## Basic Practical diesel cycle

The textbook Diesel cycle is represented by all heat addition at constant pressure. The Otto cycle which is implemented by the spark ignition internal combustion engine adds all heat at constant volume. We will model a combined or dual (Seiliger) cycle with a portion of the heat added at constant volume, the remainder at constant pressure. Setting some parameters to be defined $=1$ will reduce to either the Otto or Diesel cycle.
define some units

$$
\begin{aligned}
& \mathrm{kN}:=10^{3} \cdot \mathrm{~N} \quad \mathrm{kPa}:=10^{3} \cdot \mathrm{~Pa} \\
& \mathrm{MPa}:=10^{6} \mathrm{~Pa} \quad \mathrm{~kJ}:=10^{3} \cdot \mathrm{~J} \\
& \mathrm{kmol}:=10^{3} \mathrm{~mol}_{\mathrm{Mw}}
\end{aligned}
$$

This model will use an ideal air standard cycle with air as an ideal gas with constant specific heats and reversible processes to represent the behavior. The gas relationships are useful.
air-standard cycles ...

1. air as ideal gas is working fluid throughout cycle - no inlet or exhaust process
2. combustion process replaced by heat transfer process
3. cycle is completed by heat transfer to surroundings
4. all processes internally reversible
5. usually constant specific heat
(page 311)

## basic practical diesel cycle

Assumptions for analysis ...

1. reversible cycle with all reversible processes
2. working fluid is air assumed to be a perfect gas with constant specific heats, $\gamma=c_{p} / c_{v}=1.4$
3. mass of air in cylinder remains constant
4. combustion processes are represented by heat transfer from an external source. Constant volume or constant pressure pocesses are done.
5. cycle is completed by cooling heat transfer to the surroundings until the air temperature and pressure return to the initial conditions of the cycle (constant volume process).

1-2 isentropic compression
2-3 constant volume heat addition
3-4 constant pressure heat addition
4-5 isentropic expansion
5-1 constant volume cooling
$\square$ data for plot
this is the shape ...


volume ( $\mathrm{m} \wedge 3 / \mathrm{kg}$ )
next we will put numbers on the plots $=>$ themodynamic analysis of dual (Seiliger) cycle

The original notes are sourced from VanWylen and Sonntag. They could be revised to use the form of some of the relationships from Woud, but at considerable effort. Rather what follows is the application of the equations developed in the gas relationships lecture applied to the combined air-standard cycle deriving the relationships summarized in Table 7.3 Analytical prediction of the Selinger process on page 245 of the text.

$$
\mathrm{r}_{\mathrm{c}}, \mathrm{a}, \mathrm{~b} \quad \mathrm{r}_{\mathrm{c}}=\frac{\mathrm{v}_{1}}{\mathrm{v}_{2}} \quad \mathrm{a}=\frac{\mathrm{p}_{3}}{\mathrm{p}_{2}} \quad \mathrm{~b}=\frac{\mathrm{v}_{4}}{\mathrm{v}_{3}}
$$

## stage 1-2

## isentropic adiabatic compression (expansion)

volume ratio known

$$
\begin{array}{ll}
r_{c}=\frac{v_{\text {initial }}}{v_{\text {final }}}=\frac{v_{1}}{v_{2}} \quad p_{\text {final }}=p_{\text {initial }}\left(\frac{v_{\text {initial }}}{v_{\text {final }}}\right)^{\gamma}=\left(\frac{v_{1}}{v_{2}}\right)^{\gamma}=r_{c}^{\gamma} & T_{\text {final }}=T_{\text {initial }} \cdot\left(\frac{v_{\text {initial }}}{v_{\text {final }}}\right)^{\gamma-1}=\left(\frac{v_{1}}{v_{2}}\right)^{\gamma-1}=r_{c}^{\gamma-1} \\
\frac{v_{1}}{v_{2}}=r_{c} & \frac{\mathrm{p}_{\text {final }}}{\mathrm{p}_{\text {initial }}}=r_{c}^{\gamma}
\end{array}
$$

## stage 2-3

heat transfer at constant volume ....

$$
\begin{array}{ccc}
\frac{\mathrm{p}_{3}}{\mathrm{p}_{2}}=\mathrm{a} & \mathrm{p}_{\text {initial }}=\frac{\mathrm{R} \cdot \mathrm{~T}_{\text {initial }}}{\mathrm{v}_{\text {constant }} \cdot 100} & \mathrm{p}_{\text {final }}=\frac{\mathrm{R} \cdot \mathrm{~T}_{\text {final }}}{\mathrm{v}_{\text {constant }} \cdot 100} \quad \frac{\mathrm{~T}_{\text {final }}}{\mathrm{T}_{\text {initial }}}=\frac{\mathrm{p}_{\text {final }}}{\mathrm{p}_{\text {initial }}}=\frac{\mathrm{p}_{3}}{\mathrm{p}_{2}}=\mathrm{a} \\
\frac{\mathrm{v}_{1}}{\mathrm{v}_{2}}=1 & \frac{\mathrm{p}_{3}}{\mathrm{p}_{2}}=\mathrm{a} & \frac{\mathrm{~T}_{\text {final }}}{\mathrm{T}_{\text {initial }}}=\mathrm{a}
\end{array}
$$

## stage 3-4

heat transfer at constant pressure ....

$$
\begin{array}{lc}
\mathrm{b}=\frac{\mathrm{v}_{\text {final }}}{\mathrm{v}_{\text {initial }}}=\frac{\mathrm{v}_{4}}{\mathrm{v}_{3}} & \mathrm{v}_{\text {initial }}=\frac{\mathrm{R} \cdot \mathrm{~T}_{\text {initial }}}{\mathrm{p}_{\text {constant }} \cdot 100} \\
\frac{\mathrm{v}_{4}}{\mathrm{v}_{3}}=\mathrm{b} & \frac{\mathrm{v}_{\text {final }}}{\mathrm{p}_{3}}=1
\end{array}
$$

## stage 4-5

isentropic adiabatic compression (expansion) volume ratio known

$$
\begin{aligned}
& \frac{v_{5}}{v_{4}}=\frac{v_{5}}{v_{3}} \cdot \frac{v_{3}}{v_{4}}=\frac{v_{5}}{v_{3}} \cdot \frac{v_{3}}{v_{4}}=\frac{v_{1}}{v_{2}} \cdot \frac{v_{3}}{v_{4}}=\frac{r_{c}}{b} \quad \text { as } v_{5}=v_{1} \text { and } v_{2}=v_{3} \\
& p_{\text {final }}=p_{\text {initial }} \cdot\left(\frac{v_{\text {initial }}}{v_{\text {final }}}\right)^{\gamma}=\left[p_{4} \cdot\left(\frac{v_{4}}{v_{5}}\right)\right]^{\gamma}=p_{4} \cdot\left(\frac{1}{\frac{r_{c}}{b}}\right)^{\gamma}=p_{5}
\end{aligned}
$$

$$
\text { [W } 7.68 \& 7.69]
$$

$$
\begin{array}{ll}
T_{\text {final }}=T_{\text {initial }}\left(\frac{v_{\text {initial }}}{v_{\text {final }}}\right)^{\gamma-1}=T_{4} \cdot\left(\frac{v_{4}}{v_{5}}\right)^{\gamma-1}=T_{4} \cdot\left(\frac{1}{\frac{r_{c}}{b}}\right)^{\gamma-1}=T_{5} \\
\frac{v_{5}}{v_{4}}=\frac{r_{c}}{b} & \frac{\mathrm{p}_{4}}{\mathrm{p}_{5}}=\left(\frac{r_{c}}{b}\right)^{\gamma}
\end{array} \quad \frac{T_{4}}{T_{5}}=\left(\frac{r_{c}}{b}\right)^{\gamma-1} \quad \begin{aligned}
& \text { N.B. ratios are } \\
& \text { inconsistent } 5 / 4 \ldots 4 / 5
\end{aligned}
$$

## stage 5-1

heat transfer at constant volume ....

$$
\begin{aligned}
& \frac{v_{5}}{v_{1}}=1 \quad P_{\text {initial }}=\frac{R \cdot T_{\text {initial }}}{v_{\text {constant }} \cdot 100} \quad p_{\text {final }}=\frac{R \cdot T_{\text {final }}}{v_{\text {constant }} \cdot 100} \quad \text { so } \ldots \quad \frac{p_{\text {initial }}}{P_{\text {final }}}=\frac{T_{\text {initial }}}{T_{\text {final }}}=\frac{p_{5}}{p_{1}}=\frac{T_{5}}{T_{1}} \\
& \frac{p_{5}}{p_{1}}=\frac{p_{5}}{p_{4}} \cdot\left(\frac{p_{4}=p_{3}}{p_{1}}\right)=\frac{p_{5}}{p_{4}} \cdot \frac{p_{3}}{p_{2}} \cdot \frac{p_{2}}{p_{1}}=\frac{1}{\left(\frac{r_{c}}{b}\right)^{\gamma}} \cdot a \cdot r_{c}^{\gamma}=a \cdot b^{\gamma}
\end{aligned}
$$

$$
\frac{\mathrm{v}_{5}}{\mathrm{v}_{1}}=1 \quad \frac{\mathrm{p}_{5}}{\mathrm{p}_{1}}=\mathrm{a} \cdot \mathrm{~b}^{\gamma} \quad \frac{\mathrm{T}_{5}}{\mathrm{~T}_{1}}=\mathrm{a} \cdot \mathrm{~b}^{\gamma}
$$

this one is initial/final

## Now, applying the gas relationships to the calculation of states around the air-standard combined cycle

constants $\ldots \quad \gamma:=1.4 \quad \mathrm{c}_{\mathrm{v}}:=0.7165 \frac{\mathrm{~kJ}}{\mathrm{~kg} \cdot \mathrm{~K}} \quad \mathrm{c}_{\mathrm{p}}:=1.0035 \frac{\mathrm{~kJ}}{\mathrm{~kg} \cdot \mathrm{~K}} \quad \underset{\mathrm{kN}}{\mathrm{R}}:=0.287 \frac{\mathrm{~kJ}}{\mathrm{~kg} \cdot \mathrm{~K}}$
given ... $\mathrm{T}_{1}, \mathrm{v}_{1}$ (calc), $\mathrm{s}_{1}, \mathrm{p}_{1}, \mathrm{r}_{\mathrm{V}}, \mathrm{r}_{\mathrm{p}}, \mathrm{r}_{\mathrm{C}}$
$\mathrm{T}_{1}:=295 \mathrm{~K} \quad \mathrm{~s}_{1}:=1 \frac{\mathrm{~kJ}}{\mathrm{~kg}} \quad \mathrm{p}_{1}:=1$ bar $\quad \mathrm{r}_{\mathrm{v}}:=12.5 \quad \mathrm{r}_{\mathrm{p}}:=1.38 \quad \mathrm{r}_{\mathrm{c}}:=1.86 \mathrm{v}_{1}:=\frac{\mathrm{R} \cdot \mathrm{T}_{1}}{\mathrm{p}_{1}} \quad \mathrm{v}_{1}=0.847 \frac{\mathrm{~m}^{3}}{\mathrm{~kg}}$
$r_{v}=$ compression ratio $r_{c}$ in text
$r_{p}=$ pressure ratio during constant volume heat addition $=a$ in text $r_{c}=$ cut-off ratio. portion of stroke during which constant pressure heat addition occurs $=\mathrm{b}$ in text

$$
r_{v}=\frac{v_{1}}{v_{2}}=r_{c} \quad r_{p}=\frac{p_{3}}{p_{2}}=a \quad r_{c}=\frac{v_{4}}{v_{3}}=b
$$

## 1-2 isentropic compression of air

$\frac{T_{2}}{T_{1}}=\left(\frac{p_{2}}{\mathrm{P}_{1}}\right)^{\frac{\gamma-1}{\gamma}}=\left(\frac{\mathrm{v}_{1}}{\mathrm{v}_{2}}\right)^{\gamma-1}$
(7.35)

$$
\mathrm{s}_{2}:=\mathrm{s}_{1} \quad \mathrm{v}_{2}:=\frac{\mathrm{v}_{1}}{\mathrm{r}_{\mathrm{v}}}
$$

$$
\mathrm{T}_{2}:=\mathrm{T}_{1} \cdot\left(\frac{\mathrm{v}_{1}}{\mathrm{v}_{2}}\right)^{\gamma-1} \quad \mathrm{p}_{2}:=\frac{\mathrm{R} \cdot \mathrm{~T}_{2}}{\mathrm{v}_{2}}
$$

$$
\mathrm{v}_{2}=0.068 \frac{\mathrm{~m}^{3}}{\mathrm{~kg}} \quad \mathrm{~T}_{2}=810.188 \mathrm{~K} \quad \mathrm{p}_{2}=34.33 \text { bar }
$$

later we will plot on T-s and p-v so the relationships for intermediate states is shown. Any state value can serve as the plot parameter, but we will use temperature.
$\mathrm{T}_{1} \leq \mathrm{T}_{-}$plot $\leq \mathrm{T}_{2} \quad \mathrm{~s}=\mathrm{s}_{1}=\mathrm{s}_{2}=$ constant
$\frac{\mathrm{T}_{2}}{\mathrm{~T}_{1}}=\left(\frac{\mathrm{v}_{1}}{\mathrm{v}_{2}}\right)^{\gamma-1}$ and $\ldots \quad \frac{\mathrm{T}_{2}}{\mathrm{~T}_{1}}=\left(\frac{\mathrm{p}_{2}}{\mathrm{p}_{1}}\right)^{\frac{\gamma-1}{\gamma}} \quad$ so $\ldots \quad$ v_plot $=\mathrm{v}_{1} \cdot\left(\frac{\mathrm{~T}_{1}}{\mathrm{~T}_{-} \text {plot }}\right)^{\frac{1}{\gamma-1}}$
and...$\quad$ p_plot $=p_{1} \cdot\left(\frac{T_{\_} \text {plot }}{T_{1}}\right)^{\frac{\gamma}{\gamma-1}}$

## 2-3 constant volume heat addition using $r_{p}$ during constant volume portion of heat addition ...

| $\mathrm{v}_{3}:=\mathrm{v}_{2}$ | $\mathrm{P}_{3}:=\mathrm{p}_{2} \cdot \mathrm{r}_{\mathrm{p}}$ | $\mathrm{P}_{3}=47.37$ | nee | ulate $\mathrm{T}_{3}$ | $\mathrm{v}_{2}=\mathrm{v}_{3}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{p} \cdot \mathrm{v}=\mathrm{R} \cdot \mathrm{T}$ | (3.2) | $\frac{\mathrm{p}_{1} \cdot \mathrm{v}_{1}}{\mathrm{~T}_{1}}=\frac{\mathrm{p}_{2} \cdot \mathrm{v}_{2}}{\mathrm{~T}_{2}}$ | (3.5) | $\frac{\mathrm{P}_{3}}{\mathrm{~T}_{3}}=\frac{\mathrm{P}_{2}}{\mathrm{~T}_{2}}$ | $\mathrm{T}_{3}:=\mathrm{T}_{2} \cdot \frac{\mathrm{P}_{3}}{\mathrm{P}_{2}}$ | $\mathrm{T}_{3}=1.118 \times 10^{3} \mathrm{~K}$ |

$\mathrm{s}_{2}-\mathrm{s}_{1}=\mathrm{c}_{\mathrm{vo}} \cdot \ln \left(\frac{\mathrm{T}_{2}}{\mathrm{~T}_{1}}\right)+\mathrm{R} \cdot \ln \left(\frac{\mathrm{v}_{2}}{\mathrm{v}_{1}}\right) \quad$ (7.21) $\quad \mathrm{c}_{\mathrm{vo}}=$ constant $\quad \mathrm{s}_{3}:=\left(\mathrm{s}_{2}+\mathrm{c}_{\mathrm{v}} \cdot \ln \left(\frac{\mathrm{T}_{3}}{\mathrm{~T}_{2}}\right) \cdot \mathrm{K}\right) \quad \mathrm{s}_{3}=1.231 \frac{\mathrm{~kJ}}{\mathrm{~kg}}$
for later plotting
$\mathrm{T}_{2} \leq \mathrm{T}_{-}$plot $\leq \mathrm{T}_{3} \quad$ s_plot $=\mathrm{s}_{2}+\mathrm{c}_{\mathrm{v}} \cdot \ln \left(\frac{\mathrm{T}_{-} \text {plot }}{\mathrm{T}_{2}}\right) \cdot \mathrm{K} \quad \begin{aligned} & \mathrm{p}-\mathrm{v} \text { are end points } \mathrm{v}=\text { constant } \\ & \text { (straight lines) although intermediate } \\ & \text { states would be determined from the } \\ & \text { state equation } \ldots\end{aligned} \quad$ p_plot $=\frac{\text { R•T_plot }}{\mathrm{v}_{2}}$

## 3-4 heat added at constant pressure with $\mathrm{r}_{\underline{\mathrm{c}}}$

$$
\begin{array}{ll}
\mathrm{v}_{4}:=\mathrm{v}_{3} \cdot \mathrm{r}_{\mathrm{c}} & \mathrm{p}_{4}:=\mathrm{p}_{3} \\
\mathrm{v}_{4}=0.126 \frac{\mathrm{~m}^{3}}{\mathrm{~kg}} & \mathrm{p}_{4}=47.375 \mathrm{bar} \\
\mathrm{~s}_{2}-\mathrm{s}_{1}=\mathrm{c}_{\mathrm{po}} \cdot \ln \left(\frac{\mathrm{~T}_{4} \cdot \mathrm{v}_{4}}{\mathrm{R}} \mathrm{~T}_{1}\right)-\mathrm{R} \cdot \ln \left(\frac{\mathrm{p}_{2}}{\mathrm{p}_{1}}\right) & \mathrm{T}_{4}=2.08 \times 10^{3} \mathrm{~K} \\
\text { (7.23) } & \mathrm{c}_{\mathrm{po}}=\mathrm{constant} \quad \mathrm{~s}_{4}:=\left(\mathrm{s}_{3}+\mathrm{c}_{\mathrm{p}} \cdot \ln \left(\frac{\mathrm{~T}_{4}}{\mathrm{~T}_{3}}\right) \cdot \mathrm{K}\right) \quad \mathrm{s}_{4}=1.854 \frac{\mathrm{~kJ}}{\mathrm{~kg}}
\end{array}
$$

for later plotting

$$
\mathrm{T}_{3} \leq \mathrm{T}_{-} \text {plot } \leq \mathrm{T}_{4}
$$

$$
\text { s_plot }=\mathrm{s}_{3}+\mathrm{c}_{\mathrm{p}} \cdot \ln \left(\frac{\mathrm{~T} \_\mathrm{plot}}{\mathrm{~T}_{2}}\right) \cdot \mathrm{K}
$$

$\mathrm{p}-\mathrm{v}$ are end points $\mathrm{v}=$ constant (straight lines) although intermediate states would be determined from the state equation ...
$\underline{4-5 \text { isentropic expansion }} \quad \mathrm{v}_{5}:=\mathrm{v}_{1} \quad \mathrm{~s}_{5}:=\mathrm{s}_{4} \quad \mathrm{~s}_{5}=1.854 \frac{\mathrm{~kJ}}{\mathrm{~kg}}$
$\frac{\mathrm{T}_{4}}{\mathrm{~T}_{5}}=\left(\frac{\mathrm{v}_{5}}{\mathrm{v}_{4}}\right)^{\gamma-1}$
$\mathrm{T}_{5}:=\mathrm{T}_{4} \cdot\left(\frac{\mathrm{v}_{4}}{\mathrm{v}_{5}}\right)^{\gamma-1} \quad \mathrm{~T}_{5}=970.553 \mathrm{~K}$

$$
\mathrm{p}_{5}:=\frac{\mathrm{R} \cdot \mathrm{~T}_{5}}{\mathrm{v}_{5}} \quad \mathrm{p}_{5}=3.29 \mathrm{bar}
$$

as with isentropic compression above using T as the plot parameter ...

$$
\mathrm{T}_{4} \geq \mathrm{T}_{-} \text {plot } \geq \mathrm{T}_{5} \quad \text { decreasing } \ldots \quad \mathrm{s}=\mathrm{s}_{4}=\mathrm{s}_{5}=\text { constant }
$$

$$
\frac{\mathrm{T}_{5}}{\mathrm{~T}_{4}}=\left(\frac{\mathrm{v}_{4}}{\mathrm{v}_{5}}\right)^{\gamma-1} \text { and } \ldots \quad \frac{\mathrm{T}_{5}}{\mathrm{~T}_{4}}=\left(\frac{\mathrm{p}_{5}}{\mathrm{p}_{4}}\right)^{\frac{\gamma-1}{\gamma}} \quad \text { so } \ldots \quad \mathrm{v}_{-} \text {plot }=\mathrm{v}_{4} \cdot\left(\frac{\mathrm{~T}_{4}}{\mathrm{~T}_{-} \text {plot }}\right)^{\frac{1}{\gamma-1}} \quad \text { and } \ldots \quad \text { p_plot }=\mathrm{p}_{4} \cdot\left(\frac{\mathrm{~T}_{-} \mathrm{plot}}{\mathrm{~T}_{4}}\right)^{\frac{\gamma}{\gamma-1}}
$$

## 5-1 constant volume cooling

and for later plotting ...

$$
\mathrm{s}_{\mathrm{nd}}^{\mathrm{d}}:=\left(\mathrm{s}_{5}+\mathrm{c}_{\mathrm{V}} \cdot \ln \left(\frac{\mathrm{~T}_{1}}{\mathrm{~T}_{5}}\right) \cdot \mathrm{K}\right) \begin{aligned}
& \text { check } \\
& \begin{array}{l}
\text { closure } \\
\text { of } \mathrm{s}_{1}
\end{array}
\end{aligned} \mathrm{~s}_{1}=1 \frac{\mathrm{~kJ}}{\mathrm{~kg}}
$$

$$
\begin{array}{cll}
\mathrm{T}_{5} \geq \mathrm{T} \text { _plot } \geq \mathrm{T}_{1} & \text { s_plot }=\mathrm{s}_{5}+\mathrm{c}_{\mathrm{v}} \cdot \ln \left(\frac{\mathrm{~T}_{-} \mathrm{plot}}{\mathrm{~T}_{5}}\right) \cdot \mathrm{K} & \begin{array}{l}
\text { p-v are end points } \mathrm{v}=\text { constant } \\
\text { (straight lines) although } \\
\text { intermediate states would be }
\end{array} \\
\text { decreasing } \ldots & & \text { determined from the state }
\end{array} \quad \begin{aligned}
& \text { p_plot }=\frac{\mathrm{R} \cdot \mathrm{~T}_{-} \text {plot }}{\mathrm{v}_{2}}
\end{aligned}
$$

equation ...

$$
\left(\begin{array}{c}
\mathrm{T}_{1} \\
\mathrm{~T}_{2} \\
\mathrm{~T}_{3} \\
\mathrm{~T}_{4} \\
\mathrm{~T}_{5} \\
\mathrm{~T}_{1}
\end{array}\right)=\left(\begin{array}{c}
295 \\
810 \\
1118 \\
2080 \\
971 \\
295
\end{array}\right) \mathrm{K} \quad\left(\begin{array}{c}
\mathrm{s}_{1} \\
\mathrm{~s}_{2} \\
\mathrm{~s}_{3} \\
\mathrm{~s}_{4} \\
\mathrm{~s}_{5} \\
\mathrm{~s}_{1}
\end{array}\right)=\left(\begin{array}{c}
1 \\
1 \\
1.231 \\
1.854 \\
1.854 \\
1
\end{array}\right) \frac{\mathrm{kJ}}{\mathrm{~kg}} \quad\left(\begin{array}{c}
\mathrm{p}_{1} \\
\mathrm{p}_{2} \\
\mathrm{p}_{3} \\
\mathrm{p}_{4} \\
\mathrm{p}_{5} \\
\mathrm{p}_{1}
\end{array}\right)=\left(\begin{array}{c}
1 \\
34.33 \\
47.375 \\
47.375 \\
3.29 \\
1
\end{array}\right) \mathrm{bar} \quad\left(\begin{array}{c}
\mathrm{v}_{1} \\
\mathrm{v}_{2} \\
\mathrm{v}_{3} \\
\mathrm{v}_{4} \\
\mathrm{v}_{5} \\
\mathrm{v}_{1}
\end{array}\right)=\left(\begin{array}{c}
0.847 \\
0.068 \\
0.068 \\
0.126 \\
0.847 \\
0.847
\end{array}\right)
$$

now for plotting, including the intermediate values ... details in area below, relationships developed above

## 1- parameterization of T-s, $\mathrm{p}-\mathrm{v}$




## now for calculations of efficiency

$r_{v}=$ compression ratio $r_{v}=\frac{v_{1}}{v_{2}}$
$\underset{p}{r}=$ pressure ratio during constant volume heat addition $r_{p}=\frac{p_{3}}{p_{2}}=\frac{T_{3}}{T_{2}}$ at constant volume (ideal gas law pv=RT)
${\underset{c}{ }}_{r}^{c}=$ cut-off ratio. portion of stroke during which constant pressure heat addition occurs $r_{c}=\frac{v_{4}}{v_{3}}=\frac{T_{4}}{T_{3}}$ at constant pressure (ideal gas law pv=RT)

## 1-2 isentropic compression of air

$\frac{\mathrm{T}_{2}}{\mathrm{~T}_{1}}=\left(\frac{\mathrm{p}_{2}}{\mathrm{p}_{1}}\right)^{\frac{\gamma-1}{\gamma}}=\left(\frac{\mathrm{v}_{1}}{\mathrm{v}_{2}}\right)^{\gamma-1}$
(7.35) $\begin{aligned} & \text { this .. for reversible } \\ & \text { adiabatic process }\end{aligned} \quad \frac{\mathrm{T}_{2}}{\mathrm{~T}_{1}}=\left(\frac{\mathrm{v}_{1}}{\mathrm{v}_{2}}\right)^{\gamma-1}=\mathrm{r}_{\mathrm{v}}{ }^{\gamma-1} \quad \mathrm{p}_{2}=\mathrm{p}_{1} \cdot\left(\frac{\mathrm{v}_{1}}{\mathrm{v}_{2}}\right)^{\gamma}=\mathrm{p}_{1} \cdot \mathrm{r}_{\mathrm{v}}{ }^{\gamma}$

2-3 constant volume heat addition using $\mathrm{r}_{\mathrm{p}}$ during constant volume portion of heat addition ... $\mathrm{Q}_{\mathrm{H} 1}=\mathrm{m} \cdot \mathrm{c}_{\mathrm{v}} \cdot\left(\mathrm{T}_{3}-\mathrm{T}_{2}\right)=\mathrm{m} \cdot \mathrm{c}_{\mathrm{v}} \cdot \mathrm{T}_{2} \cdot\left(\frac{\mathrm{~T}_{3}}{\mathrm{~T}_{2}}-1\right)=\mathrm{m} \cdot \mathrm{c}_{\mathrm{v}} \cdot \mathrm{T}_{2} \cdot\left(\mathrm{r}_{\mathrm{p}}-1\right)$

## 3-4 heat added at constant pressure with $r_{\text {c- }}$

$Q_{H 2}=m \cdot c_{p} \cdot\left(T_{4}-T_{3}\right)=m \cdot c_{p} \cdot T_{3} \cdot\left(\frac{T_{4}}{T_{3}}-1\right)=m \cdot c_{p} \cdot T_{3} \cdot\left(r_{c}-1\right)=m \cdot c_{p} \cdot T_{2} \cdot \frac{T_{3}}{T_{2}} \cdot\left(r_{c}-1\right)=m \cdot \gamma \cdot c_{v} \cdot T_{2} \cdot r_{p} \cdot\left(r_{c}-1\right)$

## 5-1 constant volume cooling substituting $\gamma=\mathrm{c}_{\mathrm{p}} / \mathrm{c}_{\mathrm{v}}$ and $\mathrm{r}_{\mathrm{p}}=\mathrm{T}_{3} / \mathrm{T}_{2}$

$$
\begin{aligned}
& Q_{L}=-m \cdot c_{v} \cdot\left(T_{5}-T_{1}\right)=-m \cdot c_{v} \cdot T_{1} \cdot\left(\frac{T_{5}}{T_{1}}-1\right) \\
& \frac{T_{5}}{T_{1}}=\frac{p_{5}}{p_{1}}=\frac{p_{5}}{p_{4}} \cdot \frac{p_{4}}{P_{3}} \cdot \frac{p_{3}}{P_{2}} \cdot \frac{p_{2}}{P_{1}}=\left(\frac{v_{4}}{v_{5}}\right)^{\gamma} \cdot 1 \cdot r_{p} \cdot\left(\frac{v_{1}}{v_{2}}\right)^{\gamma}=\left(\frac{v_{4}}{v_{3}}\right)^{\gamma} \cdot r_{p}=r_{p} \cdot r_{c}^{\gamma} \quad \Rightarrow \quad Q_{L}=-m \cdot c_{v} \cdot T_{1} \cdot\left(r_{p} \cdot r_{c}^{\gamma}-1\right) \\
& \text { as } v_{5}=v_{1} \text { and } v_{2}=v_{3}
\end{aligned}
$$

combining these for thermal efficiency of the cycle ...

$$
\begin{array}{ll}
\eta_{\text {th }}=1+\frac{Q_{L}}{Q_{H}}=1-\frac{T_{1} \cdot\left(r_{p} \cdot r_{c}^{\gamma}-1\right)}{T_{2} \cdot\left[\left(r_{p}-1\right)+\gamma \cdot r_{p} \cdot\left(r_{c}-1\right)\right]}=1-\frac{r_{p} \cdot r_{c}^{\gamma}-1}{r_{v}^{\gamma-1} \cdot\left[\left(r_{p}-1\right)+\gamma \cdot r_{p} \cdot\left(r_{c}-1\right)\right]} & r_{v}=12.5 \\
\eta_{\text {th }}:=1-\frac{r_{p} \cdot r_{c}^{\gamma}-1}{r_{v}^{\gamma-1} \cdot\left[\left(r_{p}-1\right)+\gamma \cdot r_{p} \cdot\left(r_{c}-1\right)\right]} & r_{p}=1.38 \\
\eta_{\text {th }}=0.592 & r_{\mathrm{c}}=1.86
\end{array}
$$

and we could calculate the work per cycle

$$
\mathrm{W}=\mathrm{Q}_{\mathrm{H} 1}+\mathrm{Q}_{\mathrm{H} 2}+\mathrm{Q}_{\mathrm{L}}=\mathrm{m} \cdot \mathrm{c}_{\mathrm{v}} \cdot\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right)+\mathrm{m} \cdot \mathrm{c}_{\mathrm{p}} \cdot\left(\mathrm{~T}_{4}-\mathrm{T}_{3}\right)-\mathrm{m} \cdot \mathrm{c}_{\mathrm{v}} \cdot\left(\mathrm{~T}_{5}-\mathrm{T}_{1}\right)
$$

m : $=1$ for per unit mass calculation in mcd

$$
\begin{aligned}
\mathrm{Q}_{\mathrm{H} 1}:=\mathrm{m} \cdot \mathrm{c}_{\mathrm{v}} \cdot\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right) & \mathrm{Q}_{\mathrm{H} 1}=220.59 \frac{\mathrm{~kJ}}{\mathrm{~kg}} \\
\mathrm{Q}_{\mathrm{H} 2}:=\mathrm{m} \cdot \mathrm{c}_{\mathrm{p}} \cdot\left(\mathrm{~T}_{4}-\mathrm{T}_{3}\right) & \mathrm{Q}_{\mathrm{H} 2}=964.897 \frac{\mathrm{~kJ}}{\mathrm{~kg}} \\
\mathrm{Q}_{\mathrm{L}}:=-\mathrm{m} \cdot \mathrm{c}_{\mathrm{v}} \cdot\left(\mathrm{~T}_{5}-\mathrm{T}_{1}\right) & \mathrm{Q}_{\mathrm{L}}=-484.034 \frac{\mathrm{~kJ}}{\mathrm{~kg}}
\end{aligned}
$$

$$
\underset{\mathrm{w}}{\mathrm{~W}}:=\mathrm{Q}_{\mathrm{H} 1}+\mathrm{Q}_{\mathrm{H} 2}+\mathrm{Q}_{\mathrm{L}} \quad \mathrm{~W}=701.453 \frac{\mathrm{~kJ}}{\mathrm{~kg}}
$$

N.B. this specific power is more than double LM 2500
another parameter describing diesel engines
Indicated Mean Effective Pressure; Imep
Imep $=\frac{\text { work_per_stroke }}{\text { swept_volume }}=\frac{\text { work }}{\mathrm{V}_{1}-\mathrm{V}_{2}} \quad \mathrm{~m}=\frac{\mathrm{p} 1 \cdot \mathrm{~V}_{1}}{\mathrm{R} \cdot \mathrm{T}_{1}}$
$\mathrm{W}=701.453 \frac{\mathrm{~kW}}{\frac{\mathrm{~kg}}{\mathrm{~s}}}$
can also express as ...
some diesel examples ...
wartsila 32 (four stroke)
wartsila 64 (four stroke)

Imep $=\frac{W \cdot m}{V_{1}-V_{2}}=\frac{W \cdot \frac{p_{1} \cdot V_{1}}{R \cdot T_{1}}}{V_{1}-V_{2}}=\frac{W \cdot \frac{p_{1}}{R \cdot T_{1}}}{1-\frac{V_{2}}{V_{1}}}=\frac{W \cdot \frac{p_{1}}{R \cdot T_{1}}}{1-\frac{1}{r_{v}}}$

## Sulzer RT-flex96C, Sulzer RTA96C (two stroke)

in this example ... Imep $:=\frac{\mathrm{W} \cdot \frac{\mathrm{p}_{1}}{\mathrm{R} \cdot \mathrm{T}_{1}}}{1-\frac{1}{\mathrm{r}_{\mathrm{v}}}} \quad \quad$ Imep $=9.005$ bar
consider indicated power, ref: Woud 7.4.2-3 $\quad n_{e}=$ engine_rpm
power_per_cyl $=\frac{\text { work_per_cycle }}{\text { unit_time }} \quad$ time is period of power strokes $=1 /$ freq $=1 /\left(n_{e}=\right.$ engine_rpm $)$
2-stroke $n_{e}=$ \# power strokes/time

$$
\mathrm{P}_{\mathrm{i}}=\frac{\mathrm{W}_{\mathrm{i}}}{\text { period_power_stroke }}=\frac{\mathrm{W}_{\mathrm{i}}}{\frac{1}{\text { freq }}=\frac{1}{\mathrm{n}_{\mathrm{e}}=\text { engine_rpm }}}=\mathrm{W}_{\mathrm{i}} \cdot \mathrm{n}_{\mathrm{e}}
$$

4-stroke $\mathrm{n}_{\mathrm{e}}=2^{\star} \#$ power strokes/time

$$
\mathrm{P}_{\mathrm{i}}=\mathrm{W}_{\mathrm{i}} \cdot \frac{\mathrm{n}_{\mathrm{e}}}{2} \quad \mathrm{k}=\text { if }(\text { stroke }=2,1,2)
$$

with ... $\quad i=$ number_of_cylinders $\quad$ engine power is ... $\quad P_{i}=W_{i} \cdot \frac{n_{e} \cdot i}{k}=I m e p \cdot V_{S} \cdot \frac{n_{e} \cdot i}{k}$
brake power $\mathrm{P}_{\mathrm{B}}$, power at engine drive flange, after mechanical losses in engine see Woud (7.12) and 7.4.1
$\mathrm{W}_{\mathrm{e}}=\frac{\text { effective_work }}{\text { unit_mass }}=\frac{\mathrm{W}_{\mathrm{i}}}{\text { mass }} \cdot \eta_{\text {mechanical }}$
$P_{e}=W_{e} \cdot \frac{n_{e} \cdot i}{k}=$ mep $_{e} \cdot V_{S} \cdot \frac{n_{e} \cdot i}{k} \quad P_{e}=$ constant $\cdot$ mep $_{e} \cdot \mathrm{n}_{e} \quad$ for later discussion
mep $_{\mathrm{e}}=$ mean_effective_pressure $=$ brake_mean_effective_pressure $=$ BMEP
mep $_{e}=\frac{P_{e} \cdot k}{V_{S} \cdot n_{e} \cdot i}=\frac{P_{e}}{V_{S} \cdot \frac{n_{e}}{k} \cdot i}=\frac{\text { power_per_cyl }}{V_{S} \cdot \frac{n_{e}}{k}}$

## COLT-PIELSTICK PC4.2B DATA

Configuration Vee Only
Bore 570 mm
Stroke 660 mm
Engine Version 60 Hz Propulsion
Cylinder (nos) 10-12-16-18
Output Range (kW) 12,500-22,500 13,250-23,850
Speed (rpm) 400 400/430
Mean Eff. Pressure (bar) 22.3 22.3/22.0
Mean Piston Speed (m/s) 8.8 8.8/9.5
Output/cyl kW (hp) 1250 (1676) 1250 (1676)/1325 (1777)
from: page 16 of Fairbanks Morse medium speed diesel handbook
power_per_cyl $:=1250 \mathrm{~kW} \quad \mathrm{n}_{\mathrm{e}}:=\frac{400}{\min } \quad \mathrm{n} \_$stroke $:=4 \quad \mathrm{k}:=$ if (n_stroke $\left.=2,1,2\right) \quad$ bore $:=570 \mathrm{~mm} \quad$ stroke $:=660 \mathrm{~mm}$
$\mathrm{V}_{\mathrm{S}}:=\frac{\pi}{4} \cdot$ bore $^{2} \cdot$ stroke $\quad \mathrm{V}_{\mathrm{S}}=5.948 \mathrm{ft}^{3} \quad$ mep $_{\mathrm{e}}:=\frac{\text { power_per_cyl }}{\mathrm{V}_{\mathrm{S}} \cdot \frac{\mathrm{n}_{\mathrm{e}}}{\mathrm{k}}} \quad \mathrm{mep}_{\mathrm{e}}=22.266 \mathrm{bar} \quad \begin{aligned} & \text { as stated in data } \\ & \text { above }\end{aligned}$
two special cases (can be calculated above setting $r_{p}$ and $r_{c}$ appropriately ...

## Otto cycle - spark ignition engine heat added at TDC (constant volume) only

 extra subscript added for special designationair-standard Otto cycle: spark ignition internal-combustion engine
1-2 isentropic compression of air
$2-3(=4)$ heat added at constant volume (piston momentarily at rest at tdc
$4-5$ isentropic expansion
5-1 heat rejection at constant volume (piston at crank-end dead center)
from air standard dual cycle above
$\eta_{\text {th }}=1+\frac{Q_{L}}{Q_{H}}=1-\frac{T_{1} \cdot\left(\mathrm{r}_{\mathrm{p}} \cdot \mathrm{r}_{\mathrm{c}}{ }^{\gamma}-1\right)}{\mathrm{T}_{2} \cdot\left[\left(\mathrm{r}_{\mathrm{p}}-1\right)+\gamma \cdot \mathrm{r}_{\mathrm{p}} \cdot\left(\mathrm{r}_{\mathrm{c}}-1\right)\right]}=1-\frac{\mathrm{r}_{\mathrm{p}} \cdot \mathrm{r}_{\mathrm{c}}{ }^{\gamma}-1}{\mathrm{r}_{\mathrm{v}}{ }^{\gamma-1} \cdot\left[\left(\mathrm{r}_{\mathrm{p}}-1\right)+\gamma \cdot \mathrm{r}_{\mathrm{p}} \cdot\left(\mathrm{r}_{\mathrm{c}}-1\right)\right]}$

$$
\begin{aligned}
& \mathrm{r}_{\mathrm{CO}}:=1 \\
& \eta_{\text {th_otto }}:=1-\frac{\mathrm{r}_{\mathrm{po}} \cdot \mathrm{r}_{\mathrm{co}}{ }^{\gamma \mathrm{o}}-1}{\mathrm{r}_{\mathrm{vo}}{ }^{\gamma \mathrm{o}-1} \cdot\left[\left(\mathrm{r}_{\mathrm{po}}-1\right)+\gamma \mathrm{o} \cdot \mathrm{r}_{\mathrm{po}} \cdot\left(\mathrm{r}_{\mathrm{co}}-1\right)\right]}
\end{aligned}
$$

## and ... Diesel cycle ... all heat added at constant pressure

air-standard Diesel cycle: compression ignition internal-combustion engine
1-2(=3) isentropic compression of air
3-4 heat added at constant pressure (gas expanding during heat addition)
4-5 isentropic expansion
5-1 heat rejection at constant volume (piston at crank-end dead center)

$$
\begin{aligned}
& r_{\text {pd }}:=1 \quad \eta_{\text {th_diesel }}:=1-\frac{r_{p d^{\prime} \mathrm{r}_{\mathrm{cd}}}^{\gamma \mathrm{d}}-1}{\mathrm{r}_{\mathrm{vd}}{ }^{\gamma \mathrm{d}-1} \cdot\left[\left(\mathrm{r}_{\mathrm{pd}}-1\right)+\gamma \mathrm{d} \cdot \mathrm{r}_{\mathrm{pd}} \cdot\left(\mathrm{r}_{\mathrm{cd}}-1\right)\right]} \quad \eta_{\text {th_diesel }} \rightarrow 1-\frac{\mathrm{r}_{\mathrm{cd}}{ }^{\gamma \mathrm{d}}-1}{\mathrm{r}_{\mathrm{vd}}{ }^{\gamma \mathrm{d}-1} \cdot \gamma \mathrm{~d} \cdot\left(\mathrm{r}_{\mathrm{cd}}-1\right)}
\end{aligned}
$$

## Relate $r_{c}, a, b$ to efficiency

$$
r_{v}=\frac{v_{1}}{v_{2}}=r_{c} \quad r_{p}=\frac{p_{3}}{p_{2}}=a \quad r_{c}=\frac{v_{4}}{v_{3}}=b
$$

$\square$ reset variables

$$
\eta_{\text {th }}:=1-\frac{r_{p} \cdot r_{c}^{\gamma}-1}{r_{v}^{\gamma-1} \cdot\left[\left(r_{p}-1\right)+\gamma \cdot r_{p} \cdot\left(r_{c}-1\right)\right]}
$$

substitute, var1 = var2, where var1 is to be replaced
$\eta_{\text {th }}$ substitute, $r_{V}=r_{C C}, r_{p}=a, r_{C}=b, \gamma=\kappa \rightarrow 1-\frac{a \cdot b^{\kappa}-1}{r_{C C} \cdot 4 \cdot[a-1+a \cdot(b-1) \cdot \kappa]}$

