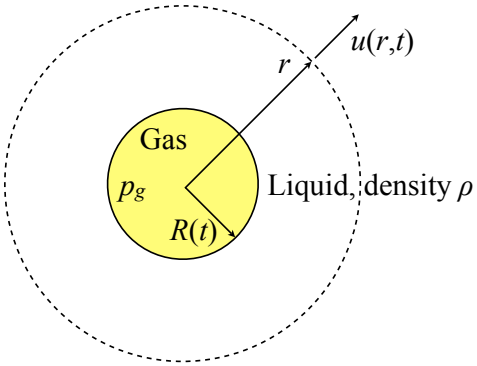
MIT Department of Mechanical Engineering
2.25 Advanced Fluid Mechanics

Problem 4.27
This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



Consider a bubble of high-pressure gas exploding in an incompressible liquid in a spherically-symmetrical fashion. The gas is not soluble in the liquid, and the liquid does not evaporate into the gas. At any instant R is the radius of the bubble, dR/dt is the velocity of the interface, p_g is the gas pressure (assumed uniform in the bubble), u is the liquid velocity at the radius r, and p_∞ is the liquid pressure at a great distance from the bubble. Gravity is to be neglected. The following questions pertain to the formulation of an analysis which will lead to the details of the pressure and velocity distributions and to the rate of bubble growth in the limit of inviscid liquid flow.

(a) Show that at any instant

$$u = \frac{R^2}{r^2}\frac{dR}{dt} \tag{4.27a}$$

(b) Show that the rate of growth of the bubble is described by the equation

$$R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + \frac{2\sigma}{\rho R} = \frac{p_g - p_\infty}{\rho} \tag{4.27b}$$

where σ is the surface tension at the gas-liquid interface.

(c) What additional information or assumptions would be necessary in order to establish the bubble radius R as a funciton of time? Explain how you would use this information.

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