Demo #2: Far-Field Diffraction

(Fraunhofer Diffraction: Tale of Two Germans?)

Joseph von Fraunhofer (1787-1826)

Gustav Kirchhoff (1824-1887)

2.71/2.710 - Optics

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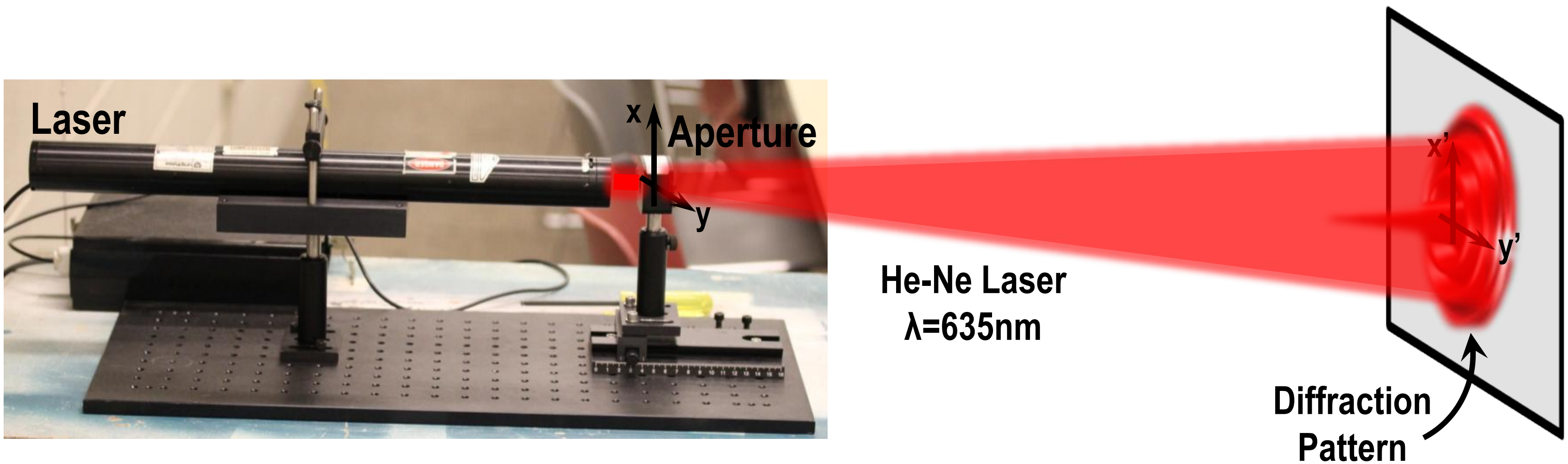
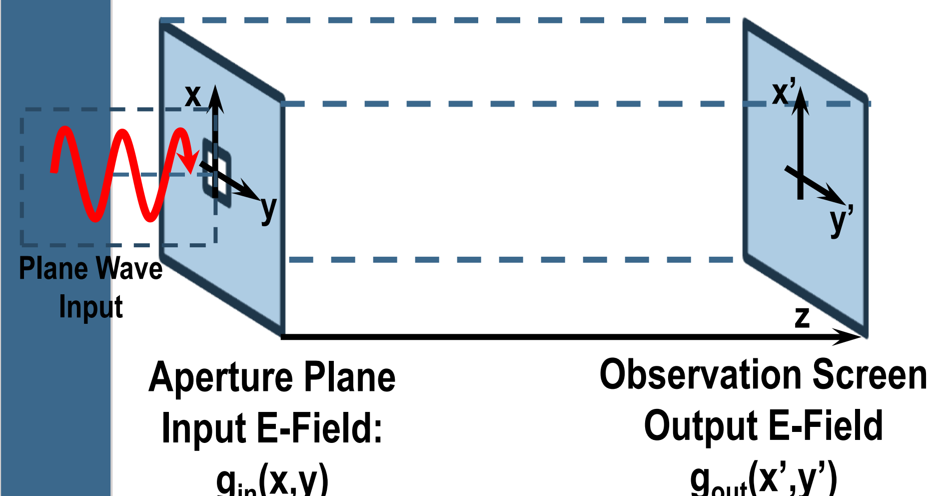
Demo #2: Far-field Diffraction





Fraunhofer Diffraction Integral:

$$g_{out}(x',y';z)=\frac{e^{i2\pi z/\lambda}}{i\lambda z}e^{i\pi(x'^2+y'^2)/\lambda z}\iint_{-\infty}^{\infty}g_{in}(x,y)e^{-i2\pi(xx'+yy')/\lambda z}dxdy$$

Definition of Fourier Transform:

$$G_{in}(u,v)=\mathscr{F}\{g_{in}(x,y)\}=\iint_{-\infty}^{\infty}g_{in}(x,y)e^{-i2\pi(xu+yv)}dxdy$$

Spatial Frequencies: $u=\frac{x'}{\lambda z}$ *and* $v=\frac{y'}{\lambda z}$

Field of Diffraction Field

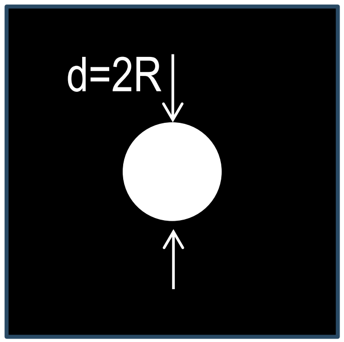
$$g_{out}(x',y';z)=\frac{e^{i2\pi z/\lambda}}{i\lambda z}e^{i\pi(x'^2+y'^2)/\lambda z}Gin\left(\frac{x'}{\lambda z},\frac{y'}{\lambda z}\right)$$

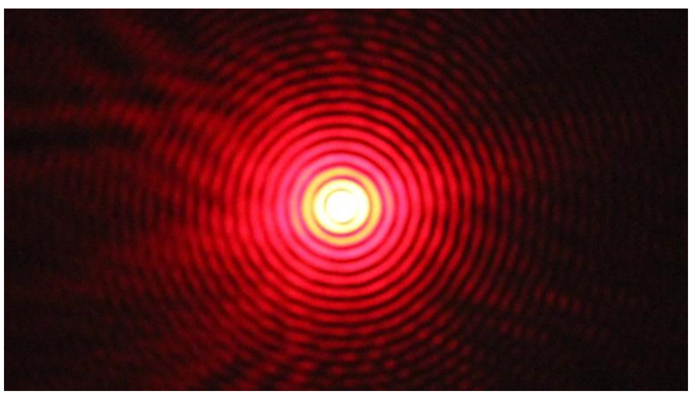
$$g_{out}(x',y';z)\propto Gin\left(\frac{x'}{\lambda z},\frac{y'}{\lambda z}\right)$$

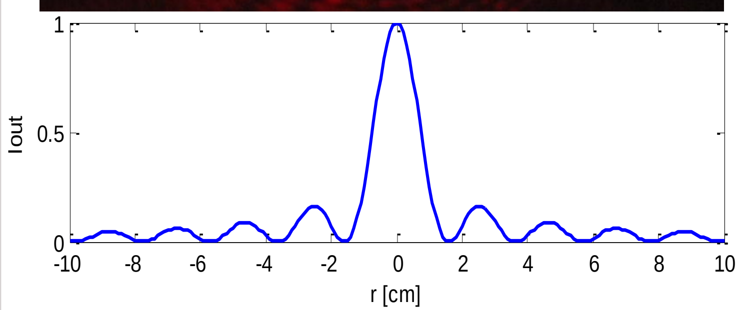
Observed Pattern is **INTENSITY**: $I_{out}(x',y';z)\propto|g_{out}(x',y';z)|^2=g_{out}^{*}\cdot g_{out}$

Demo #2: Circular Aperture

- Circular Aperture
- Radius: R = 230 um
- Distance to screen: z ~ 15m







Aperture Function:

$$g_{in}(x,y) = \frac{\sqrt{x^2+y^2}}{R} = \mathrm{circ}\left(\frac{r}{R}\right)$$

E-Field Diffraction Pattern in k-space:

$$G_{out}(u,v) \propto \mathscr{F}\left\{\mathrm{circ}\left(\frac{\sqrt{x^2+y^2}}{R}\right)\right\} = \frac{RJ_1\left(2\pi\sqrt{u^2+v^2}\right)}{\sqrt{u^2+v^2}}$$

Spatial Frequencies: $u = \frac{x'}{\lambda z}$ *and* $v = \frac{y'}{\lambda z}$;

E-Field Diffraction Pattern in real-space:

$$G_{out}(x',y';z) \propto \frac{RJ_1\left(\frac{2\pi R}{\lambda z}\sqrt{x'^2+y'^2}\right)}{\frac{R}{\lambda z}\sqrt{x'^2+y'^2}} = \frac{RJ_1\left(\frac{2\pi R}{\lambda z}r'\right)}{\frac{R}{\lambda z}r'} = R\,\mathrm{jinc}\left(\frac{Rr'}{\lambda z}\right)$$

Note: $\mathrm{jinc}(x) = J_1(2\pi x)/x$

Observed Intensity Pattern:

$$I_{out}(r';z) \propto \left|R\cdot\mathrm{jinc}\left(\frac{Rr'}{\lambda z}\right)\right|^2 = \left|\frac{RJ_1\left(\frac{2\pi Rr'}{\lambda z}\right)}{\frac{Rr'}{\lambda z}}\right|^2$$

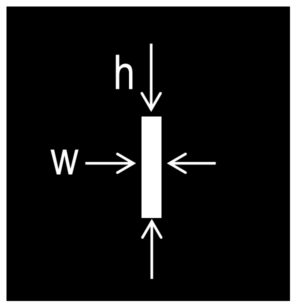
First Null occurs: $J_1(\pi x) = 0$ *when* $x = 1.220,\ 2.233, etc.$

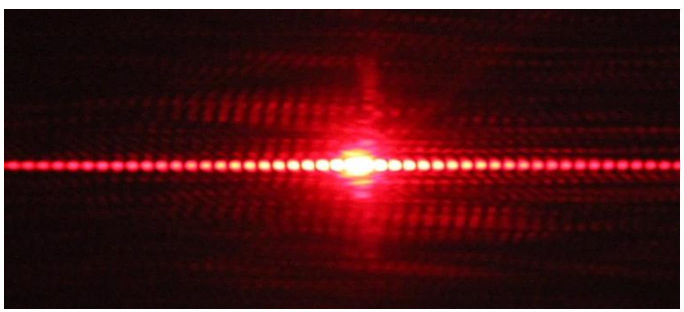
Width of Central Lobe: $\frac{2\pi R r_0}{\lambda z} = \pi(1.220) \Rightarrow r_0 = \frac{1.220\lambda z}{2R}$

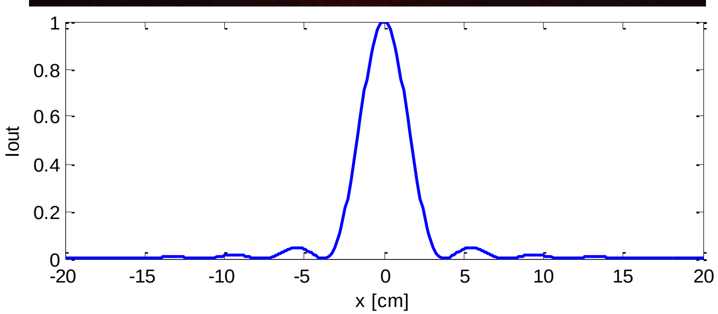
$$r_0 = \frac{1.220(635nm)15m}{2\cdot 230um} = 2.5cm$$

Demo #2: Slit Aperture

- Single Slit Aperture
- Variable Width: w = 250µm
- Variable Height: h = 5mm
- Distance to screen: z ~ 15m







Aperture Function:

$$g_{in}(x,y) = \mathrm{rect}\left(\frac{x}{w}\right)\mathrm{rect}\left(\frac{y}{h}\right)$$

E-Field Diffraction Pattern in k-space:

$$(G_{out})u,v \propto \mathscr{F}\left\{\mathrm{rect}\left(\frac{x}{w}\right)\mathrm{rect}\left(\frac{y}{h}\right)\right\} = wh\cdot\mathrm{sinc}(wu)\mathrm{sinc}(hv)$$

Spatial Frequencies: $u = \dfrac{x'}{\lambda z}$ *and* $v = \dfrac{y'}{\lambda z}$

E-Field Diffraction Pattern in real-space:

$$G_{out}(x',y';z) \propto \mathrm{sinc}\left(\frac{x'w}{\lambda z}\right)\mathrm{sinc}\left(\frac{y'h}{\lambda z}\right)$$

Note: $\mathrm{sinc}(x) = \sin(\pi x)/\pi x$

Observed Intensity Pattern:

$$I_{out}(x',y';z) \propto \left|\mathrm{sinc}\left(\frac{x'w}{\lambda z}\right)\mathrm{sinc}\left(\frac{y'h}{\lambda z}\right)\right|^2$$

First Null occurs: $\mathrm{sinc}(x) = 0$ *when* $x = 1$

Half Width of Central Lobe: $\dfrac{wx'_0}{\lambda z} = 1 \Rightarrow x'_0 = \dfrac{\lambda z}{w}$

Half Height of Central Lobe: $\dfrac{hy'_0}{\lambda z} = 1 \Rightarrow y'_0 = \dfrac{\lambda z}{h}$

$$x'_0 = \frac{(635nm)15m}{250um} = 3.8cm \qquad y'_0 = \frac{(635nm)15m}{5mm} = 1.9mm$$

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