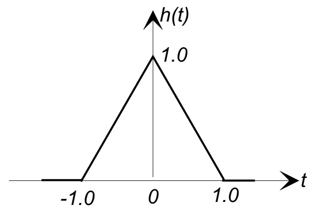
Assigned: Sept. 9, 2008 **Due:** Sept. 18, 2008

Problem 1:

(a) Plot the result of convolving a one-dimensional function consisting of three impulses with a triangular function as shown below:

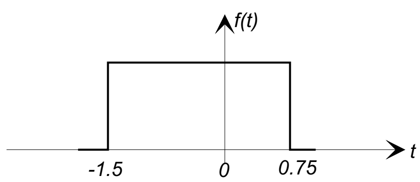


(a) Impulsive waveform

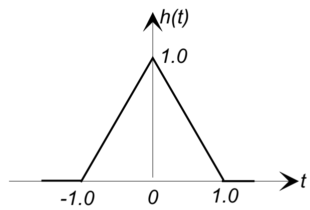


(b) Impulse response

(b) Plot the result of convolving a one-dimensional function consisting of a rectangular function with the same triangular function used in part(a), as shown below:



(a) Rectangular input function



(b) Impulse response

(c) Plot the result of convolving a pair of identical even "top-hat" (pulse) functions: $f(t) = 1$ for $|t| < T/2$ and $f(t) = 0$ otherwise. Use your result show that the Fourier transform of a triangular pulse (such as used in parts (a) and (b)) is of the form $(\sin(x)/x)^2$.

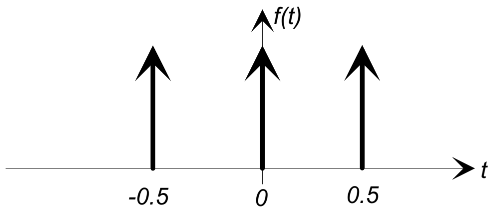
Problem 2: Show that when two gaussian functions

$$f_1(x) = e^{-ax^2}$$
$$f_2(x) = e^{-bx^2}$$

are convolved, the result is another gaussian function.
Hint: Complete the square in the exponent, change the variable of integration, and recognize that

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Problem 3: Find the Fourier Transform of three equally spaced impulses as shown below:



Plot the result (real and imaginary parts).

Problem 4: (The following is taken from the Signal Processing PhD Quals written exam for January 2007. Note: this is not the complete exam.)
Assume we have a signal $x(t)$ with a Fourier transform $X(j\Omega)$ given by

$$X(j\Omega) = \begin{cases} 0 & \Omega < -2W \\ X_0/2 & -2W \leq \Omega < 0 \\ X_0 & 0 \leq \Omega < W \\ 0 & \Omega \geq W \end{cases}$$

where X_0 is some real valued number, W is a real valued positive number, and Ω is specified in units of radians/second.

(a) What is the value of $x(t)$ at $t = 0$?

(b) For an arbitrary t, what is the relationship between $x(t)$ and $x(-t)$?

(c) What is the value of $\int_{-\infty}^{\infty} x(t)dt$?

(d) What is the value of $\int_{-\infty}^{\infty} |x(t)|^2 \, dt$?

Problem 5: An impulse $\delta(t)$ is passed through an ideal low-pass filter with frequency response $H(j\Omega) = 1$ for $|\Omega| < \Omega_c$ and $H(j\Omega) = 0$ otherwise. Find and sketch $y(t)$, the output of the filter. Is this a causal filter?

Problem 6: After measurement and curve-fitting it is determined that a causal signal processing filter has an impulse response $h(t) = 5e^{-3t}$ for $t > 0$.. What is the filter's (a) transfer function, and (b) its frequency response function? Determine the -6 dB cut-off frequency, that is the frequency at which the output amplitude is one half of the low frequency response.