2.20 Problem Set 3A

Name: ____________________

1. Recall from Lecture 3 that the Kinematic Transport Theorem (KTT) is:

$$(1)\quad \frac{d}{dt}\iiint_{V(t)} f(\vec{x},t)\,dV = \iiint_{V(t)} \frac{\partial f}{\partial t}\,dV + \iint_{S(t)} f\vec{U}(\vec{x},t)\cdot\hat{n}\,dS$$

where *V(t)* is an arbitrary time-varying control volume, *S(t)* is its bounding surface, $\vec{U}(\vec{x},t)$ is the absolute velocity of the surface *S* with respect to a fixed frame, and $\hat{n}$ is the normal to the surface *S* pointing out of the volume *V*.

(a) In many applications, the control volume for a flow system can be chosen as fixed in space. Choose the property *f* to be the mass per unit volume, or density $\rho(\vec{x},t)$. Denote the fixed control volume as V_C and the corresponding control surface as S_C. Write the simplified KTT for this case, leaving it in terms of general integrals as above:

(b) Now assume the flow is also *steady*. This further simplifies the KTT to the following:

(c) Now, start again with equation (1) but let the control volume be a *material* volume $V_m(t)$ moving with the fluid and having a material surface $S_m(t)$ whose points move with the fluid velocity $\vec{V}(\vec{x},t)$. Write equation (1) for *unsteady* flow, again leaving it in terms of general integrals:

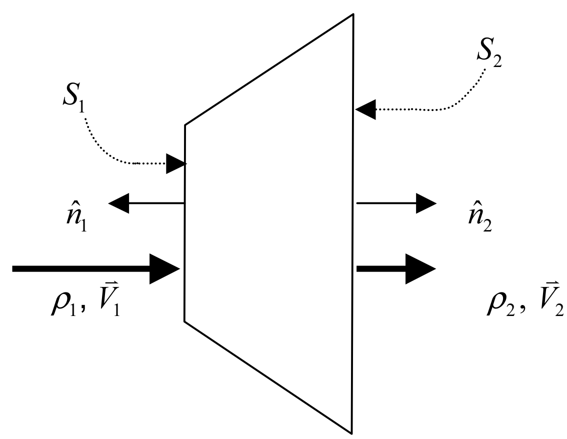
(d) Why must the right and left-hand sides of the expression you derived in (c) be equal to zero?

(e) Simplify the version of the KTT obtained in (c) for the case of *steady* flow, remembering to set the expression equal to zero:

(f) The surface integral in (e) over the moving material surface $S_m(t)$ is done at a *specific instant of time*. But this means we can then choose *any* control surface *S* in applying this version of the KTT, because any closed surface in the fluid *at a given instant* is a material surface. Rewrite the expression in (e) and denote the moving surface as S :

IMPORTANT POINT: An equivalent interpretation of this expression is that the surface *S* that we instantaneously choose is actually *fixed* in space, and the velocity $\vec{V}$ is then the velocity of the fluid relative to the fixed control surface. This often makes more sense from a problem-solving standpoint. We use it next.

(g) Now apply the steady flow version of the KTT obtained in (f) to a specific case where the fixed control volume and surface are shown below. The fluid enters normal to a surface S_1 with an area A_1, and the fluid exits normal to a surface S_2 with and area A_2. No fluid passes through the top or bottom of the control volume. Assume the fluid velocity and density are uniform over the surfaces S_1 and S_2. Simplify all integrals to get an expression relating the inlet and outlet mass flow rates.



2. There is a steady flow of water through a horizontal pipe that has a bend in it as shown. The water enters normal to the inlet area A_1 and exits normal to the area A_2. Using a fixed control volume and the momentum theorem (see text SAH, Section 4.2), find the components R_x and R_y of the force reaction of the pipe bend ON the water in terms of pressures p_1 and p_2, areas A_1 and A_2, velocity magnitudes V_1 and V_2, the constant density ρ, and the bend angle θ. Pressures and velocities are uniform over A_1 and A_2.