Outline:

- *Light polarization and Jones Matrix*
- *Birefrigence*
- *Photoelasticity*

A. Light polarization and Jones Matrix

Recall that in an isotropic medium, electromagnetic field travelling along z direction can be further divided into 2 independent Polarizations:

(E_x, H_y only)

$$i\omega\mu\mu_0 H_y = -\frac{\partial}{\partial z} E_x \quad (1)$$

And $$i\omega\varepsilon\varepsilon_0 E_z = \frac{\partial}{\partial z} H_y \quad (2)$$

Or (E_y, H_x only)

$$i\omega\mu\mu_0 H_x = -\frac{\partial}{\partial z} E_y \quad (3)$$

And $$i\omega\varepsilon\varepsilon_0 E_y = \frac{\partial}{\partial z} H_x \quad (4)$$

So far, our discussion in the wave optics does not include polarization effect. In another word, we assumed the interference and diffraction effect take place within the same polarization of light field. However, polarization of light might be converted as the field travel through an optical medium such as birefringence. In our daily life, we may find methods of actively controlling the polarization via electro-optical and acousto-optical effects.

The polarization state of an E field of incoming light can be defined as a 2D vector—**Jones vector**—containing the two complex amplitudes (with common factor $\exp(ikz - i\omega t)$:

$$\widetilde{\boldsymbol{E}} = \begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \end{pmatrix} \quad (5)$$

Thus the Jones vector for linearly polarized light is given by:

$$\widetilde{\boldsymbol{E}} = E_0 \begin{pmatrix} cos\alpha \\ sin\alpha \end{pmatrix} \quad (6)$$

And for the elliptically polarized light, the Jones matrix form is:

$$\widetilde{\boldsymbol{E}} = E_0 \begin{pmatrix} cos\alpha \\ sin\alpha \exp(i\Delta\phi) \end{pmatrix} \quad (7)$$

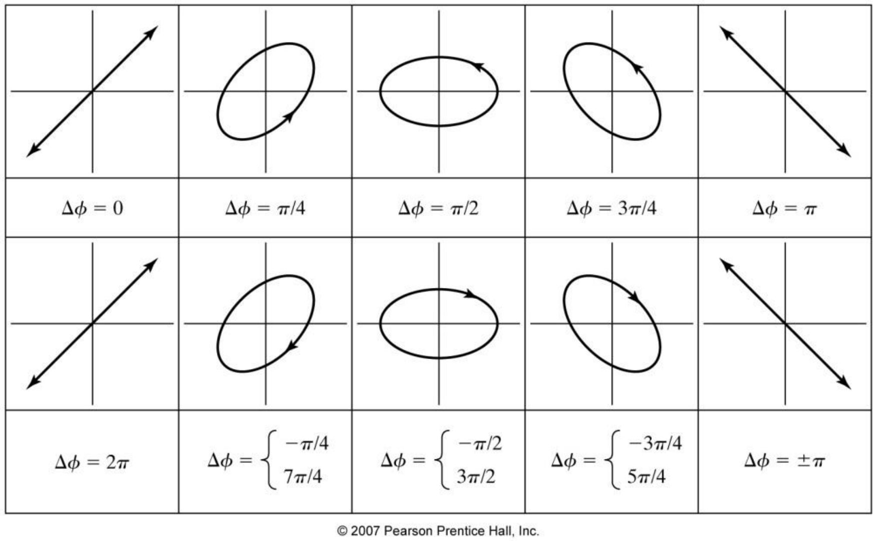
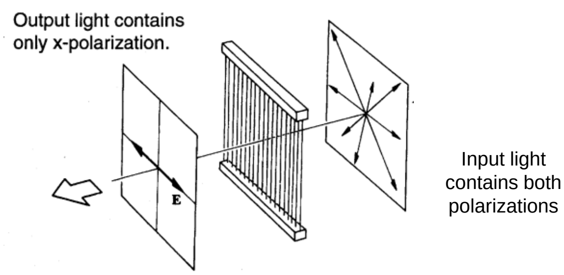


Figure 14.4 of Pedrotti: Lissajous Figures of light polarization, $\Delta\phi = \phi_y - \phi_x$

Similarly, **Jones matrices** are used to model the effect of a medium on light's polarization state.

$$\widetilde{\boldsymbol{E}}_{out} = \underline{M}\widetilde{\boldsymbol{E}}_{in} \tag{8}$$

Example: Wire grid polarizer



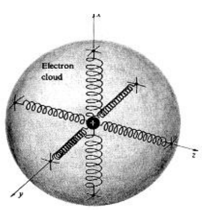
The light can excite electrons to move along the wires, which then emit light that cancels the input light. This cannot happen to light field perpendicular to the wires. Such polarizers work best in the visible to IR wavelength.

In this case, the polarizer gives an output field $\tilde{\boldsymbol{E}}_{out} = \begin{pmatrix} \tilde{E}_x \\ 0 \end{pmatrix}$, regardless of the input (random polarization). Therefore the Jones matrix of this x-polarizer is:

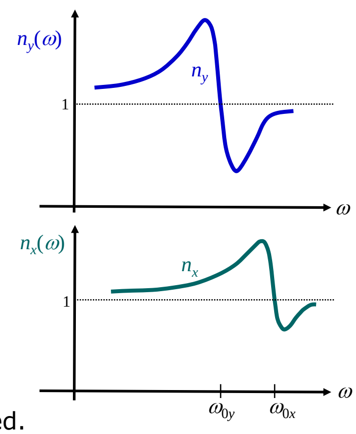
$$\underline{M} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{9}$$

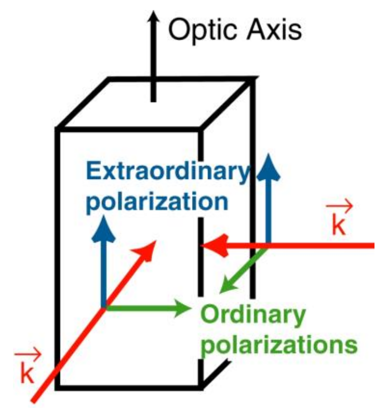
B. Birefringence

Crystal materials often exhibit birefringence or known as **double refraction**. This is because the anisotropic atomic structure of the lattice can respond to different polarization of light waves, giving rise to anisotropic index of refraction at a given frequency range. Due to Snell's Law, light of different polarizations will bend by different amounts at an interface.



The x- and y-polarizations can see different resonances and hence different refractive index curves.



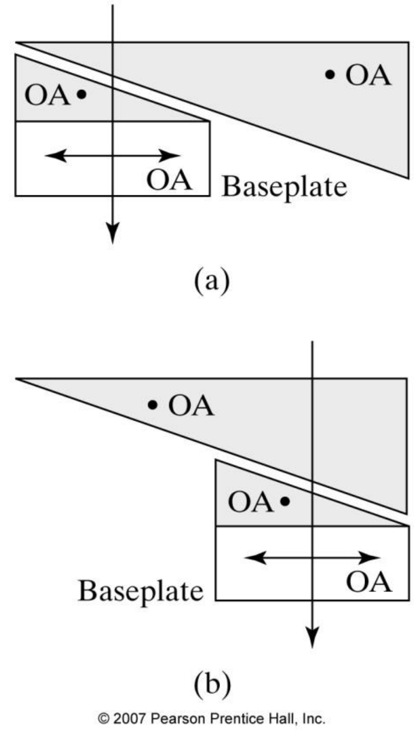


Uniaxial crystals have one refractive index for light polarized along the optic axis(n_e) and another for light polarized in either of the two directions perpendicular to it (n_o). Light polarized along the optic axis is called the **extraordinary** ray, and light polarized perpendicular to it is called the **ordinary** ray. These polarization directions are the **crystal principal axes**. For example, calcite is one of the most birefringent materials known in the world.(n_0=1.6584, n_e=1.4864@589.3nm)

The properties of double refraction are often used for application of *wave plates* or *retarders*. For example, ***a quarter-wave plate*** can be used to convert a linearly polarized light beam to a circularly polarized light.

In order to compensate the phase difference, two sliding wedges made of the uniaxial crystals are often used to vary the thickness.

$$\Delta\phi = \frac{2\pi d}{\lambda}(n_e - n_o) \qquad (10)$$



Exercise:
For quartz at wavelength of 589nm, ne=1.5533, no=1.5442. Let's estimate the following two cases:

(a) Minimum thickness of the quartz window to be used as quarter wave plate;
(b) The thickness of the quartz window when the phase shift is (200+1/4)π.

C. **Photoelasticity**:

Polymers can make efficient polarizers, as partial alignment of the long chain polymer molecules can cause birefringence. In fact, this is often used to measure stress distribution of a plastic element, as you will see a quite beautiful display of colors as you place these samples between two crossed linear polarizers.

Image of plastic protractor removed due to copyright restrictions.

Example of Photoelasticity: a plastic protractor placed between two crossed polarizers. (Source: Wikipedia; http://en.wikipedia.org/wiki/File:Plastic_Protractor_Polarized_05375.jpg)

For convenience, let's set the optic axis of the (stressed) plastic in the y-direction, and the Jones matrix of a thin slab of the plastic is given by:

$$\underline{\mathrm{M}} = \begin{bmatrix} 1 & 0 \\ 0 & \exp(i\Delta\phi) \end{bmatrix} \tag{11}$$

For a pair of polarizers placed at $\pm 45^o$, the transmitted E field can be expressed as:

$$\tilde{E}_{out} = \frac{\tilde{E}_0}{2\sqrt{2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \exp(i\Delta\phi) \end{bmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{12}$$

$$\tilde{E}_{out} = \frac{\tilde{E}_0 (1 - \exp(i\Delta\phi))}{2\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{13}$$

The intensity is given by

$$\boxed{I \propto \left| \frac{1 - \exp(i\Delta\phi)}{2} \right|^2 = sin^2 \left(\frac{\Delta\phi}{2} \right)} \tag{14}$$

This element works as a compact Fabry-Perot interferometer!

Note: If you are interested in converting the measured phase difference to quantitative stress maps, please read the section "Photoelasticity," by Krishnamurthi Ramesh, in Springer Handbook of Experimental Solid Mechanics, 2008".