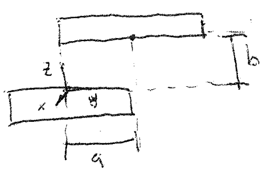
3B.1 Eccentric Disk Rheometer

a) This is a shear flow because

i) there is a one parameter family of material surfaces at constant z that move isometrically

ii) Separation of neighboring surfaces is constant

b)



In a rigid rotation

$$\underline{v} = \underline{w} \times (\underline{r} - \underline{r_c})$$

For the eccentric disk rheometer

$$\underline{r} = x\,\underline{e_x} + y\,\underline{e_y} + z\,\underline{e_z}$$

$$\underline{r_c} = 0\,\underline{e_x} + f(z)\,\underline{e_y} + z\,\underline{e_z}$$

$f(z)$ is line from $(0,0,0)$ to $(0,a,b)$

$$f(z) = \frac{a}{b} z = Az$$

$$\underline{r} - \underline{r_c} = x\,\underline{e_x} + y\,\underline{e_y} + z\,\underline{e_z} - Az\,\underline{e_y} - z\,\underline{e_z}$$

$$= x\,\underline{e_x} + (y - Az)\,\underline{e_y}$$

$$\underline{w} = w\,\underline{e_z}$$

$$\underline{v} = \underline{w} \times (\underline{r} - \underline{r_c}) = w\,\underline{e_z} \times (x\,\underline{e_x} + (y - Az)\,\underline{e_y})$$

$$= wx\,\underline{e_y} + w(y - Az)(-\underline{e_x})$$

$$\therefore \boxed{\begin{aligned} v_x &= -w(y - Az) \\ v_y &= wx \\ v_z &= 0 \end{aligned}}$$

3B.3 STRESS TENSOR FOR UNIDIRECTIONAL SHEAR FLOW

It is postulated that,

$$\hat{v}_1 = \hat{\dot{\gamma}}_{21}\,\hat{x}_2$$
$$\hat{v}_2 = \hat{v}_3 = 0$$

a. Make the change in coordinate system $\hat{x} \to \bar{x}$

$$\bar{x}_1 = -\hat{x}_1$$
$$\bar{x}_2 = -\hat{x}_2$$
$$\bar{x}_3 = \hat{x}_3$$

This corresponds to a 180° rotation about the x_3 axis. Simple shear flow is symmetric w.r.t such a rotation, because

$$\bar{v}_1 = \frac{d\bar{x}_1}{dt} = -\frac{d\hat{x}_1}{dt} = -\hat{\dot{\gamma}}_{21}\,\hat{x}_2 = \dot{\gamma}_{21}\,\bar{x}_1$$

$$\bar{v}_2 = \bar{v}_3 = 0$$

$$\therefore\ v_i(\hat{x}_1, \hat{x}_2, \hat{x}_3, t) = v_i(\bar{x}_1, \bar{x}_2, \bar{x}_3, t)$$

b. For an isotropic fluid the stress field preserves this symmetry.

$\therefore\ \bar{\tau}_{ij} = \hat{\tau}_{ij}$ —(1) Due to symmetry w.r.t. to the 3-axis $\left.\begin{matrix}\hat{\tau}_{13} = \hat{\tau}_{31} \\ \hat{\tau}_{23} = \hat{\tau}_{32}\end{matrix}\right\}$ —(2)

$$[\delta_3 \cdot \hat{\tau}] = \hat{\delta}_1\,\hat{\tau}_{31} + \hat{\delta}_2\,\hat{\tau}_{32} + \hat{\delta}_3\,\hat{\tau}_{33} \qquad \text{—(3)}$$

$$[\delta_3 \cdot \tau] = \bar{\delta}_1\,\bar{\tau}_{31} + \bar{\delta}_2\,\bar{\tau}_{32} + \bar{\delta}_3\,\tau_{33} = -\hat{\delta}_1\,\bar{\tau}_{31} - \hat{\delta}_2\,\bar{\tau}_{32} + \hat{\delta}_3\,\tau_{33} \qquad \text{—(4)}$$

Equating the components of the force on the plane normal to δ_3,

$$\left.\begin{matrix}\bar{\tau}_{31} = \hat{\tau}_{31} = -\hat{\tau}_{31} \\ \bar{\tau}_{32} = \hat{\tau}_{32} = -\hat{\tau}_{32}\end{matrix}\right. \qquad \text{(Making use of (1), (3) and (4))} \qquad \text{—(5)}$$

$\therefore$ From (2) and (5),

$$\underline{\hat{\tau}_{31} \quad \hat{\tau}_{13} \quad \hat{\tau}_{32} \quad \hat{\tau}_{23} = 0}$$

3B.5 Steady Radial Creeping Flow Between Two Circular Disks

No. Consider the definition given on p. 155. This flow certainly violates part i) since there are no shearing surfaces which move isometrically. Two neighboring particles in a plane parallel to the disks will be separated as they move towards the edge of the disks. Two neighboring points in a radial plane will move at different velocities towards the edge since v_r is a function of height z.

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