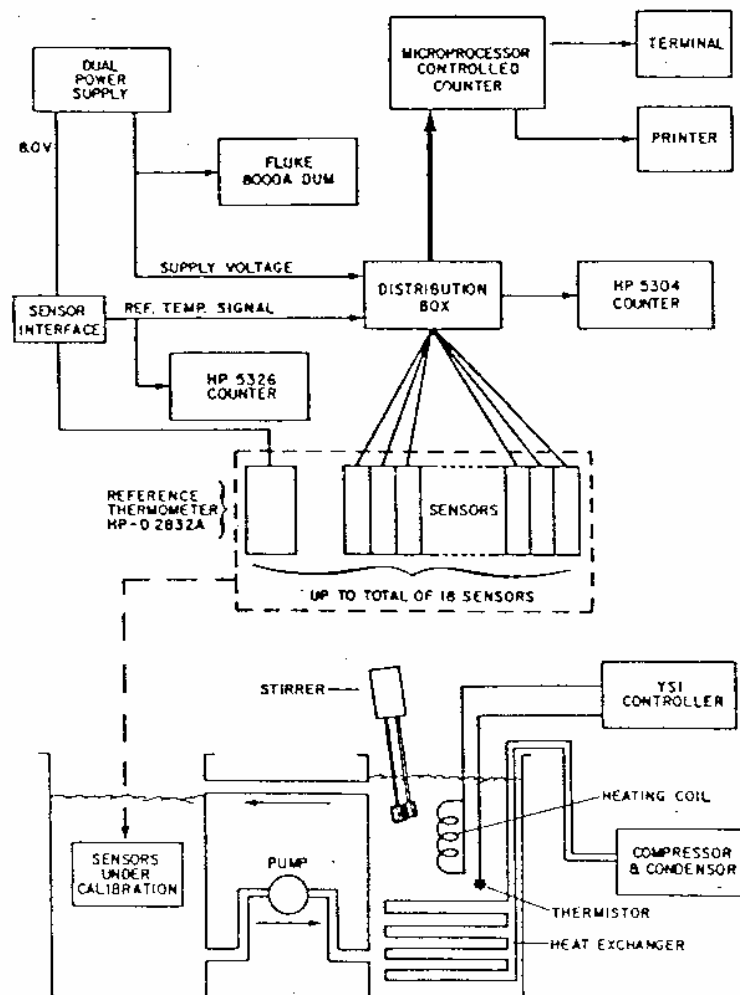


Calibrations: Static and Dynamic

Sensor response: As stated above, the sensor is the ocean scientist and engineer's interface with the real world. Any sensor acts as a filter on the data it processes. The observer must understand these filter effects in order to be able to remove them and truly estimate the statistics of the environment. We will discuss these effects in the following sections on calibrations, the frequency response of the sensors and how to correct the spectra and series for their effects, and finally the limit imposed on measurements by the sensor noise itself.

Static Calibrations: Everyone is aware that sensors must be calibrated against some standard in order that the results can be relied upon. However, some of the details as to how this is done are a bit vague. For example let us take the case of the temperature sensor on the CTD system used for vertically profiling the water column. It is calibrated in a temperature controlled water bath. The temperature of this bath is cooled by a water-cooling heat exchanger that constantly removed heat from the tank. A temperature controller measures the temperature of the bath with a thermistor, and puts heat into the water by an immersion heater to maintain the desired temperature. The water in the tank is will mixed by a stirring motor. This whole procedure is illustrated below for a calibration setup at Univ. New Hampshire.



The temperature sensor or sensors to be calibrated are placed in this well mixed tank, and allowed to come to thermal equilibrium. A reference thermometer is placed with the sensors under calibration, and the readings from the reference thermometer and sensors being calibrated are recorded.

The reference thermometer is regularly sent back to a national calibration facility that calibrates it with standards traceable to the National Institute of Standards and Technology. This reference thermometer is generally a platinum resistance thermometer, and with repeated calibrations accumulates a history on how it changes with time. In order to check its operation before each calibration, the reference thermometer is generally “standardized” in a triple-point-of-water cell to check one point in absolute temperature and a Gallium melt cell to check a second point. The triple point is defined in terms of the temperature where gas, water and ice phase all exist as $0.010,00\text{ }^{\circ}\text{C} \pm 0.000,01\text{ }^{\circ}\text{C}$. The Gallium melt cell (29.7646°C) provides an upper point for oceanographic range of temperatures. The PRT is used to interpolate between these two points based on its calibration. We will discuss the temperature standard, currently the IPT90, when we discuss water properties, and temperature measurement.

The actual temperature of the water is then calculated and plotted versus the output from each sensor. When a number of points at different temperatures have been taken, plotted, and appear consistent, the results are then fit to the functional form of the sensor. If the sensor were linear, then a least squares linear fit is done on the actual temperature and the sensor output, and the derived coefficients can then be used to normalize any data collected with the sensor. The least squares fitting removes some of the random statistical error associated with a single calibration point, and gives a smoothed, consistent summary of the calibration results. It is obvious that other functional forms can be used to fit the data. It is best to study the sensor's physical behavior, and choose a functional form that best represents the type of sensor being used, rather than just expand the calibration in a power series. As an example, the Vibrotron pressure sensor (used in early bottom pressure measurements) is best fit by the frequency output squared is a linear function of pressure

$$P = A + B f^2 .$$

and not f . The Sea Bird temperature sensors used on moorings in the Gulf of Maine and on Georges Bank (which we will discuss in more detail below) was fit by the following relationship

$$T[{}^{\circ}\text{C}] = (A + B \ln(F) + C \ln^2(F) + D \ln^3(F))^{-1} - 273.15$$

where $F = f_0/f$, f is the sensor frequency, f_0 , A , B , C , and D are the calibration constants. The natural logarithms in this case take into account the kind of sensitivity of the thermistor used in the sensor. (See Sea Bird Electronics temperature sensor calibration on following page.)

Calibration history: Calibration history is an important part of knowing data quality. Normally sensors are calibrated before and after each cruise. If a calibration record is maintained, and the sensors are not adjusted at calibration time, then the long-term drift of the sensors can be observed, and any sudden changes in behavior used to determine sensor problems before failure. Some of the Sea Bird temperature sensors have a 30-year calibration history that shows a smooth drift that decreases with time to less than $1\text{ m}^{\circ}\text{C}$ per year at present. This means that I can believe the temperatures that the sensor sees to better than $1\text{ m}^{\circ}\text{C}$ in absolute value. And the calibration history on the sensor gives me the information to make such a statement. This

information is required to justify observations made in the field, and is generally summarized in a data quality or quality assurance document that we will examine in more detail later.

SEA-BIRD ELECTRONICS, INC.

1808 136th Place N.E., Bellevue, Washington 98005 USA
 Phone: (206) 643 - 9866 Fax: (206) 643 - 9954 Internet: seabird@seabird.com

SENSOR SERIAL NUMBER = 1629
 CALIBRATION DATE: 27-Sep-96s

TEMPERATURE CALIBRATION DATA
 ITS-90 TEMPERATURE SCALE

ITS-90 COEFFICIENTS

g = 4.83406520e-03
 h = 6.77564998e-04
 i = 3.26730215e-05
 j = 3.25482938e-06
 f₀ = 1000.000

IPTS-68 COEFFICIENTS

a = 3.68160997e-03
 b = 5.90725810e-04
 c = 1.47974337e-05
 d = 3.25638963e-06
 f₀ = 6254.902

BATH TEMP (ITS-90 °C)	INSTRUMENT FREQ (Hz)	INST TEMP (ITS-90 °C)	RESIDUAL (ITS-90 °C)
-1.5292	6254.902	-1.5293	-0.00018
1.0319	6630.528	1.0320	0.00013
4.6052	7181.545	4.6054	0.00024
8.1109	7753.409	8.1110	0.00008
11.6142	8356.639	11.6141	-0.00014
15.1741	9002.974	15.1738	-0.00028
18.6371	9664.780	18.6370	-0.00015
22.1382	10367.650	22.1383	0.00007
25.6657	11110.829	25.6659	0.00026
29.1370	11877.103	29.1373	0.00026
32.6116	12679.260	32.6113	-0.00028

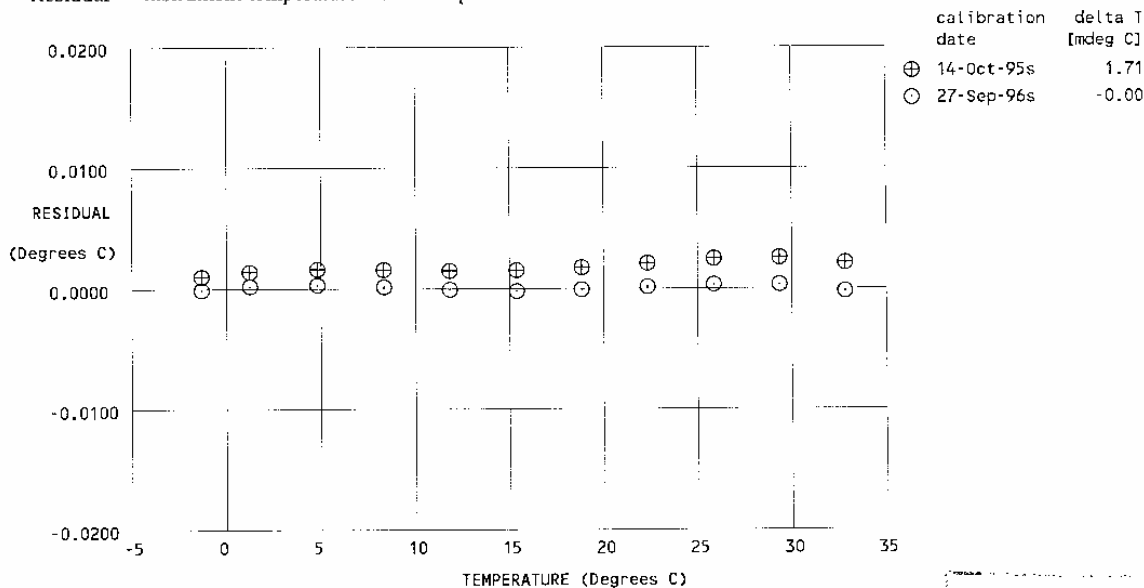
Handwritten note:
 a = 3.68160997e-03
 b = 5.90725810e-04

Temperature ITS-90 = 1/{g + h[ln(f₀/f)] + i[ln²(f₀/f)] + j[ln³(f₀/f)]} - 273.15 (°C)

Temperature IPTS-68 = 1/{a + b[ln(f₀/f)] + c[ln²(f₀/f)] + d[ln³(f₀/f)]} - 273.15 (°C)

Following the recommendation of JPOTS: T₆₈ is assumed to be 1.00024 * T₉₀ (-2 to 35 °C).

Residual = instrument temperature - bath temperature



Dynamic Calibrations or Frequency Response: A sensor is a filter that affects the frequency content of the data that passes through it. It is up to the experimentalist to make sure that these filtering effects are understood and do not effect the results that he is trying to obtain.

To measure the frequency response of a sensor, one could input an impulse, measure the response, transform it and obtain the frequency response function. An impulse is a good input since it contains all frequencies. To see this, consider our gate function again

$$\delta(t) \approx 1/\tau \Pi(t/\tau) \supset \text{Sinc}(f\tau) = \text{Sin}(\pi f\tau)/\pi f\tau$$

as $\tau \rightarrow 0$, the gate function goes to an impulse. The first zero crossing of the SINC function occurs at $f = 1/\tau$, where the Sin goes to zero. This is also observed to be the inverse of the length of the gate function, τ . So as τ goes to zero, the zero crossing moves to higher and higher frequency and the Sinc function broadens. See graph below for three different width gate functions and associated Sinc function. In the limit of a delta function, the Sinc function goes to constant function of frequency. The transform of the impulse, is a nonzero constant so contains equal energy at all frequencies!

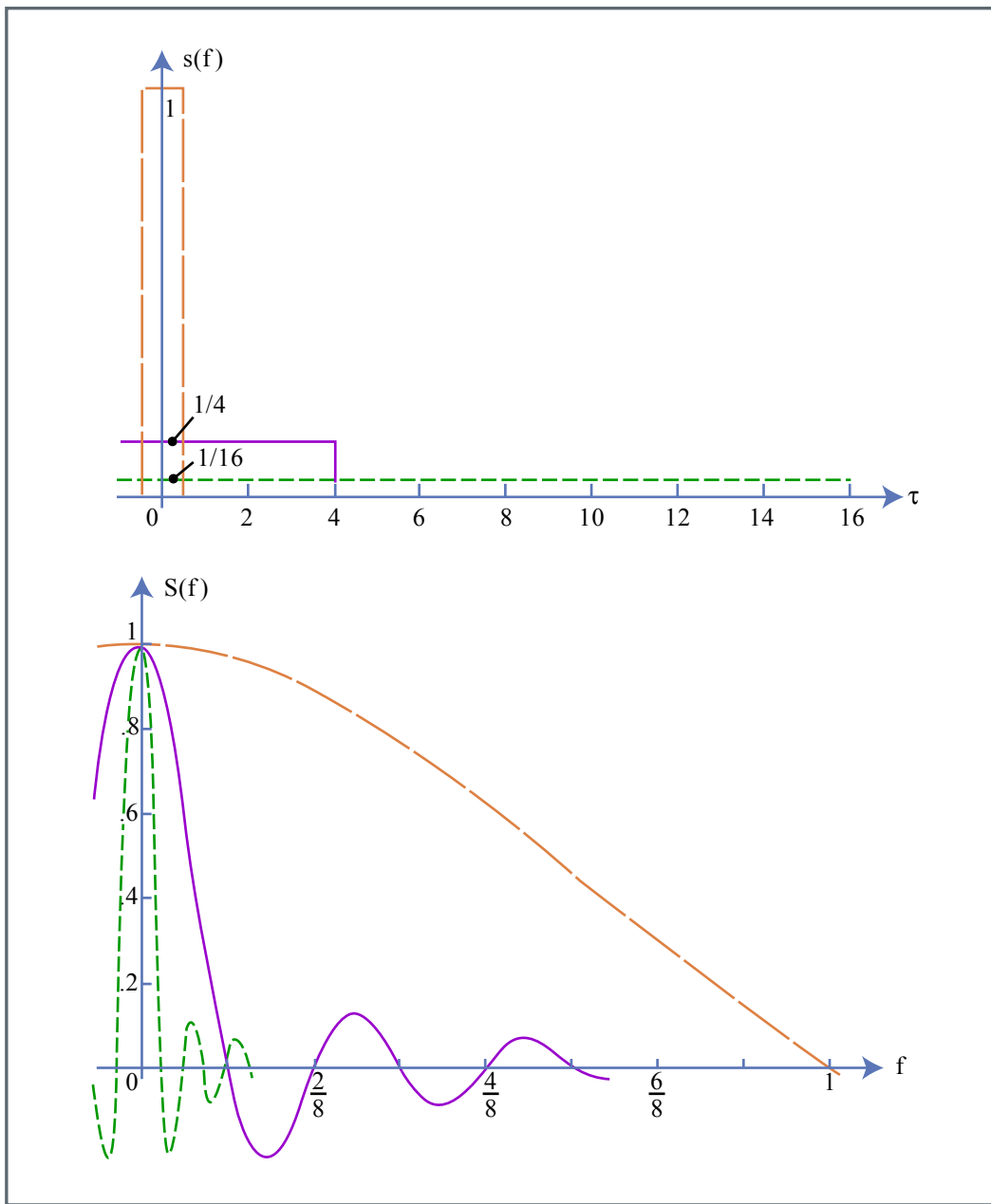


Figure by MIT OCW.

The gate function with varying widths (16, 4, 1) on top, and the associated transform (Sinc function) on the bottom, showing that the Sinc gets broader as the gate gets narrower.

If we are considering a temperature sensor, it is difficult to input an impulse, because it is an infinitely short, infinitely high pulse. However, to get around this problem, consider the case of the step function.

$$x(t) = 1 \text{ for } t > 0$$

$$x(t) = 0 \text{ for } t < 0.$$

This is easy to make, just plunge the sensor quickly from warm air into cold water or between a warm and cold-water bath in a time much faster than the response time of the sensor. In this case we have a scaled step function where we can write

$$T(t) = T_{\text{air}} \text{ for } t < 0$$

$$T(t) = T_{\text{water}} \text{ for } t > 0$$

Now the derivative of the step function is an impulse which has an amplitude, A , equal to the change in temperature. We assume that the time for the sensor to move through the interface is short compared with the response time of the sensor, so relative to the sensor's response time, the change in temperature does look like a true step. Thus, we have a known input or forcing function. If the sample interval, δt , is small compared with the response time of the sensor, then we can resolve the sensor's response which looks a bit more like a slow ramp, rather than a step. Hence, with a known input, and a measure of the output as a function to time, $y(t)$, we can calculate the response function. From the definition of the convolution product we have

$$Y(t) = T(t) * 1(t)$$

$$y(t) = T(t) * L(\omega)$$

and transforming into frequency space, the convolution theorem allows us to replace the convolution product in time space with a multiplication in frequency space. Therefore, we can divide the transforms to get the frequency response function,

$$L(\omega) = \frac{y(t)}{T(t)}$$

Note that we want the impulse narrow so that it contains energy at all frequencies. If our step were indeed a true impulse, then its transform would be just 1 and the response would be the transform of the output, $y(t)$. However, our step function, and hence our first difference, has an amplitude, A . Note that we are ignoring any phase shifts here as it relies on knowing very accurately the timing of the impulse or step.

We will consider three examples of this:

1. The moored Wein bridge oscillator temperature sensor serial number 201 which has had a plastic thermal mass added to the sensing element to "prefilter" the data to remove unwanted high frequency energy before digitizing. These sensors became the Sea Bird Electronics temperature sensors when the company was formed.
2. The Bessett Berman (Neil Brown designed) 9040 CTD system and UW digitizer used in the Mid-Ocean Acoustics Transmission Experiment off the coast of Washington state.
3. The platinum film temperature sensor used on the APL/UW AUV SPURV to measure turbulence fluctuations. (And also a bare thermistor flake in a 'towed' application.)

Moored Wein Bridge Oscillator Temperature Sensor: A plastic thermal mass added to the sensing element of the sensor to "prefilter the data and eliminate unwanted high frequency fluctuations. The time constant of this mass (the time for the temperature to diffuse through the material and the center reach 1/e of the final temperature) was supposed to be on the order of 2 minutes to act as a prefilter to prevent aliasing with a 2-minute sample interval. The mass was added to prevent aliasing, but it could also contaminate the results. Also, as part of an internal waves experiment, it was desired to measure the high frequency portion of the spectrum where any sensor frequency response effects would be important. Therefore, the response was estimated by the method discussed above. (See Levine, Irish, Ewart and Reynolds, 1986 and Levine and Irish, 1981)

The sensor was held in air above the well-stirred calibration tank with cold water, and at time zero plunged into the tank and the output recorded. This produced the step response shown in the top panel on the following page. The change in temperature was 15.5° C, and measured by allowing the sensor to reach equilibrium, making a temperature reading and subtracting it from the initial reading. The output was digitized to an equivalent 1 m°C with a δt of 5.489 seconds. The data was normalized to temperature in deg C, and the first difference taken to obtain the graph at the bottom of the following page, which is an estimation of the impulse response, where the impulse was input at $t = 0$. Note that the sensor response lagged the impulse considerably, and reaches a peak value 50 to 60 seconds after the impulse (or step).

This signal was then transformed, and scaled by the magnitude of the step (i.e. 15.5°C) to get the frequency response function also shown on the top of the following page. The sensor's spectral response is about 1 (or 0 db where $db = 20 \log A_{out}/A_{in}$) at low frequency as the sensor follows low frequency temperature changes well. The discrete points in the transform are shown by the stars, and the response of a simple exponential filter with a time constant of 90 seconds is shown as the solid line for comparison.

The single exponential filter has an impulse response,

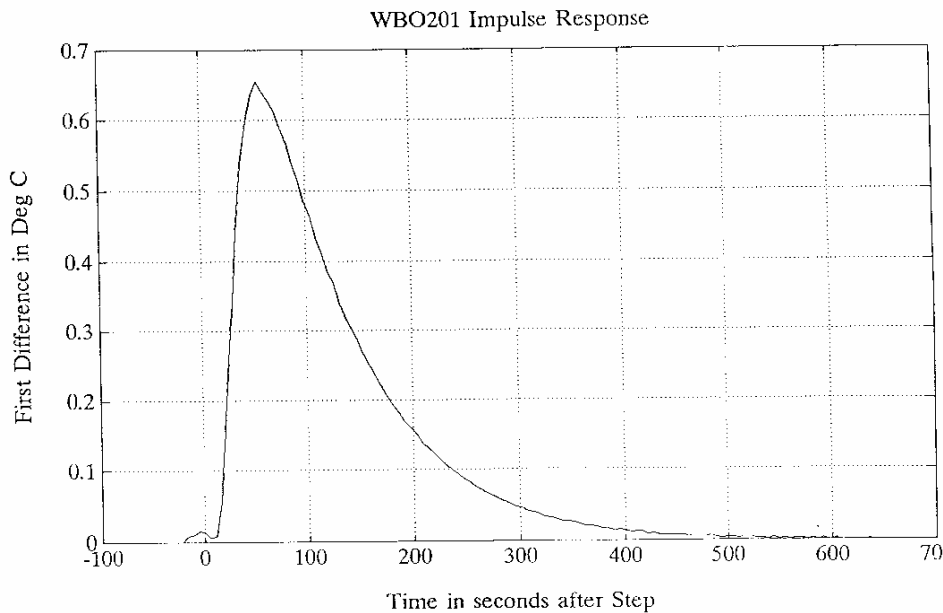
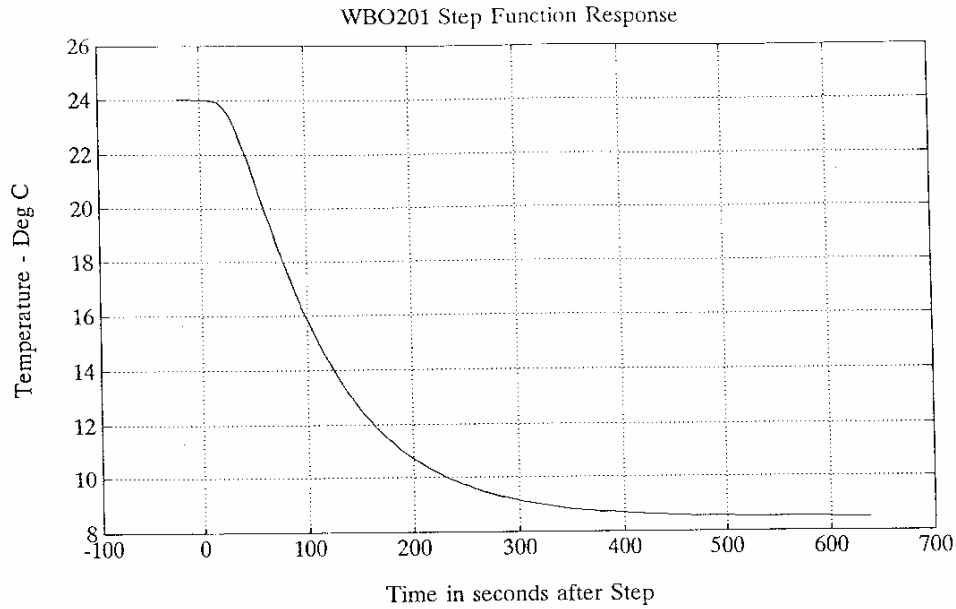
$$l(t) = 1/\tau e^{-t/\tau}$$

where τ is the time constant of the filter. This has a frequency response

$$\begin{aligned} L(\omega) &= (1 + i 2\pi f\tau) \\ &= [1 + (2\pi f\tau)^2]^{-1/2} e^{i[\text{Atan}(2\pi\tau)]} \end{aligned}$$

Out to the Nyquist frequency (15 cph), the measured frequency response is well fit by this simple 90-second exponential filter, and this result was used to correct the spectra for the response of the sensor. Note that we have plotted the amplitude response only and have not considered the phase shift. Thus, the prefilter was adequate and attenuated high frequencies at the Nyquist by almost one decade, and suppressed higher frequencies to a greater extent.

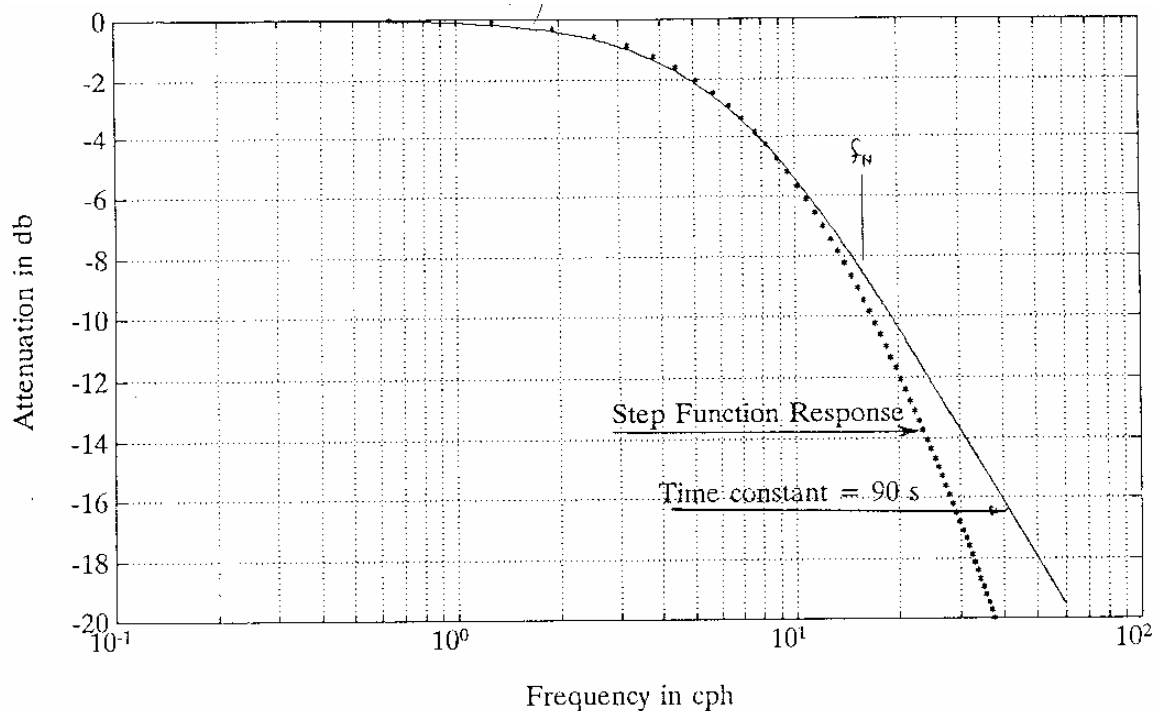
For a paper on fine structure and internal waves in the ocean, Levine and Irish (1981) wanted to get accurate spectra out to the Nyquist frequency, and used this response function to correct moored spectra near the Nyquist frequency for sensor response effects and obtain an estimate of the true frequency dependence. An example of this is shown on page 63. Also note that at the Nyquist frequency, the spectra tend to level out or have a hook. This is due to remaining aliasing effects.



The step function response (top) and corresponding estimate of the impulse response (bottom) for WBO S/N 201 with thermal mass added.

To study these aliasing effects relative to our sensor's response with and without the prefilter, consider the case of a typical internal wave temperature spectrum which decreases with frequency as $f^{-2.5}$ as shown on page 64 below. When sampled with a δt of 2 minutes, the spectrum is folded about $f_N = 15$ cph (cycles per hour), and this aliased spectrum shown. Note that the percent error is marked on the plot and at f_N this error is 100% or the signal is twice what it should be. This aliasing causes a characteristic leveling out at high frequencies observed in many spectra. Note that if geophysical spectra did not tend to fall off with frequency as fast, that

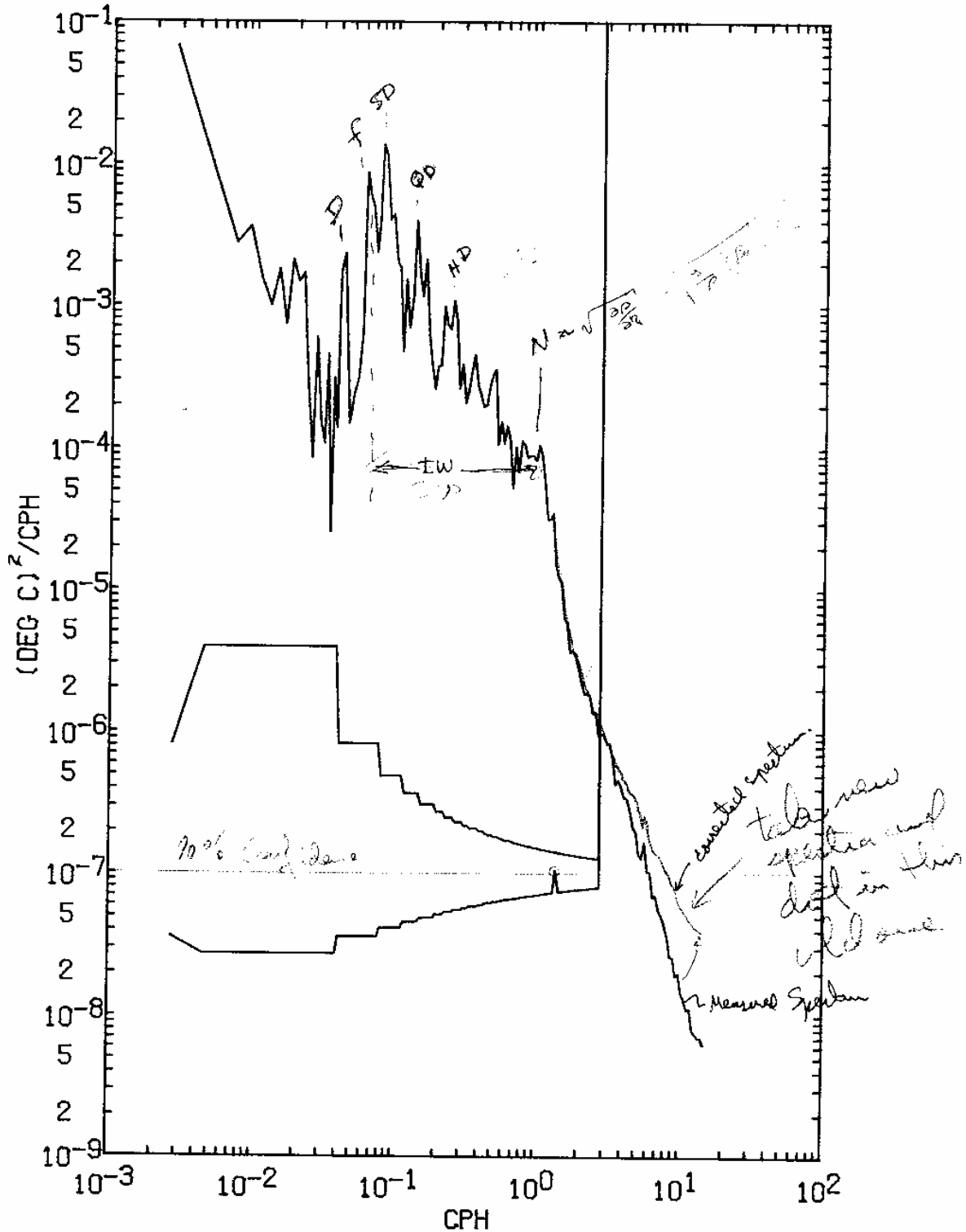
the error would be more significant. One reason people have generally gotten away without considering these spectral aliasing effects is that they were after the larger, low frequency signals, and they oversampled and ignored the high frequency portion of the spectrum.



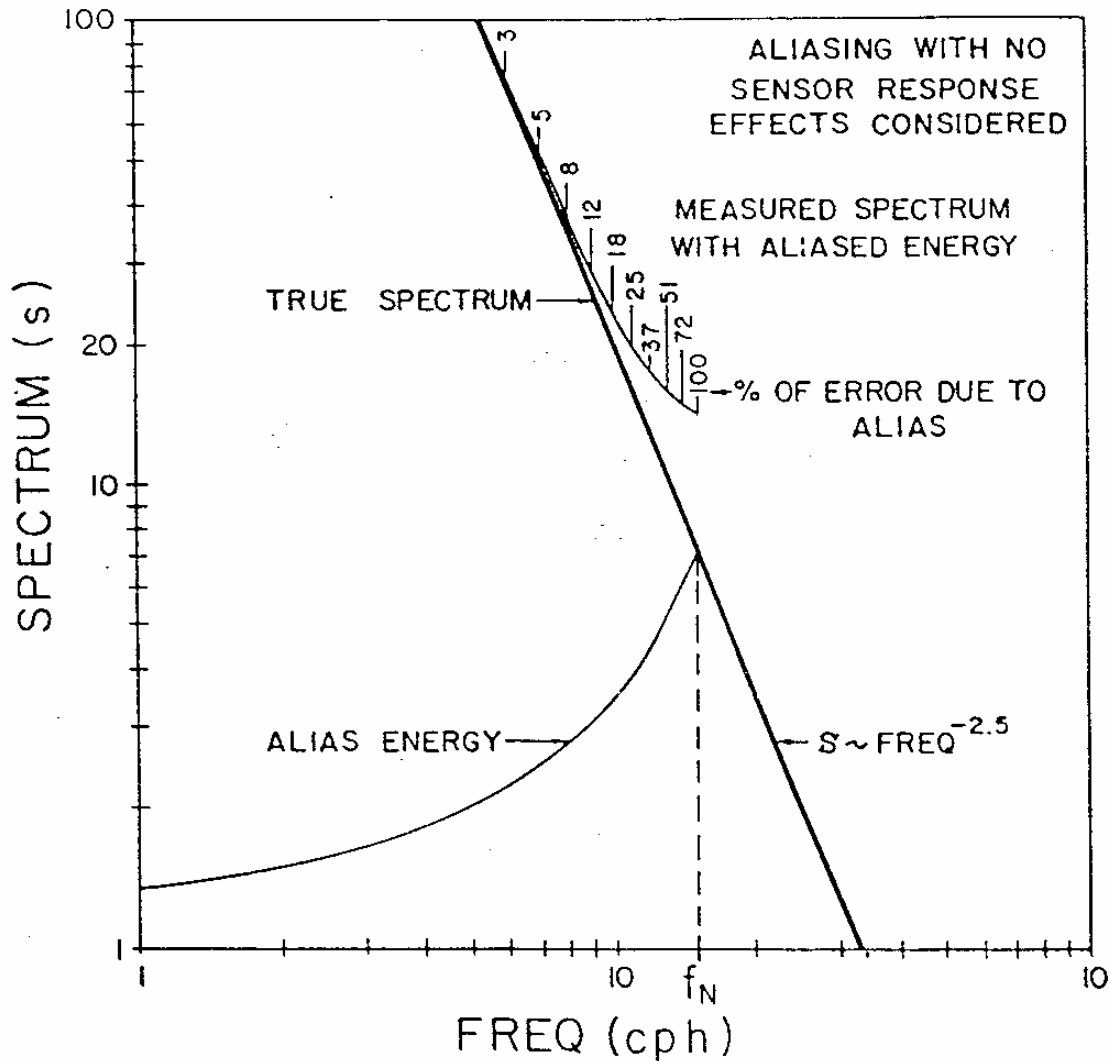
Frequency response function measured for WBOT 201.

Now to reduce the aliasing effects, we put the prefilter mass discussed above on the sensor, and this acts as a mechanical prefilter to reduce unwanted the high frequency energy. This is illustrated on page 65 where the spectrum previously shown has been multiplied by the measured frequency response to obtain the spectrum with sensor response applied. Again we sample the series with a 2-minute sample interval and obtain the uncorrected spectrum with alias. However, we know the sensor's response and can correct the spectrum for this effect by dividing the uncorrected spectrum with alias by the frequency response function. We then obtain the corrected spectrum with alias shown by the dots. The corrected spectrum is closer to the actual spectrum than in the un-prefiltered case. The error at f_N is still 100%, but as you decrease the frequency, the prefilter removed the higher frequency energy which caused the alias, so the spectrum approaches the real spectrum quicker. Actual corrected spectra do show this characteristic "hook" near the Nyquist, and knowing that this is not real, one can restrict analysis to frequencies a bit lower where this aliasing is now negligible.

Bessett-Berman 9040 CTD: Our temperature spectrum example above was the simple case where we just wanted to correct a frequency spectrum. A more complicated case is response of a CTD instrument where the temperature sensor has a time constant longer than the conductivity sensor, and we have a mismatch when calculating the salinity. This results in the classical case of salinity spiking (see Figure on page 66). The artificial spikes are due to calculating the salinity from temperature and conductivity without taking into account the different frequency response of the temperature sensor (a platinum resistance thermometer in a pressure protective shief) and the inductive conductivity cell.



Moored temperature spectrum from the Mid Ocean Acoustics Transmission Experiment at about 1200 m depth in the north Pacific ocean.

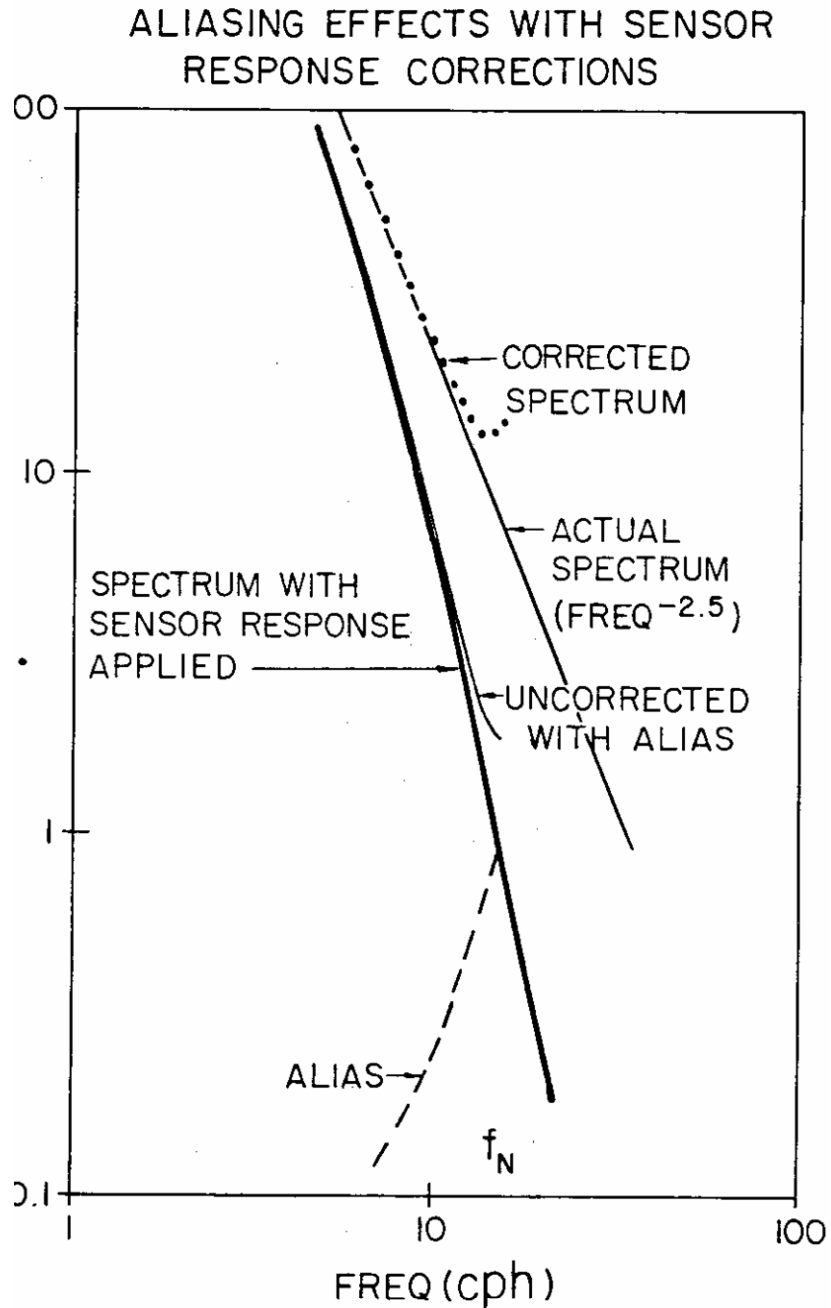


Aliasing effects on a frequency spectrum with no prefiltering.

If one were to do things properly, he should measure the frequency response of both sensors (remember this is a complex number with a real and imaginary part or an amplitude and phase part), and then transform the observations into frequency space and divide the transform of the observations by the complex response function. Finally the records should be back transformed into the time domain to obtain corrected versions of both conductivity and temperature time series. Then and only then, should one proceed to calculate derived quantities such as salinity and density from these two records and the pressure time series. If the response functions of the sensors are not well matched, the response correction could be large. This mismatch in time constant results in what is known as salinity spiking, where the mismatch in time constant results in erroneous results when calculating salinity from temperature and conductivity. More examples of this will be shown later when we discuss CTDs.

Let us examine these two sensor responses a bit further in light of internal wave and fine structure theory. Levine and Irish (1981) wanted to determine the vertical wavenumber spectra for internal waves (from CTD temperature profile series) for comparison with the moored temperature spectra to compare the results for consistency with internal wave theory, and to

produce a model of the fine structure effects on these two spectra. This time we were concerned with the response of a Bessett Berman 9040 CTD system (inductive profiling system designed by Neil Brown). The temperature sensor is a platinum resistance thermometer with a faster response time than the moored Sea Bird sensor. However, we must now also consider the effects of the period counter used to digitize the CTD's output. This acts as a simple averaging filter to suppress unwanted high frequencies. Its response and that of a SINC function are shown on page 67. The digitizer's response has a first notch or minimum at 5.78 Hz. For our sampling program in the field, the Nyquist frequency was 2.5 Hz.



Aliasing effects with thermal mass prefiltering

Graph removed for copyright reasons.

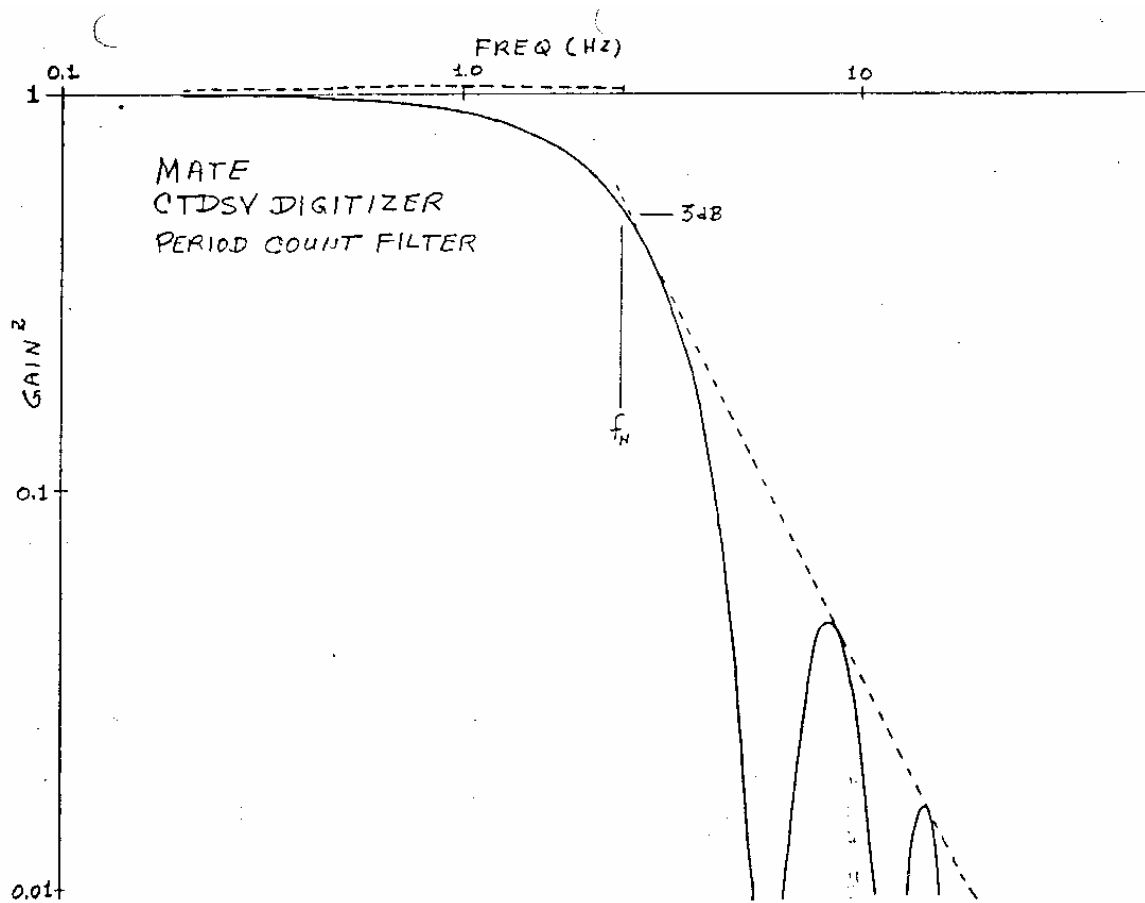
In addition to the digitizer filtering effects, we want to look at the CTD's platinum resistance thermometer response function. We do this in a similar manner to what we did for the simple moored temperature sensor. We held the CTD over a well mixed calibration tank and let the temperature sensor come to thermal equilibrium with the air. Then we quickly lowered the CTD into the calibration tank and recorded the step function response. This operation was performed four times, and the four estimates of the impulse response by the step input is shown in the top figure on page 68.

The four different measurements of the response, which were averaged to increase confidence in the response function estimate. The response time is much shorter than the moored sensor. A concern was that the response time was much longer than the time it took to

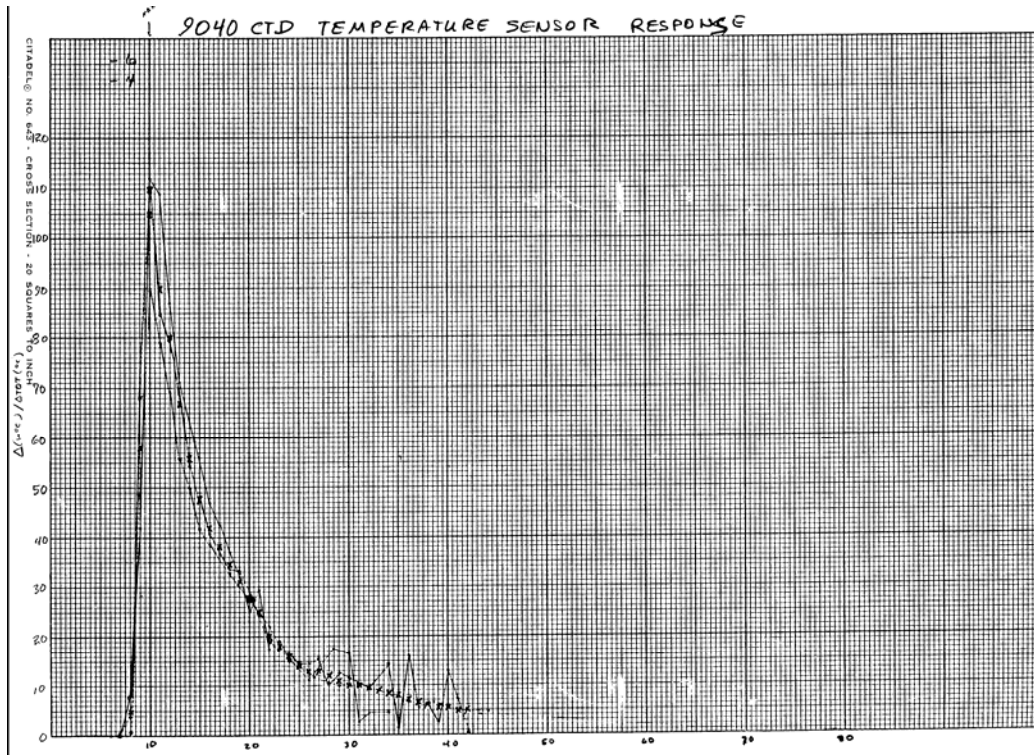
apply the step change in temperature. We estimated that we were able to do this within a fraction of a second, so it was probably OK, but if the response time of the instrument is near the time to apply the step, then this approach can introduce larger errors than acceptable.

The normalized transformed sensor response function is shown on the bottom figure on pg 98. Also shown on this plot is a simple exponential filter with a time constant of 1 second. The response is not as smooth as the moored sensor, showing that there are multiple factors contributing to the response. (That is thermal diffusion from additional pieces of metal making up the sensing system). A time constant of 1 second was selected by Roden and Irish (1975) for the 9040 CTD system to reduce salinity spikes in calculated salinity. This number was selected by comparing up and down profiles and adjusting the time constant until the series "looked" right. This result is in good agreement with the measure step response at first, and then the system appears to behave with shorter time constant. The spectra must be corrected by the inverse of the dotted circles to correct for the temperature sensor's response.

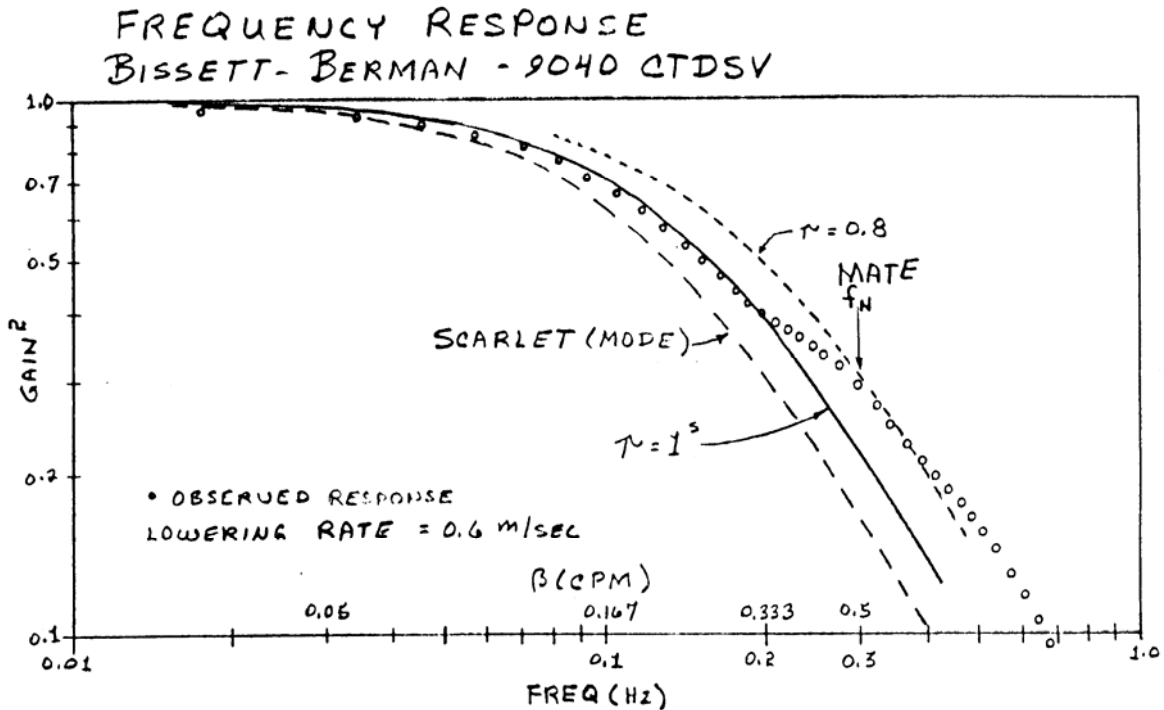
However, the actual response of the system is not just the CTD sensor, but the combined response of the CTD sensor and the digitizer. These two individual response functions and the combined response are shown in the figure on page 69 and this combined response was used to correct the vertical spectra and discussed below.



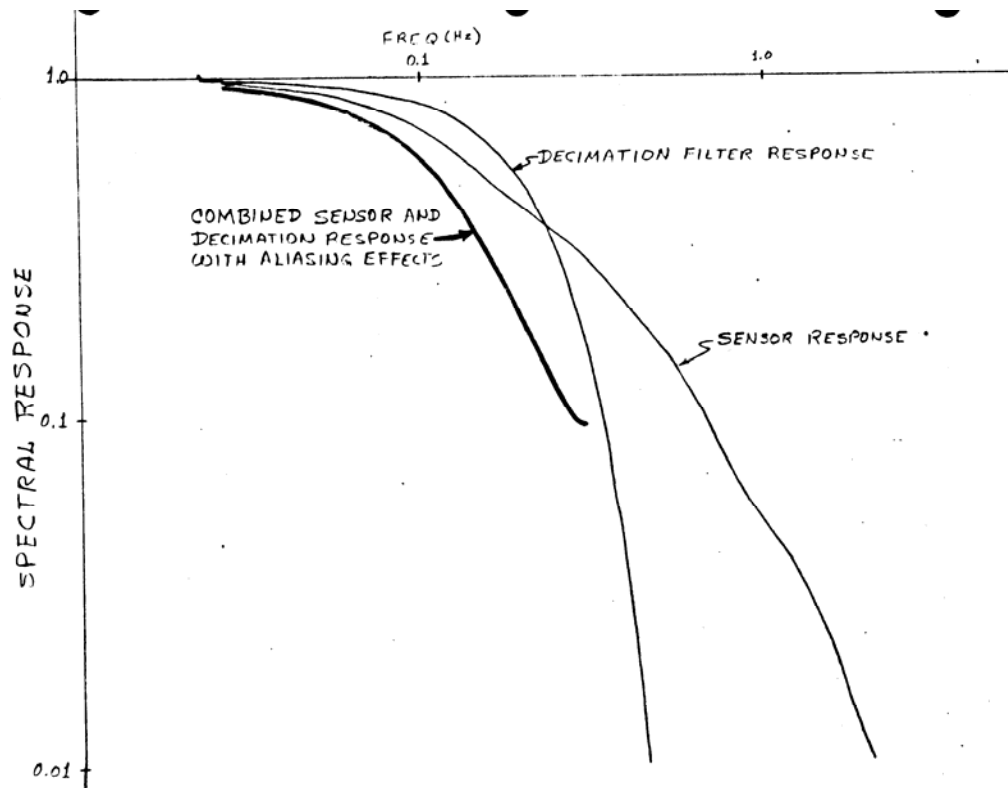
Averaging filter due to APL/UW CTD's period counting digitizer.



Four estimates of the impulse response made by taking the first difference of the step function response of a 9040 CTD temperature sensor in a well mixed water bath.



Frequency response of the 9040 CTD's temperature sensor along with a simple exponential filter with a 1 second time constant, and the estimated response selected by Scarlet to reduce salinity spiking in the 9040 CTD in oceanic data.

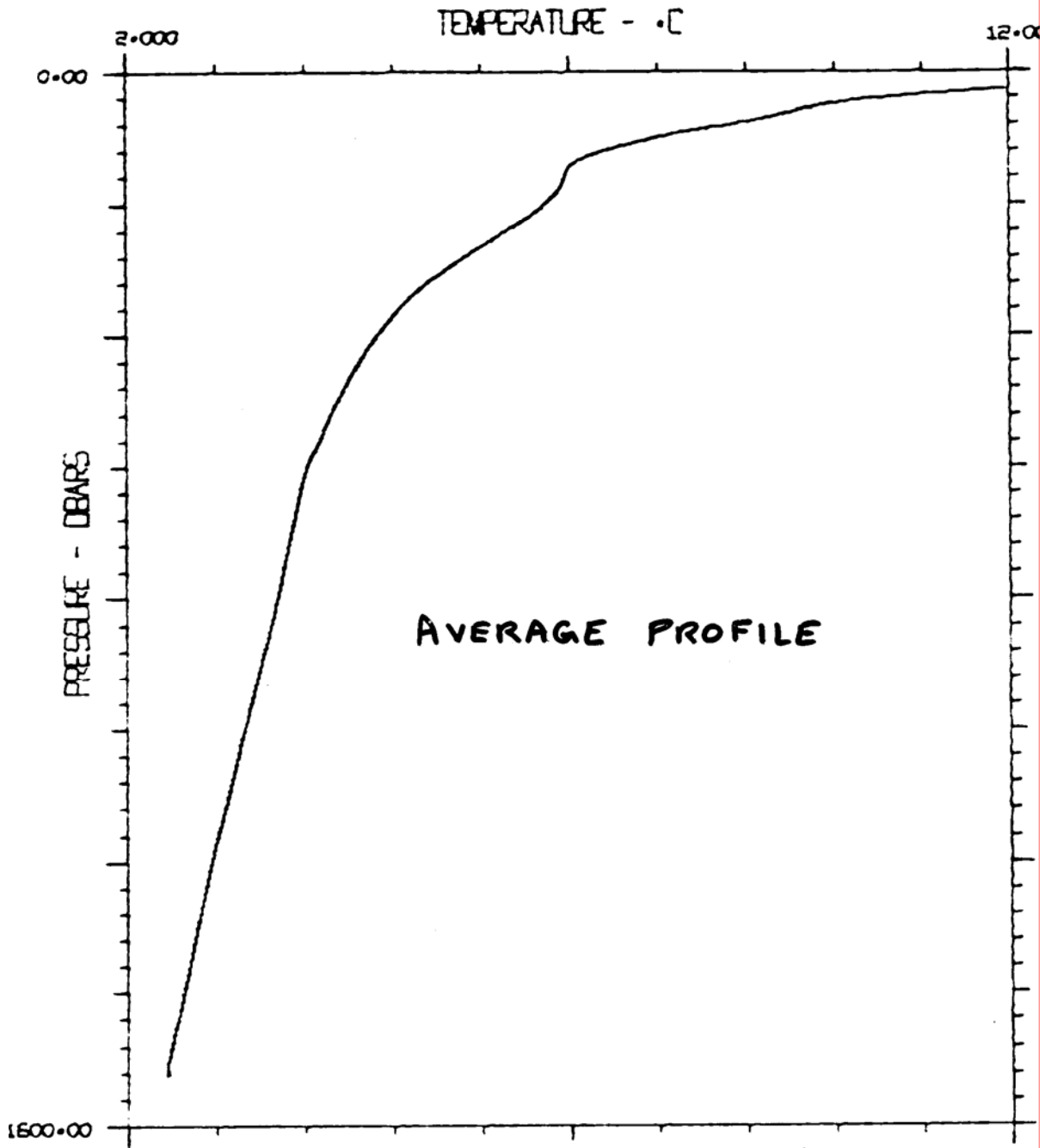


Combined sensor response and digitizer response for Bisset-Berman 9040 CTD and APL/UW CTD period count digitizer.

A vertical CTD profile from the Mid-Ocean Acoustic Transmission Experiment (MATE) is shown on the following page. This is the average of all profiles taken during the experiment and used as the average water conditions with the internal wave effects averaged out. The deviation of a single profile from this mean is assumed to be the distortion of the mean profile by the internal waves. Also the mean profile was used to determine the averaged temperature gradient which was then used to convert the temperature time series to vertical displacement time series and the spectra of a series of moored observations is shown on page 70.

The vertical wave number spectra were then calculated from the individual deviation profiles. These vertical wave number spectra were then compared with the internal wave model with fine structure predictions of the vertical temperature spectra from the moored displacement observations. This comparison is shown in page 73 and is in good agreement. These observations agree only because the correction was made to the moored temperature series for the high frequency portion of the spectrum, and the correction was made to the vertical profiles of temperature measured by the CTD for the sensor response and the digitizer response.

These data formed a basis for a Ph.D. thesis by Dr. Murry Levine at the Univ of Washington. He is presently at Oregon State University studying internal waves, mixing along shelf edges and in the arctic.



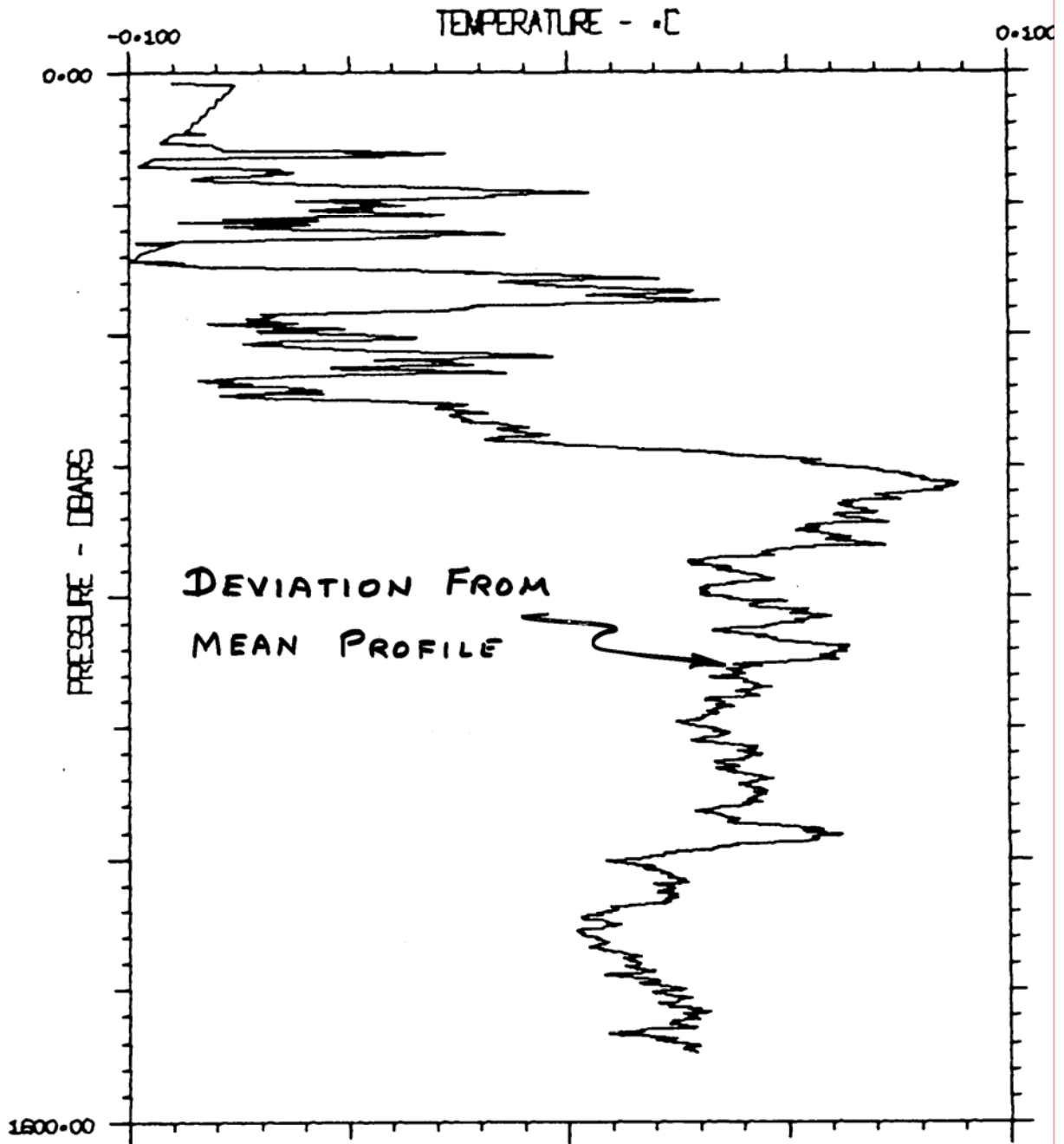
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1486. POINTS

Average profile of temperature from all the profiles collected during the three weeks of operations at the site.



77M012

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 STOP 21.51.02

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 130 54.76
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1344. POINTS

The deviation of one profile from the average profile shown on the preceding page.

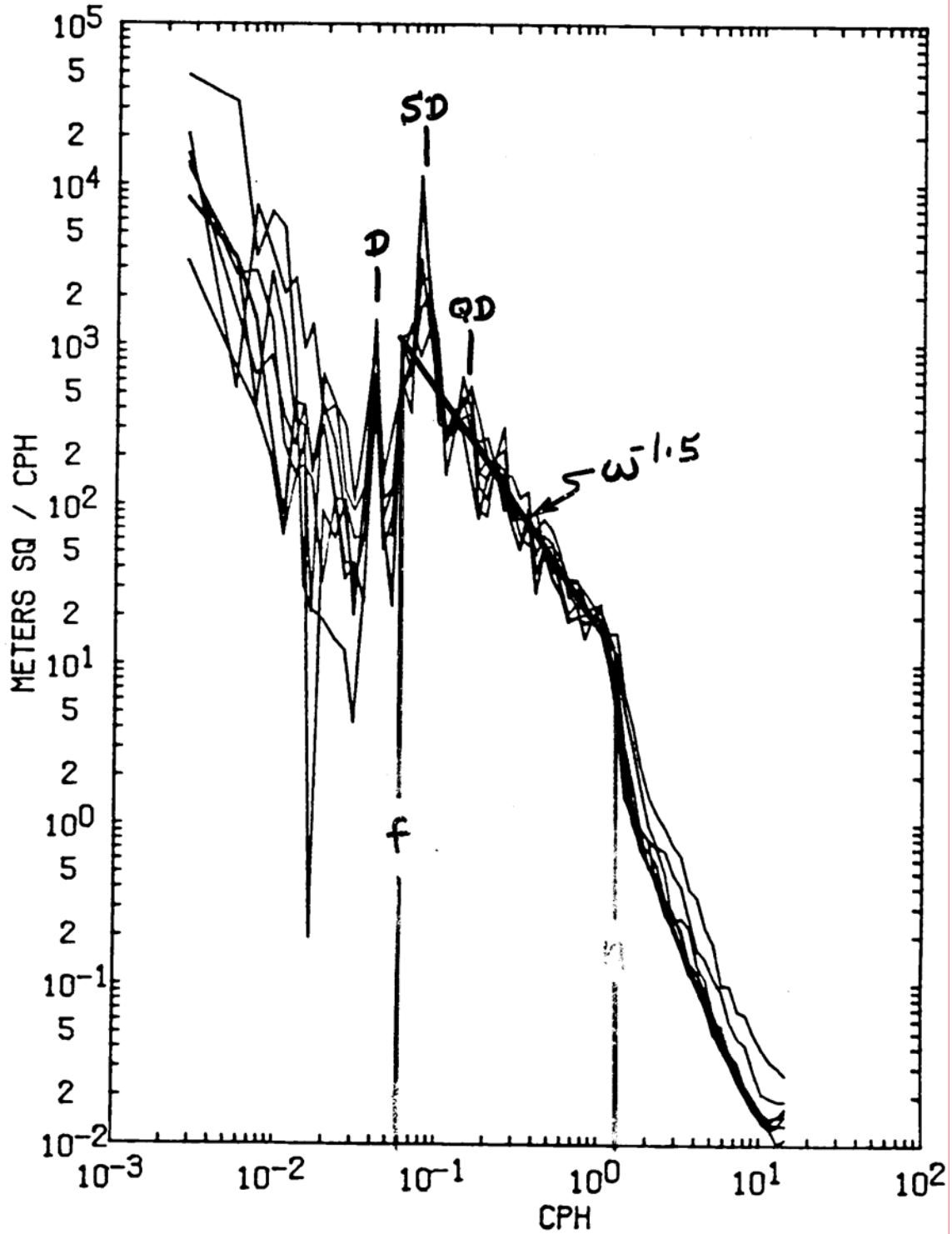
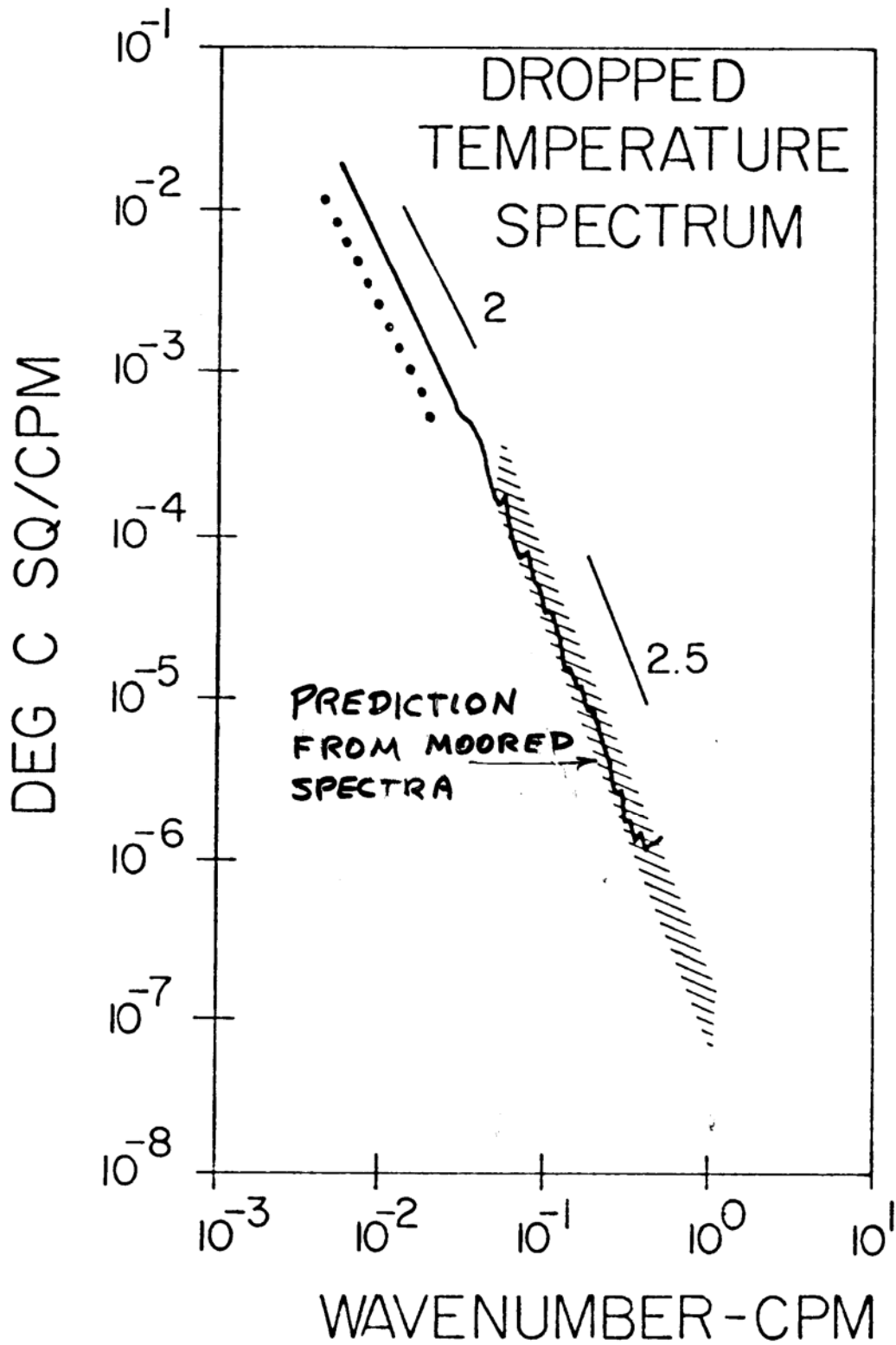


Figure 11. Moored displacement spectra -- ARRAY TWO

The moored temperature time series spectra show the expected internal wave behavior, and higher frequency fine structure effects being studied in detail. Note that these corrected spectra show the 'hook' at high frequencies due to aliasing effects.



Comparing the vertical wavenumber spectra from the CTD observations (the solid line) with the predicted vertical wave number spectrum estimated by the model from the moored spectra on the preceding page (the shaded region). The comparison is good.

Turbulence scale Sensor Response: Measurements of sensor response using a thermal plume tank.

Sensor Noise Estimation: - Besides digitizing noise, there is another limitation in a measurement due to the inherent noise in a sensor itself. Today we have the technology to reduce the digitizing interval or digitizing noise to below the sensor noise, so the limiting factor in a measurement is generally the physics of the sensor itself. Ideally, one would put the sensor in a noise free environment and measure the spectra of the sensor's output to get the sensor noise. However, this would only work if the sensor's noise was not related to signal level, and if you could find a noise free environment.

Consider the case where several sensors are measuring the same geophysical signal, $s(t)$, and that in addition to this signal each sensor sees its own inherent noise, $n_i(t)$. Therefore, the signal that we record from sensor i is,

$$x_i(t) = s(t) + n_i(t)$$

The cross-covariance function then becomes

$$\begin{aligned} R_{xy}(\tau) &= \{x_i(t) x_j(t-\tau)\} \\ &= \{[s(t) + n_i(t)][s(t-\tau) + n_j(t-\tau)]\} \\ &= \{s(t) s(t-\tau)\} + \{s(t) n_j(t-\tau)\} + \\ &\quad \{n_i(t) s(t-\tau)\} + \{n_i(t) n_j(t-\tau)\} \end{aligned}$$

where $\{ \}$ is the expected value of the quantity or the integral of the quantity in brackets over all time divided by the integral of time. Assuming that the signal and the noise are uncorrelated, e.g. $\{n(t) s(t)\} = 0$ and that the noise between the sensors is also uncorrelated $\{n_i(t) n_j(t)\} = 0$, then

$$R_{xy}(t) = \{s(t) s(t-\tau)\} + \delta_{ij} \{n_i(t) n_j(t-\tau)\}$$

Transforming, the cross spectrum is

$$C_{ij} = \int_{-\infty}^{\infty} R_{ij}(t) e^{-i\omega\tau} d\tau = S(\omega) + \delta_{ij} N_{ij}$$

where S is the transform of the signal and N is the transform of the noise. For the case of only two sensors, 1 and 2,

$$C_{11}(\omega) = S(\omega) + N_{11}(\omega)$$

$$C_{12}(\omega) = S(\omega)$$

$$C_{22}(\omega) = S(\omega) + N_{22}(\omega)$$

The co-spectra extracts the signal from the noise of the sensors and can be used to improve the signal to noise when it is poor with only one sensor.

If we define N as the sum of the two noises,

$$N(\omega) = N_{11}(\omega) + N_{22}(\omega)$$

$$\begin{aligned}
&= [C_{11}(\omega) - S(\omega)] + [C_{22}(\omega) - S(\omega)] \\
&= C_{11}(\omega) + C_{22}(\omega) - 2 C_{12}(\omega)
\end{aligned}$$

Consider the difference between the two signals,

$$d(t) = x_1(t) - x_2(t),$$

then the auto-covariance of this difference function is

$$\begin{aligned}
R(\tau) &= \{d(t) d(t-\tau)\} \\
&= \{[x_1(t) - x_2(t)] [x_1(t-\tau) - x_2(t-\tau)]\} \\
&= \{x_1(t) x_1(t-\tau)\} - \{x_1(t) x_2(t-\tau)\} \\
&\quad - \{x_2(t) x_1(t-\tau)\} + \{x_2(t) x_2(t-\tau)\}
\end{aligned}$$

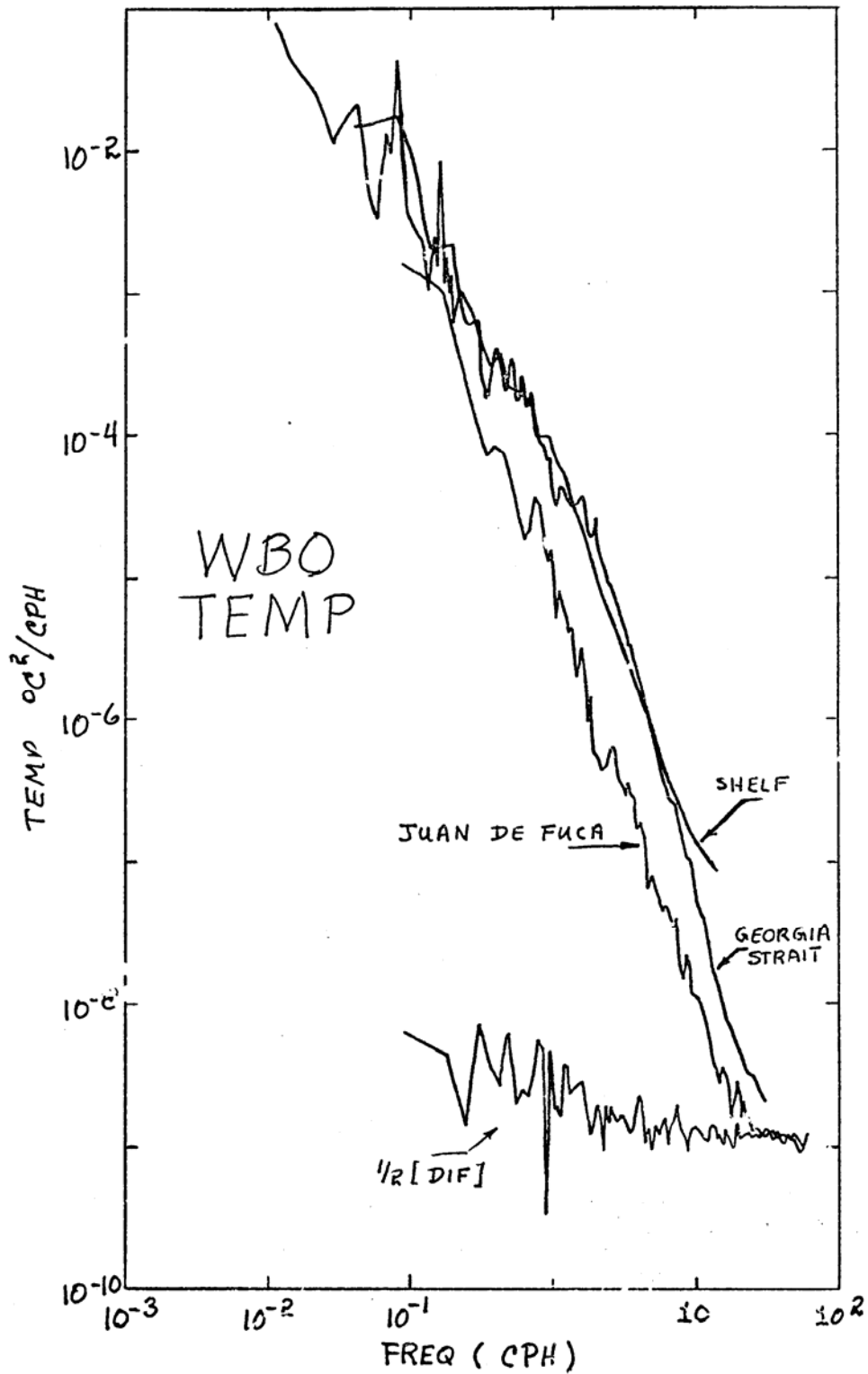
Again assuming that the cross terms are zero as above, and transforming, the spectrum of the difference is

$$\begin{aligned}
D(\omega) &= S(\omega) + N_{11}(\omega) - S(\omega) - S(\omega) + S(\omega) + N_{22}(\omega) \\
&= N_{11}(\omega) + N_{22}(\omega) = N(\omega)
\end{aligned}$$

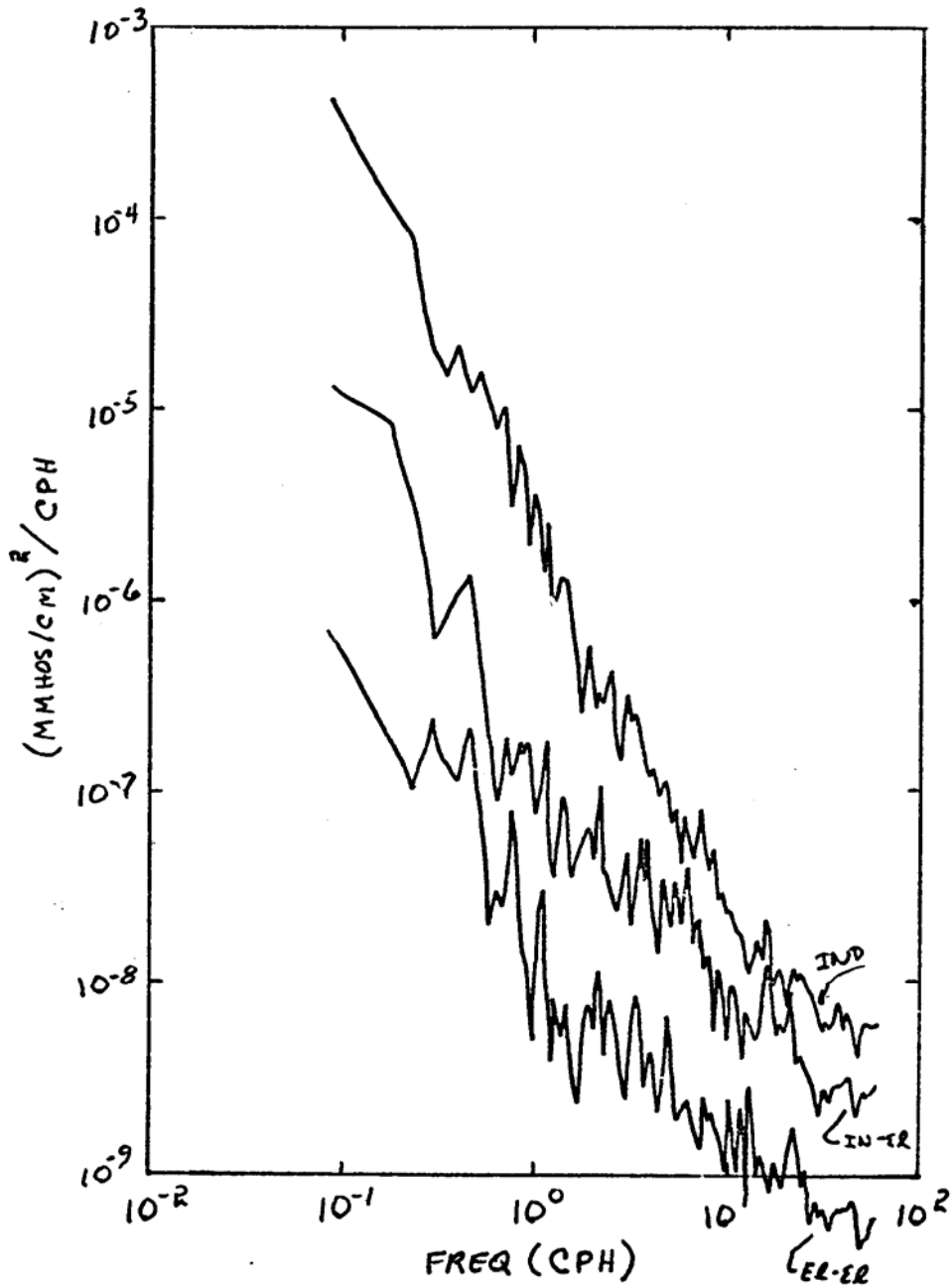
Therefore the difference of two signals measuring the same geophysical signal and different uncorrelated noises, is just the sum of the noises in the two sensors. The noise also contains any calibration errors, etc. in our normalization of the sensor's data. It is obvious that the same exercise can be carried out with three sensors, and cross spectral techniques used separate the three individual sensor noise spectra. It should be clear that this noise spectra is the limiting quantity in any spectral measuring process i.e. we can not get any better than our sensors.

An example of this is seen in spectra on the following page. Several temperature spectra are shown from moored temperature sensors in various locations on the west coast of the US. The record taken in the Strait of Juan De Fuca shows a spectrum which tends to level out at high frequencies, and this level is above the digitizing noise level discussed above. Since this mooring had dual sensors at that depth, a cross spectrum and a spectrum of the difference could be taken and the sum of the noise levels of the sensors estimated. It was then assumed that the instrument noise of both sensors contributed equally to the results, so the noise spectrum was divided by 2 and is shown to be the cause of the spectrum leveling out. This noise cannot be reduced without changing sensors to ones with lower noise level. In this case we need not have sampled as often. A 30 cph (3 minute) sample would match the sampling interval and sensor noise to the environmental signal. This reduced sample rate will have the effect of reducing the required data storage space, or allow the experiment to be lengthened.

This instrument noise limit is also seen the Acoustic Doppler Current Profiler. The statistical uncertainty in the velocity estimate is a function of the acoustic frequency, the vertical spatial resolution and the number of pings averaged in the estimate. The figure below shows the ADCP and VACM spectra from Massachusetts Bay, and shows the leveling out of the ADCP spectra. The data should have been recorded as hourly averages, since the high frequency energy recorded is just noise, and contains no geophysical information. To reduce the noise, considerably more pings (at greater power) would need to be averaged into the ensemble.



Moored temperature spectra from the Pacific Northwest, with the power spectra of the record in the Strait of Juan De Fuca and one half the difference spectra representing the sensor noise illustrates the noise limitation of these sensors at high frequencies.



Conductivity sensor difference, implies the electrode sensor noise is lower than the inductive sensor noise, but that we really haven't resolved the sensor noise yet.

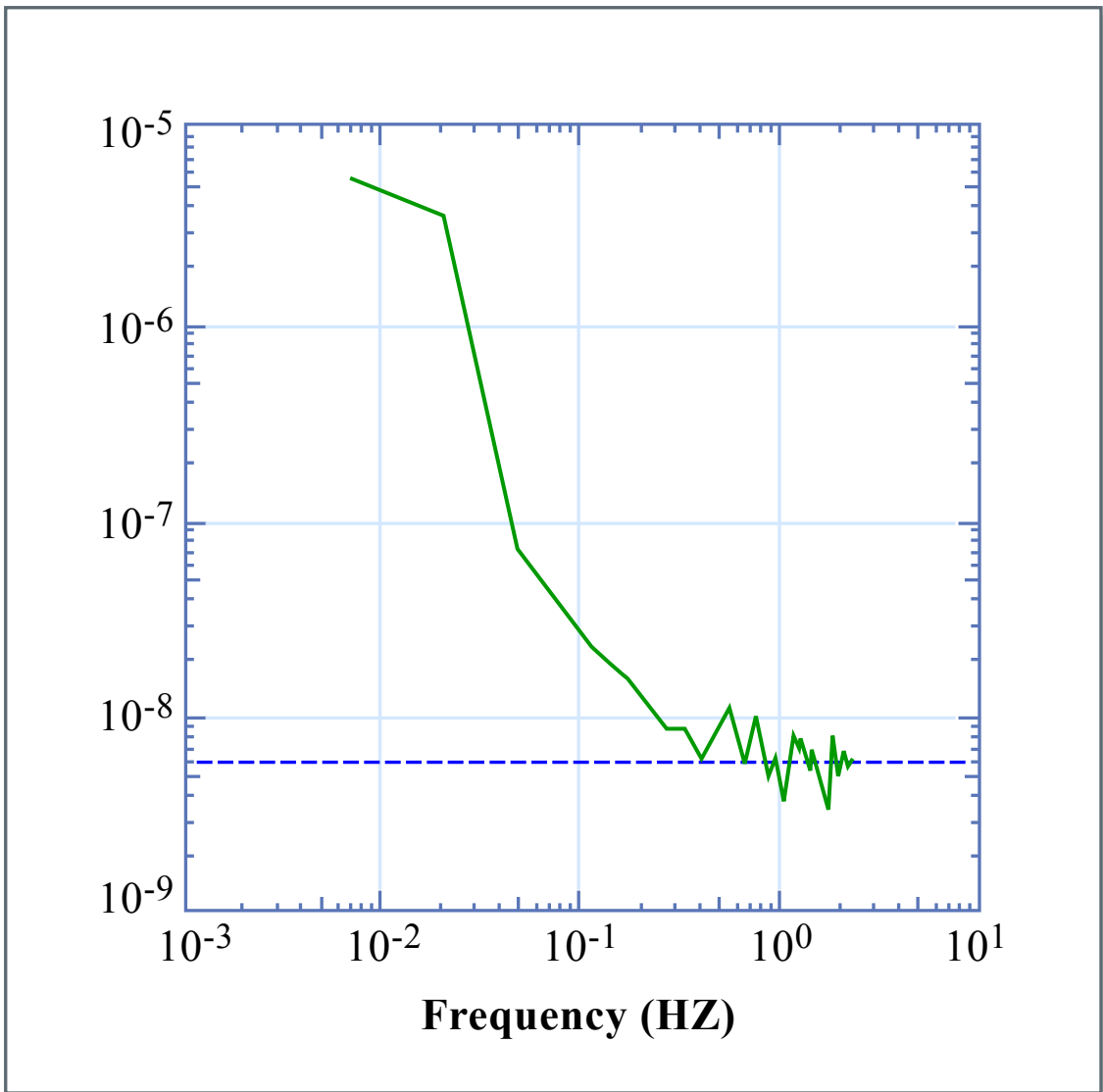


Figure by MIT OCW.

Noise spectrum of a Sea Bird SBE-03-01 temperature spectrum made by the APL/UW Microstructure Structure Recorder in a fjord below the sill depth where little environmental signal exists, so the sensor noise dominates at higher frequencies (>0.5 Hz)

Properly Sampling the Environment: Lets summarize the needs to be considered in properly sampling the environment as discussed above:

1. Suppress unwanted high frequencies by such techniques as prefiltering. (e.g. Placing a thermal mass on a temperature sensor to filter out high frequency fluctuations before digitizing. One could average a frequency over the sample interval, which is the same as applying a "boxcar" filter to the data before digitizing.) Also, the sensor could be sampled at a high rate to resolve high frequencies and the storage system average the

data to suppress high frequency fluctuations, or you could resolve the higher frequency (if enough storage space were available, and then take care of things in post-processing.)

2. Sample at least twice the highest frequency present after any prefiltering to prevent aliasing. (Normally one would sample several times the highest frequency desired after taking the proper steps to prefilter the data.)
3. Sample long enough to give the required
 - a. resolution in frequency - $\delta f = 1/T$ (This is also the lowest frequency, and is often dictated by data storage or battery capacity as well as ship schedules and funding.)
 - b. confidence in spectral results - necessary. (Note that the confidence limits for the cross-spectrum or the coherence are different than the auto-spectrum, and fairly complicated.) There is a trade off in resolution and confidence - generally we can not get the resolution and confidence that we would like.
4. Know the calibration and frequency response of the sensor to
 - a. make sure that the uncertainty in the absolute calibration of the sensor is appropriate i.e. compare results between instruments, and calculate derived quantities.
 - b. be sure that there are no sensor response effects in the frequency region of interest or if so, that
 - c. corrections for sensor response are made.
5. Know the digitizing effects of the recording system so that any effects it puts into the data are negligible. It is easy to reduce the digitizing noise to below the sensor noise, however again too much below means that you are recording too many bits and wasting data storage space.
6. Understand what statistics are needed, and what type of processing needs to be done, so
 - a. the experiment can be designed to give the data and
 - b. analysis techniques can be tailored to reducing the data.
 - c. remove trends as non-stationarities, i.e. tides on surface wave spectra.

Warning: You really need to know what is there to design a properly sampled experiment. There can be aliasing due to improper sampling in space and/or time.