2.75
CONSTRAINT
LECTURE

Design of Constraints in Precision Systems

Background

- History
- Reasons
- Requirements
- Problems

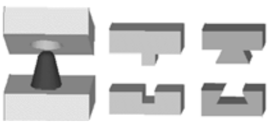
Classes of Constraint

- Kinematic
- Quasi-Kinematic
- Variable Geometry
- Partial/Compliance

Hardware/discussion time

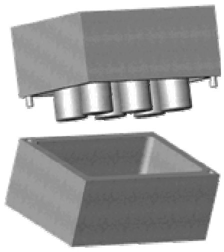
Elastic Averaging will be done next lecture

Common Coupling Methods



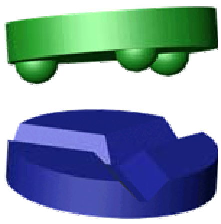
Elastic Averaging

Non-Deterministic



Pinned Joints

No Unique Position

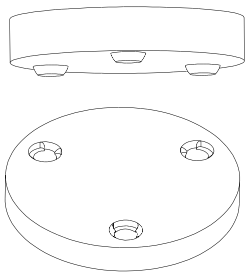


Kinematic Couplings

Kinematic Constraint

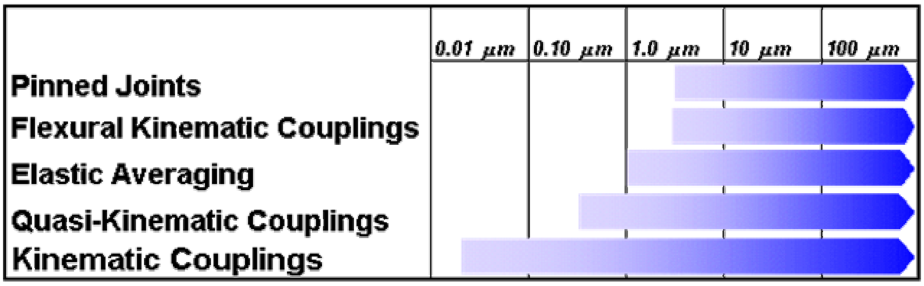
Flexural Kin. Couplings

Kinematic Constraint



Quasi-Kinematic Couplings

Near Kinematic Constraint



Perspective: What the coupling designer faces…

APPLICATION	SYSTEM SIZE	REQ'D PRECISION
Fiber Optics	Meso	Nano
Optical Resonators	Meso	Nano
Large array telescopes	60 ft diam.	Angstrom
Automotive	3 ft	1 micron

Problems due to strain affects

- Thermal Affects — Air, hands, sunlight
- Gravity — Sagging
- Stress Relief — Time variable assemblies
- Loads — Stiffness

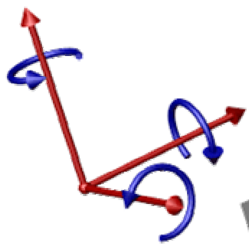
Problems due to sub-optimal designers

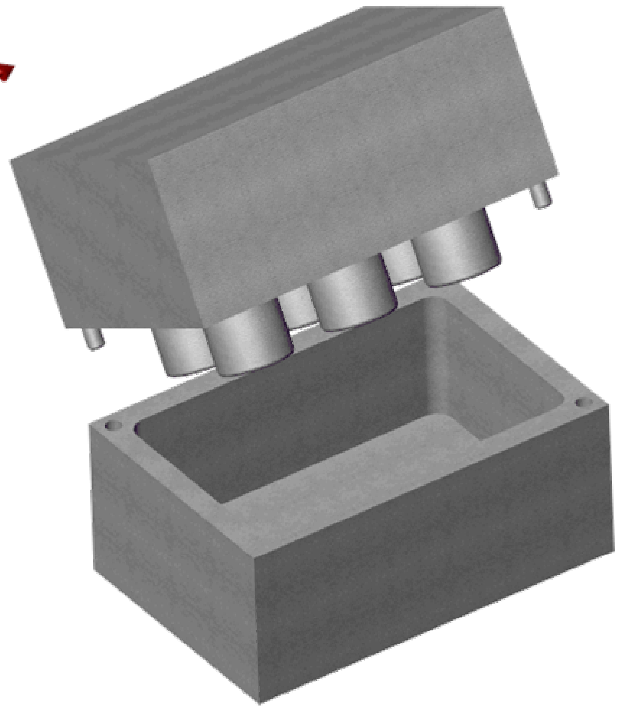
- Competing cost vs performance
- Automotive (no temperature control, large parts, resistance to change)

General Service Requirements & Applications

Ideal couplings:

- Inexpensive
- Accurate & Repeatable
- High Stiffness
- Handle Load Capacity
- Sealing Interfaces
- Well Damping





Example Applications:

- Grinding
- Optic Mounts
- Robotics
- Automotive

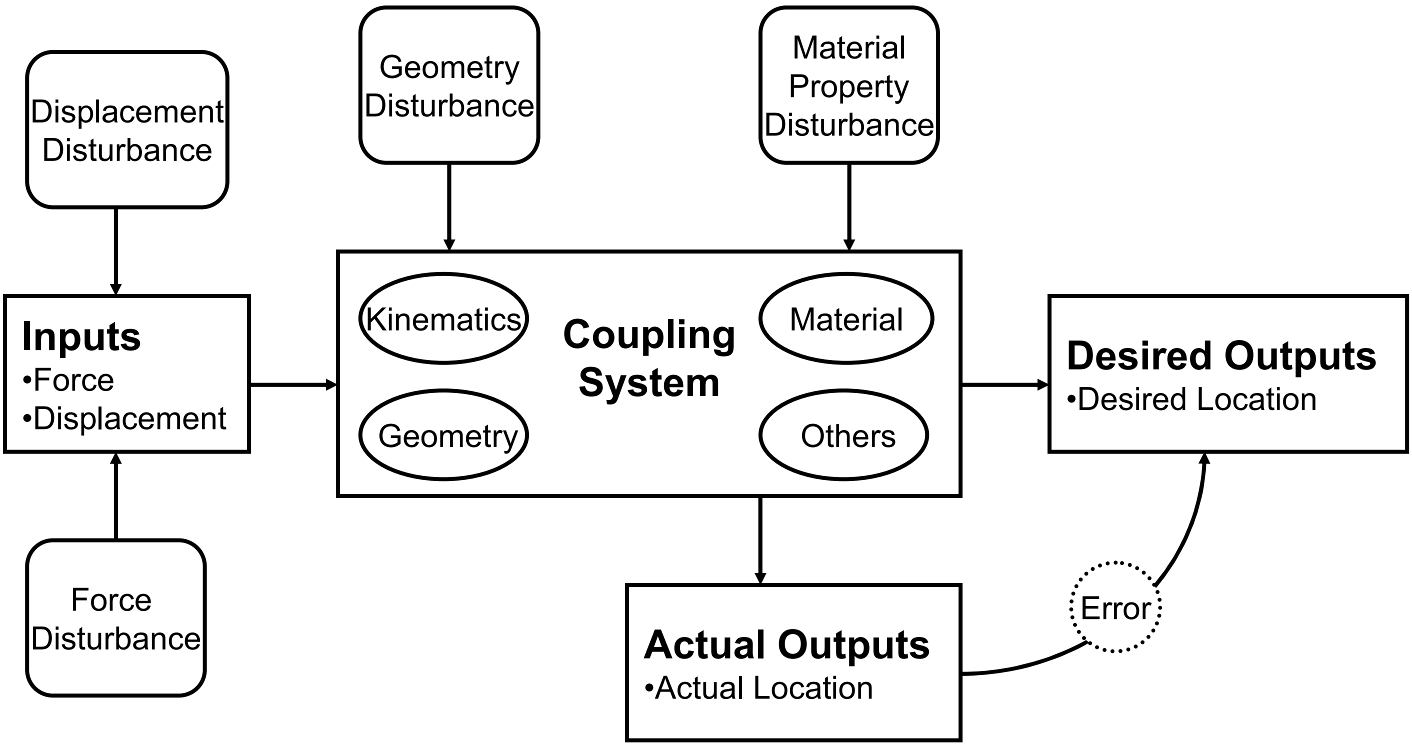
Sensitivity

- What are the sensitive directions?!?!?!?!?

Couplings Are Designed as Systems

You must know what is going on (loads, environment, thermal)!

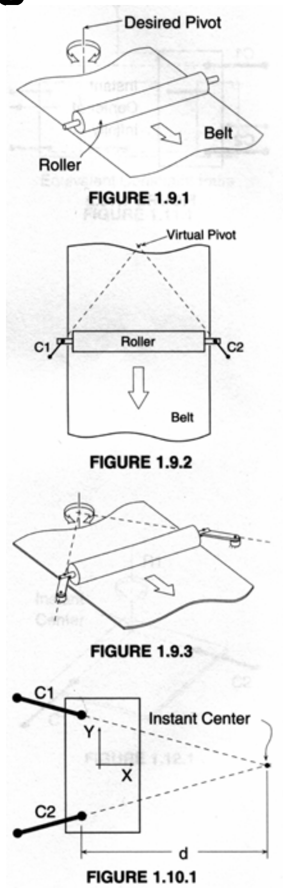
Shoot for determinism or it will "suck to be you"



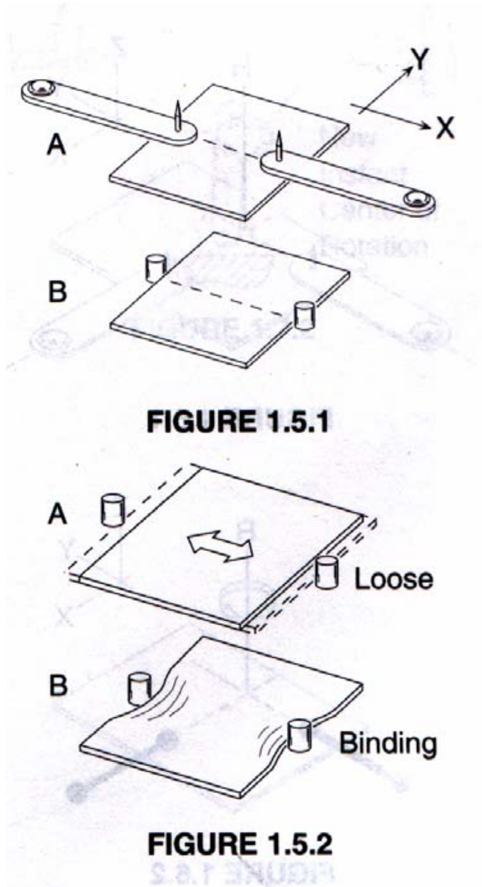
KINEMATIC COUPLINGS

The good, the bad, the ugly….

Defining Constraint



Clever use of constraint



Penalties for over constraint

Exact Constraint (Kinematic) Design

Exact Constraint: Number of constraint points = DOF to be constrained

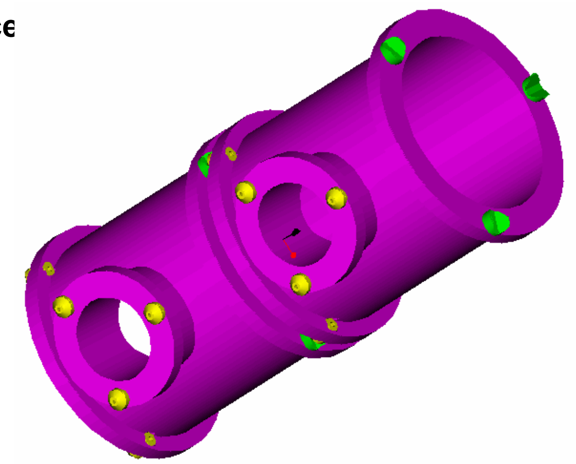
These constraints must be independent!!!

Assuming couplings have rigid bodies, equations can be written to describe movements

Design is deterministic, saves design and development $

KCs provide repeatability on the order of parts' surface finish

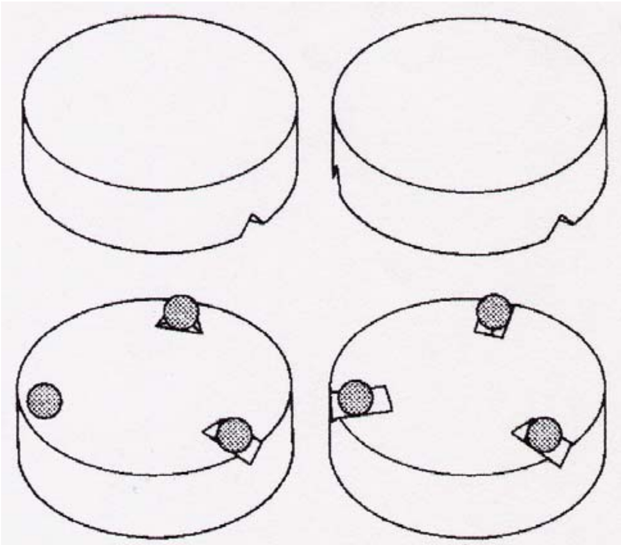
- ¼ micron repeatability is common
- Managing contact stresses are the key to succe



Making Life Easier

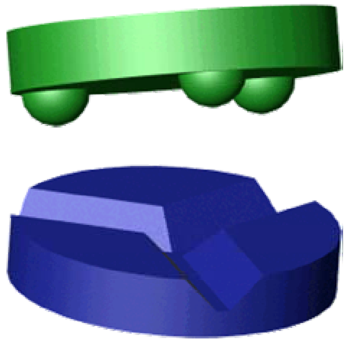
"Kinematic Design", "Exact Constraint Design"…..the issues are:

- **KNOW what is happening in the system**
- **Manage forces and deflections**
- **Minimize stored energy in the coupling**
- **Know when "Kinematic Design" should be used**
- **Know when "Elastic Averaging" should be used (next week)**



Kelvin Clamp Boyes Clamp

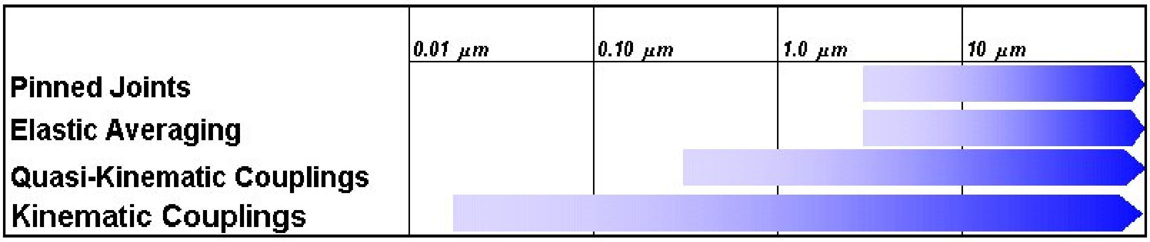
Kinematic couplings



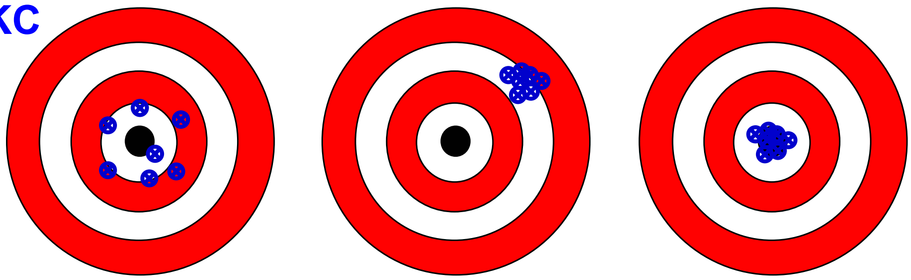
Kinematic Couplings:

- Deterministic Coupling
- # POC = # DOF
- Do Not Allow Sealing Contact
- Excellent Repeatability

Performance



Power of the KC

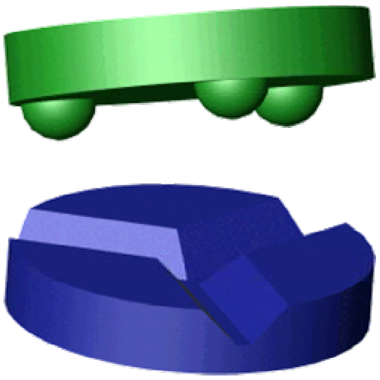


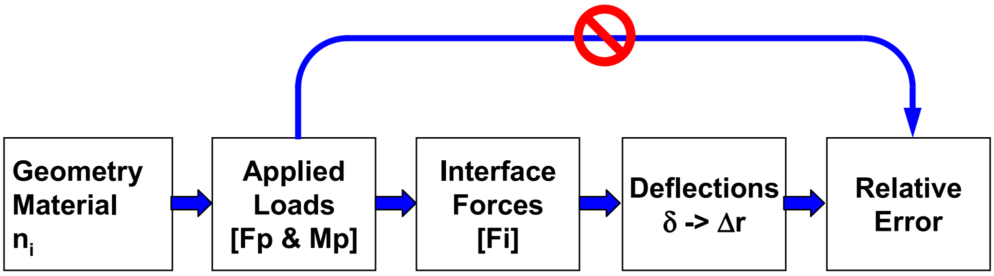
Accuracy

Repeatability

Accuracy & Repeatability

Modeling Kinematic Coupling Error Motions



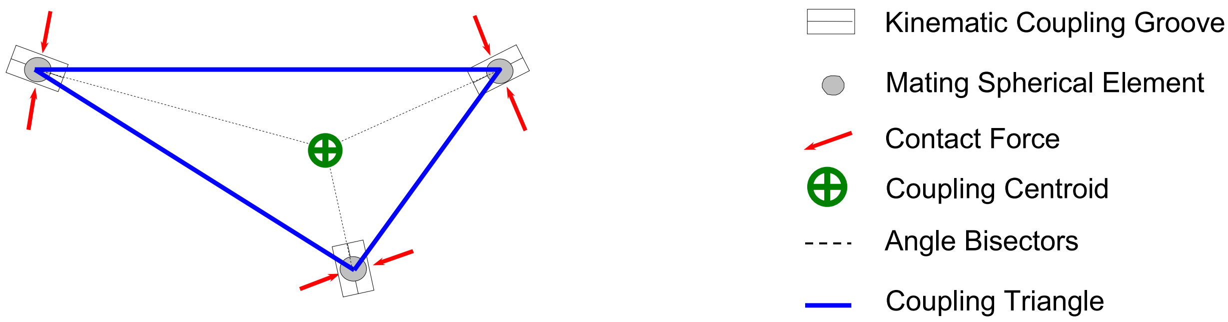


6 Unknown Forces & 6 Equilibrium Equations

$$\Sigma F_i = F_P \qquad \Sigma M_i = M_P$$

Hertzian Point Contact for Local Displacements

$$\delta_i = f(E_B, E_G, \nu_B, \nu_G, R_B, R_G)$$



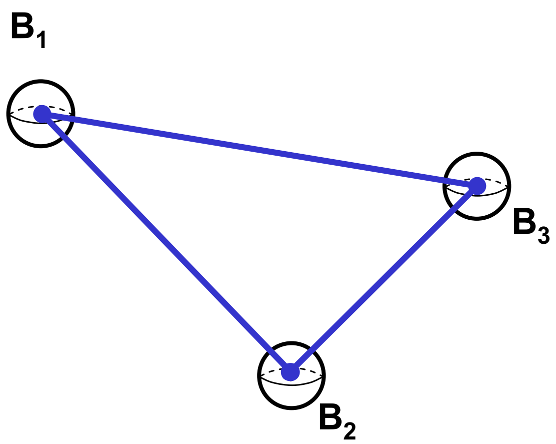
KC Error Motion Analysis

Need δ_x, δ_y, δ_z, ε_x, ε_y , ε_z to predict effect of non-repeatability

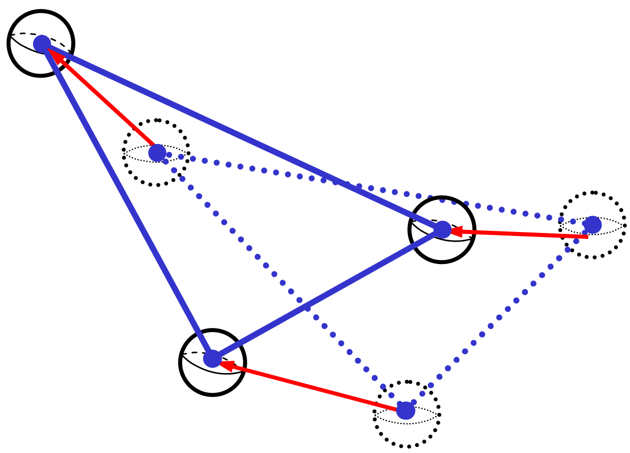
Hertz deflections -> displacements of ball centers

Three ball centers form a plane

Analyze relative position of “before” and “after” planes for error motions



Original Positions

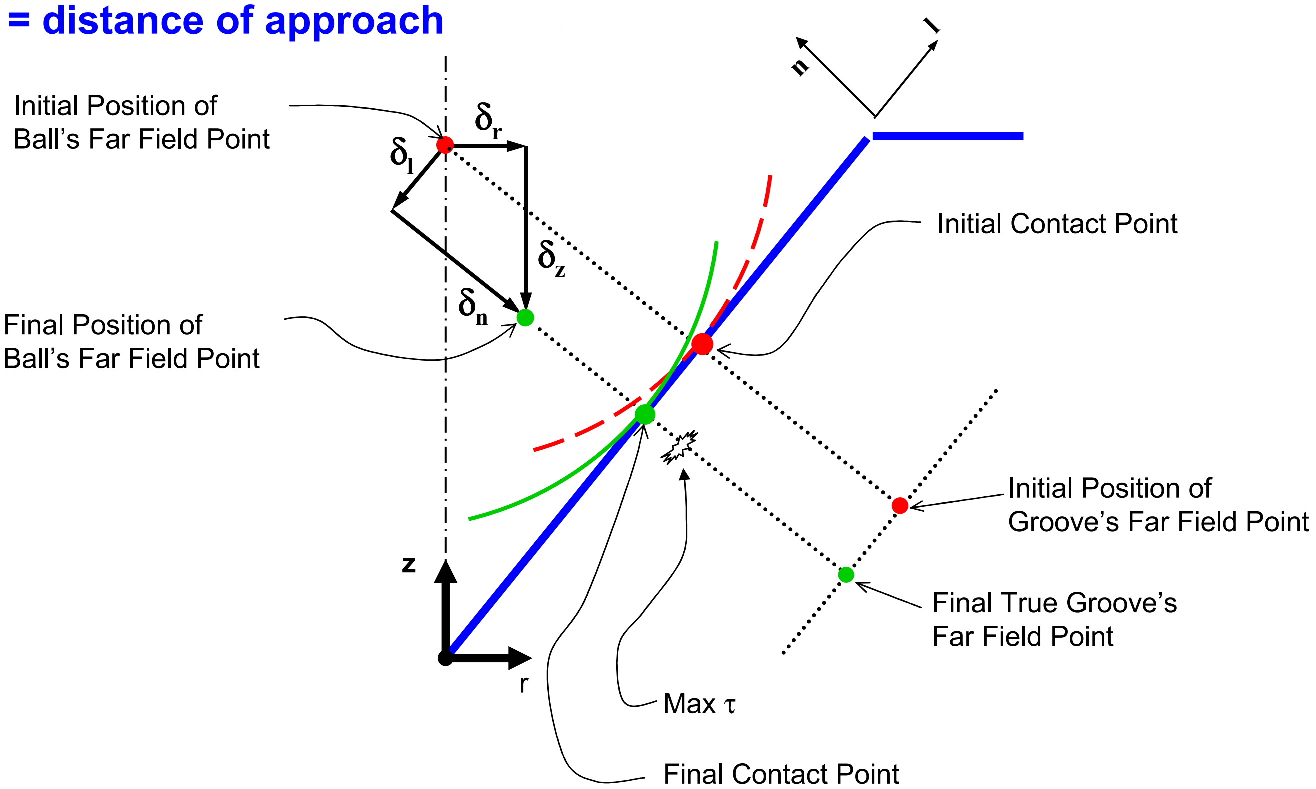


Final Positions

Kinematic Couplings and Distance of Approach

How do we characterize motions of the ball centers?

δ_n = distance of approach



Max shear stress occurs below surface, in the member with larges R

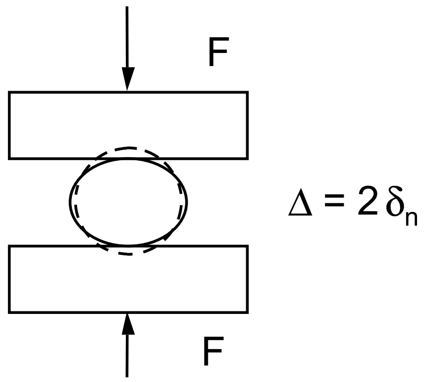
Contact Mechanics – Hertz Contact

Heinrich Hertz – 1st analytic solution for “near” point contact

KC contacts are modeled as Hertz Contacts

Enables us to determine stress and distance of approach, δ_n

Radii	Ronemaj	**1.00E+06**
	Ronemin	**0.06250**
	Rtwomaj	**0.25000**
	Rtwomin	**-0.06500**
Load	Applied load F	**13**
	Phi (degrees)	**0**
	Max contact stress	**10,000**
Modulus	Elastic modulus Eone	**3.00E+07**
	Elastic modulus Etwo	**4.40E+05**
ν Ratio	Poisson's ratio vone	**0.3**
	Poisson's ratio vtwo	**0.3**
	Equivalent modulus Ee	4.77E+05
	Equivalent radius Re	0.2167
Stress	Contact pressure	12,162
	Stress ratio (must be less than 1)	1.22
	Deflection at the one contact interface	
Deflection	Deflection (μunits)	829



Key Hertz Physical Relations

Equivalent radius and modulus:

$$R_e = \frac{1}{\frac{1}{R_{1major}} + \frac{1}{R_{1minor}} + \frac{1}{R_{2major}} + \frac{1}{R_{2minor}}} \qquad E_e = \frac{1}{\frac{1-\eta_1^2}{E_1} + \frac{1-\eta_2^2}{E_2}}$$

cos(θ) function (φ is the angle between the planes of principal curvature of the two bodies)

$$\cos\theta = R_e\left[\left(\frac{1}{R_{1major}} - \frac{1}{R_{1minor}}\right)^2 + \left(\frac{1}{R_{2major}} - \frac{1}{R_{2minor}}\right)^2 + 2\left(\frac{1}{R_{1major}} - \frac{1}{R_{1minor}}\right)\left(\frac{1}{R_{2major}} - \frac{1}{R_{2minor}}\right)\cos 2\phi\right]^{1/2}$$

Solution to elliptic integrals estimated with curve fits

$$\alpha = 1.939e^{-5.26\theta} + 1.78e^{-1.09\theta} + 0.723/\theta + 0.221$$

$$\beta = 35.228e^{-0.98\theta} - 32.424e^{-1.0475\theta} + 1.486\theta - 2.634$$

$$\lambda = -0.214e^{-4.95\theta} - 0.179\theta^2 + 0.555\theta + 0.319$$

Contact Pressure	Distance of Approach	Major Contact Axis	Minor Contact Axis
$q = \frac{3F}{2\pi cd} \approx 1.5\sigma_{tensile\ for\ metals}$	$\delta = \lambda\left(\frac{2F^2}{3R_e E_e^2}\right)^{1/3}$	$c = \alpha\left(\frac{3FR_e}{2E_e}\right)^{1/3}$	$\beta\left(\frac{3FR_e}{2E_e}\right)^{1/3}$

KEY Hertz Relations

Contact Pressure is proportional to:

- Force to the 1/3rd power
- Radius to the –2/3rd power
- Modulus to the 2/3rd power

Distance of approach is proportional to:

- Force to the 2/3rd power
- Radius to the –1/3rd power
- Modulus to the –2/3rd power

Contact ellipse diameter is proportional to:

- Force to the 1/3rd power
- Radius to the 1/3rd power
- Modulus to the –1/3rd power

DO NOT ALLOW THE CONTACT ELLIPSE TO BE WITHIN ONE DIAMETER OF THE EDGE OF A SURFACE!

Calculating Errors Motions in Kinematic Couplings

Motion of ball centers -> Centroid motion in 6 DOF -> Δx, Δy, Δz at X, Y, Z

- **Coupling Centroid Translation Errors**

$$\delta_{\zeta c} = \left(\frac{\delta_{1\zeta}}{L_{1c}} + \frac{\delta_{2\zeta}}{L_{2c}} + \frac{\delta_{3\zeta}}{L_{3c}} \right) \cdot \frac{L_{1c} + L_{2c} + L_{3c}}{3}$$

- **Rotations**

$$\varepsilon_x = \frac{\delta_{z1}}{L_{1,23}} \cdot \cos(\theta_{23}) + \frac{\delta_{z2}}{L_{2,31}} \cdot \cos(\theta_{31}) + \frac{\delta_{z3}}{L_{3,12}} \cdot \cos(\theta_{12})$$

$$\varepsilon_y = \frac{\delta_{z1}}{L_{1,23}} \cdot \sin(\theta_{23}) + \frac{\delta_{z2}}{L_{2,31}} \cdot \sin(\theta_{31}) + \frac{\delta_{z3}}{L_{3,12}} \cdot \sin(\theta_{12})$$

$$\varepsilon_{z1} = \frac{\sqrt{(\alpha_{B1} \cdot \delta_1 + \alpha_{B2} \cdot \delta_2)^2 + (\beta_{B1} \cdot \delta_1 + \beta_{B2} \cdot \delta_2)^2}}{\sqrt{(x_1 - x_c)^2 + (y_1 - y_c)^2}} \cdot \mathrm{SIGN}(\alpha_{B1} \cdot \delta_1 - \alpha_{B2} \cdot \delta_2) \longrightarrow \varepsilon_z = \frac{\varepsilon_{z1} + \varepsilon_{z2} + \varepsilon_{z3}}{3}$$

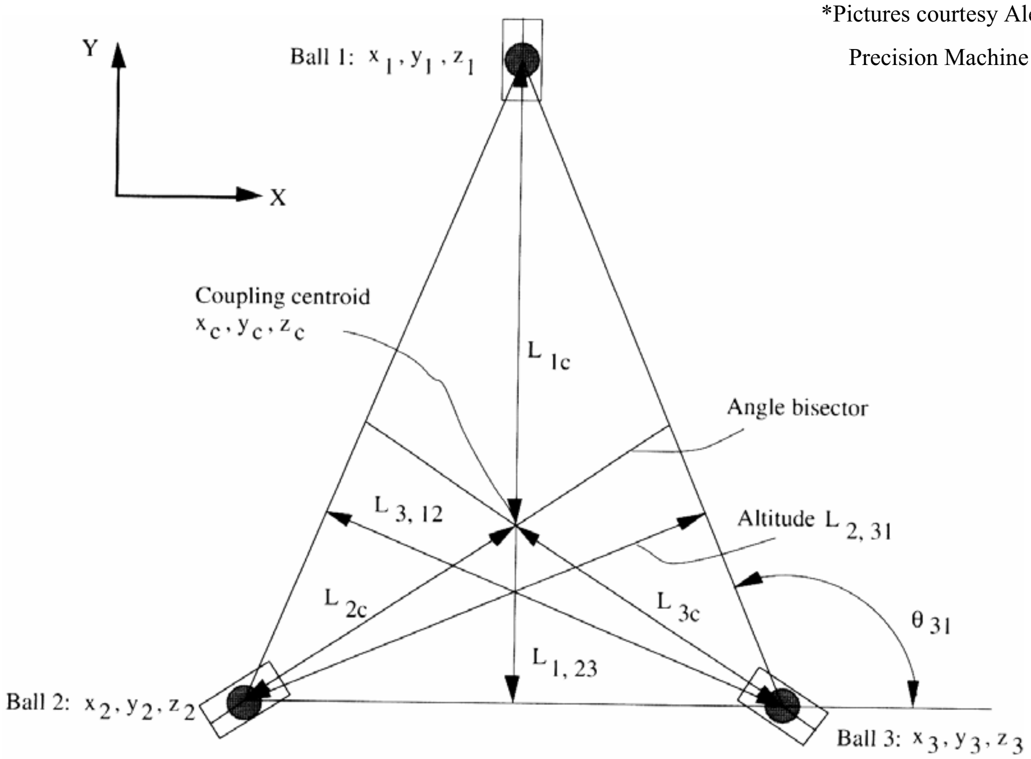
- **Error At X, Y, Z (includes translation and sine errors)**

$$\begin{pmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -\varepsilon_z & \varepsilon_y & \delta_x \\ \varepsilon_z & 1 & -\varepsilon_x & \delta_y \\ -\varepsilon_y & \varepsilon_x & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X - x_c \\ Y - y_c \\ Z - z_c \\ 1 \end{pmatrix} - \begin{pmatrix} X - x_c \\ Y - y_c \\ Z - z_c \\ 1 \end{pmatrix}$$

Kinematic Coupling Centroid Displacement

*Pictures courtesy Alex Slocum

Precision Machine Design



Centroid Displacement: $\delta_{\xi c} = \delta_{1\xi}\dfrac{L_{1c}}{L_{1,23}} + \delta_{2\xi}\dfrac{L_{2c}}{L_{2,31}} + \delta_{3\xi}\dfrac{L_{3c}}{L_{3,12}}$

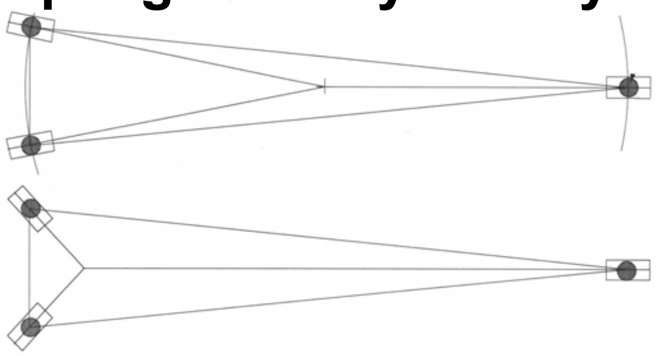
General Design Guidelines

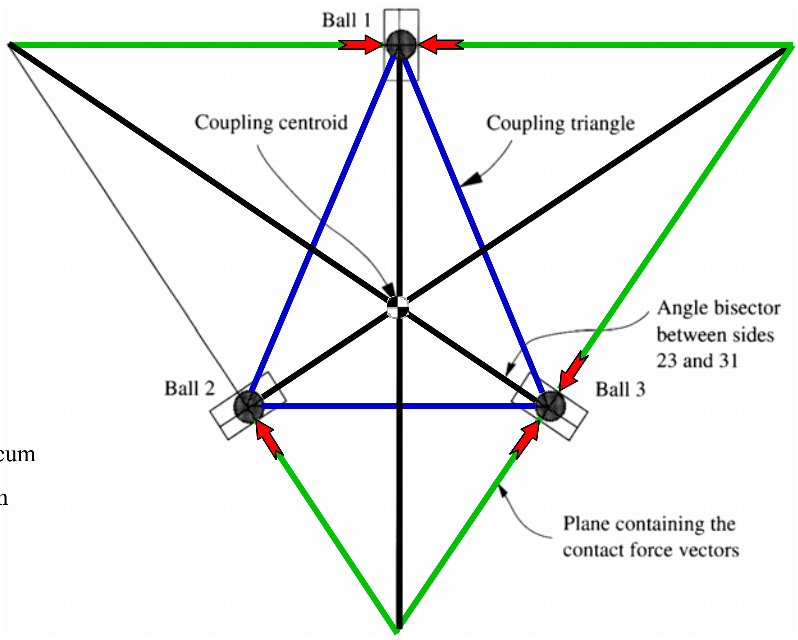
1. Location of the coupling plane is important to avoid sine errors
2. For good stability, normals to planes containing contact for vectors should bisect angles of coupling triangle
3. Coupling triangle centroid lies at center circle that coincides with the three ball centers
4. Coupling centroid is at intersection of angle bisectors
5. These are only coincident for equilateral triangles
6. Mounting the balls at different radii makes crash-proof
7. Non-symmetric grooves make coupling idiot-proof

Kinematic Coupling Stability Theory

Poor Design

Good Design

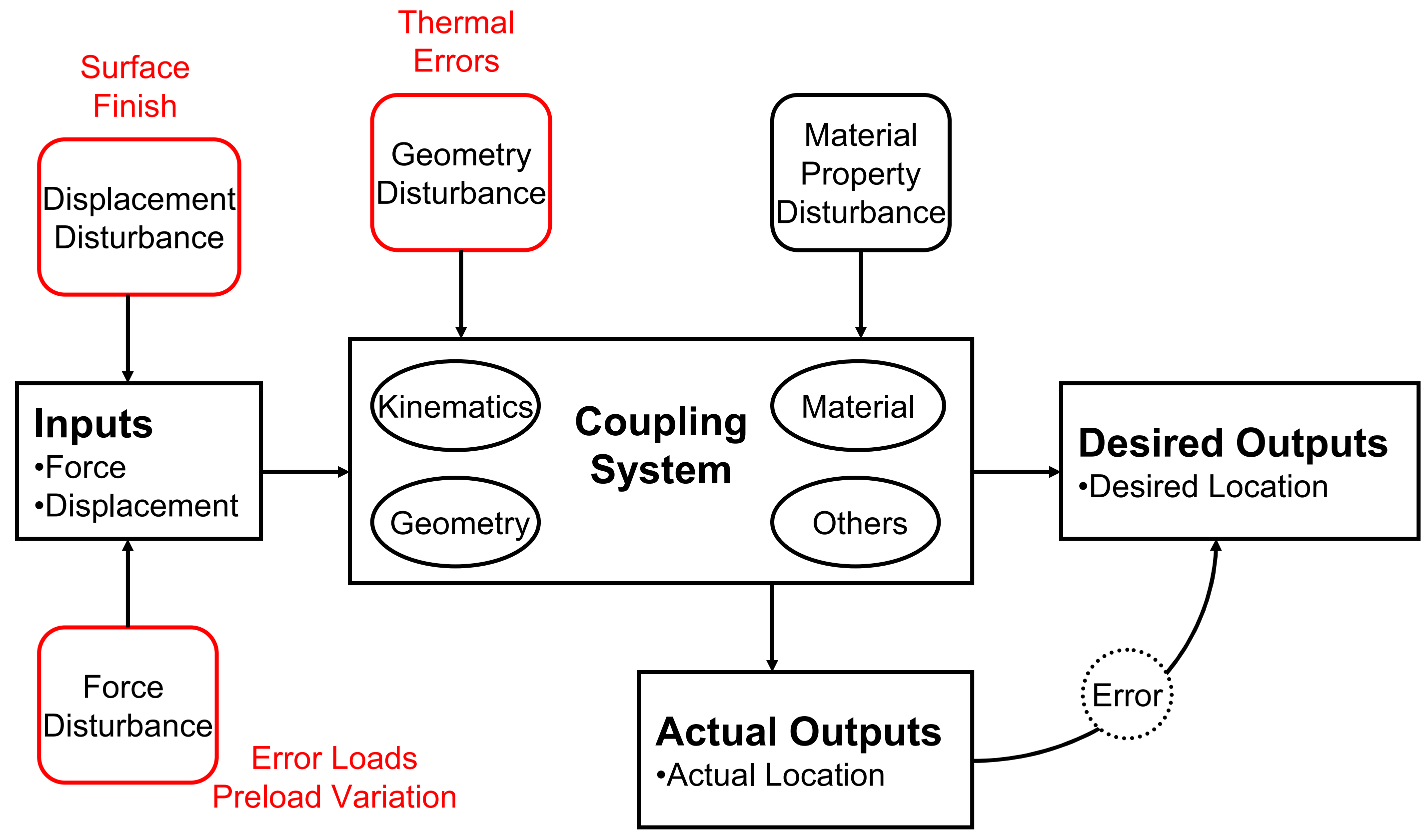




*Pictures courtesy Alex Slocum

Precision Machine Design

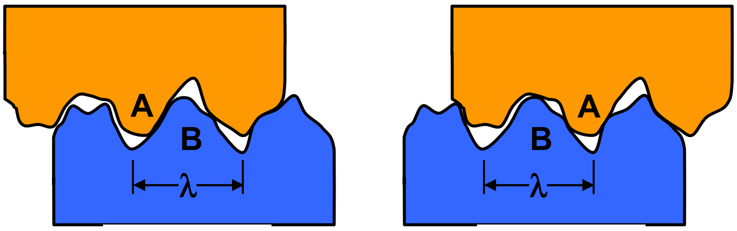
Sources of Errors in KCs



Problems With Physical Contact (and solutions)

Surface topology (finish):

- 50 cycle repeatability ~ 1/3 µm Ra
- Friction depends on surface finish!
- Finish should be a design spec
- Surface may be brinelled if possible

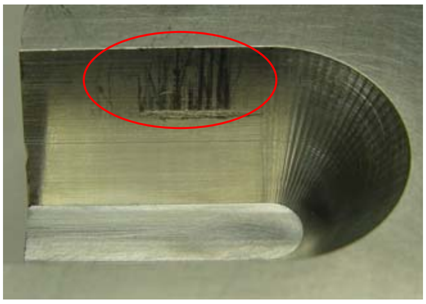


Mate n   Mate n + 1

Wear and Fretting:

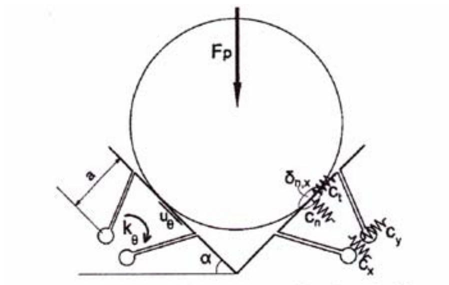
- High stress + sliding = wear
- Metallic surfaces = fretting
- Use ceramics if possible (low µ and high strength)
- Dissimilar metals avoids “snowballing”

Wear on Groove



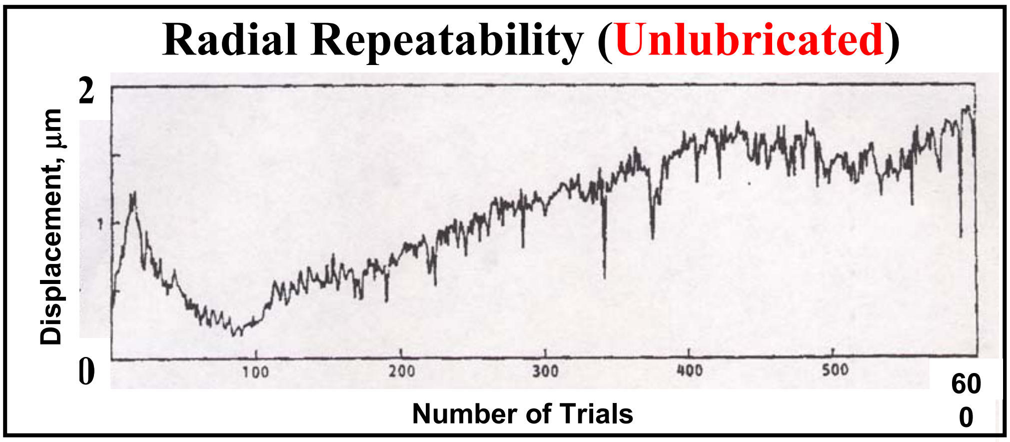
Friction:

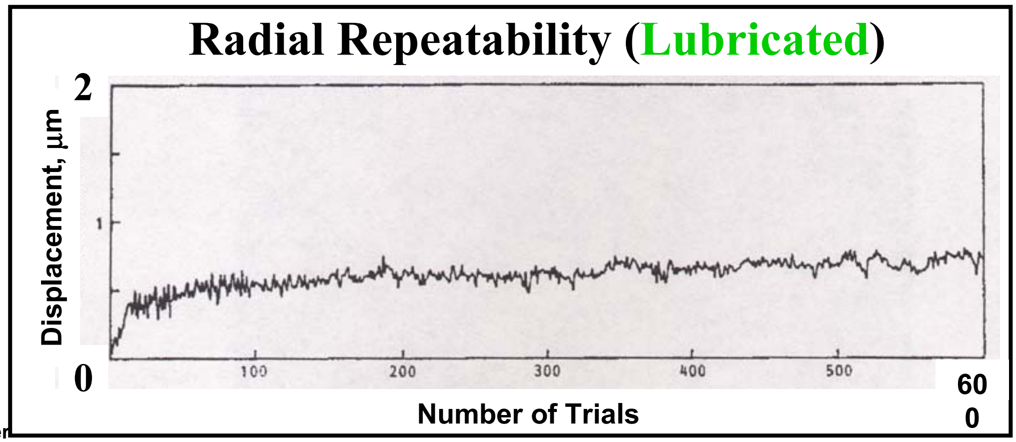
- Friction = Hysteresis, stored energy, overconstraint
- Flexures can help (see right)
- Lubrication (high pressure grease) helps
  - Beware settling time and particles
- Tapping can help if you have the “magic touch”



Ball in V-Groove with Elastic Hinges

Experimental Results –Repeatability & Lubrication





Practical Design of Kinematic Couplings

Design

- Specify surface finish or brinell on contacting surfaces
- Normal to contact forces bisect angles of coupling triangle!!!

Manufacturing & Performance

- Repeatability = f (friction, surface, error loads, preload variation, stiffness)
- Accuracy = f (assembly) unless using and ARKC

Precision Balls (ubiquitous, easy to buy)

- Baltec sells hardened, polished kinematic coupling balls or…..

Grooves (more difficult to make than balls)

- May be integral or inserts. Inserts should be potted with thin layer of epoxy

Materials

- Ceramics = low friction, high stiffness, and small contact points
- If using metals, harden
- Use dissimilar materials for ball and groove

Preparation and Assembly

- Clean with oil mist
- Lubricate grooves if needed

Example: Servo-Controlled Kinematic Couplings

- Location & automatic leveling of precision electronic test equipment
- Teradyne has shipped over 500 systems

Example: Canoe-Ball Kinematic Interface Element

The "Canoe Ball" shape is the secret to a highly repeatable design

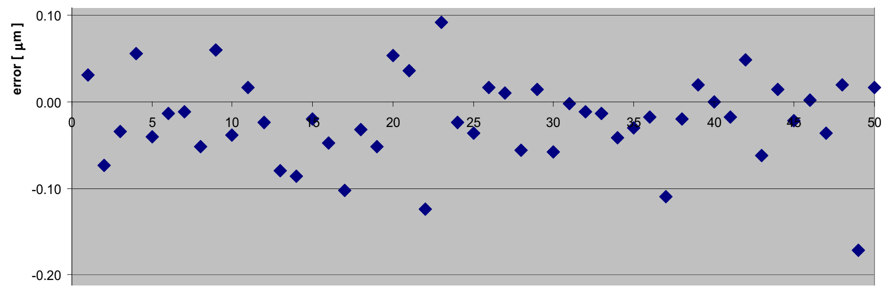
- **It acts like a ball 1 meter in diameter**
- **It has 100 times the stiffness and load capacity of a *normal* 1" ball**

Large, shallow Hertzian zone is *very (i.e. < 0.1 microns)* repeatable

Canoe Ball Repeatability Measurements

Test Setup

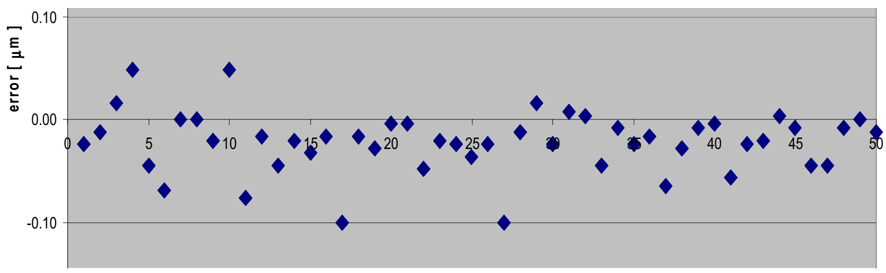
Coupling



0.1 µm

-0.2 µm

Meas. system



0.1 µm

-0.1 µm

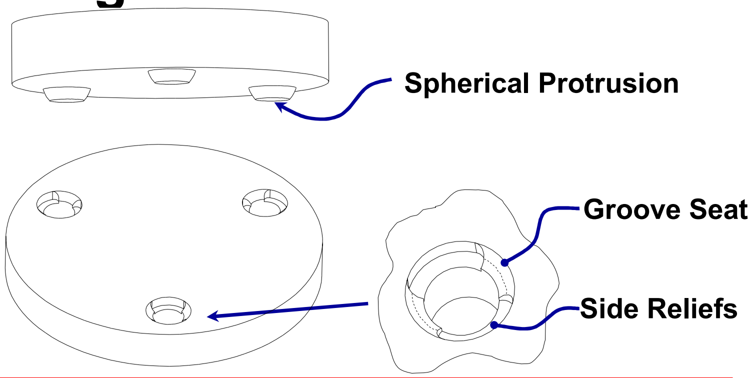
QKCs

Why do it the easy way when you can do it the lazy way?

Quasi-Kinematic (QKC) Alignment

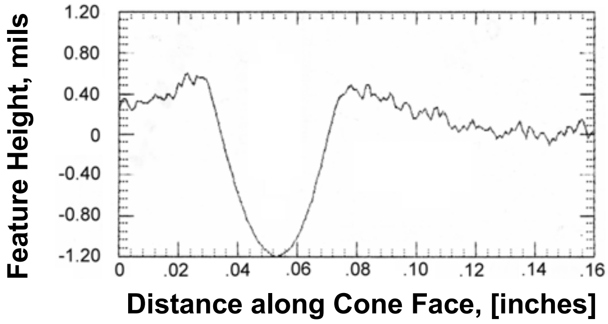
QKC characteristics:

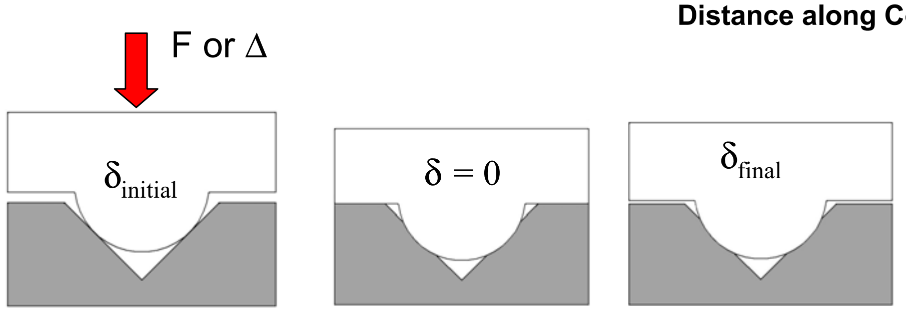
- Arc contact
- Submicron repeatability
- Stiff, sealing contact
- Less expensive than KCs
- Easier to make than KCs



QKC Function:

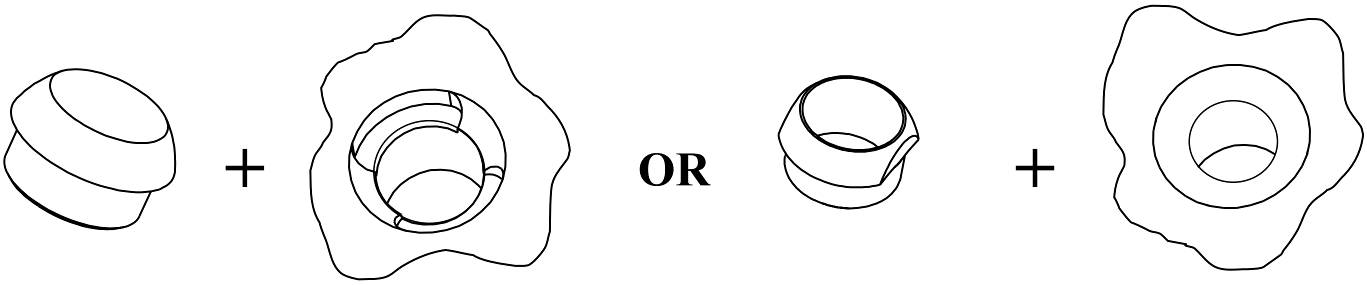
- Ball & groove comply
- Burnish surface irregularities
- Elastic recovery restores gap





Details of QKC Element Geometry

PAIRS OF QKC ELEMENTS

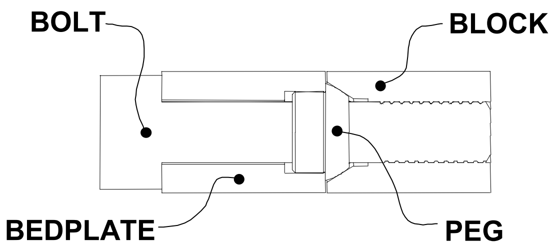


TYPE 2 GROOVE MFG.

+ =

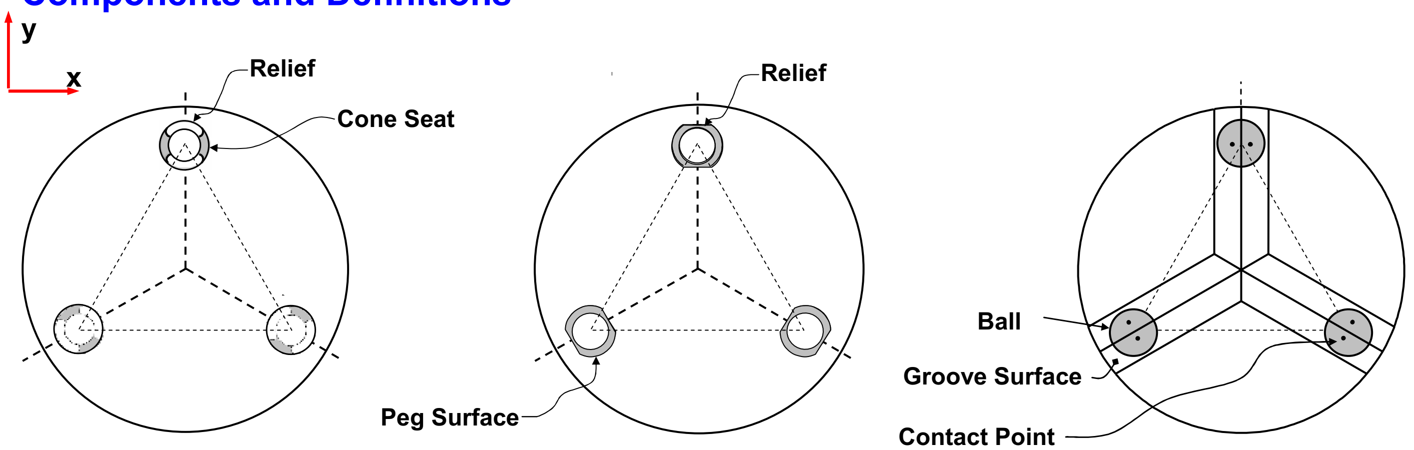
CAST FORM TOOL FINISHED

ASSEMBLED JOINT

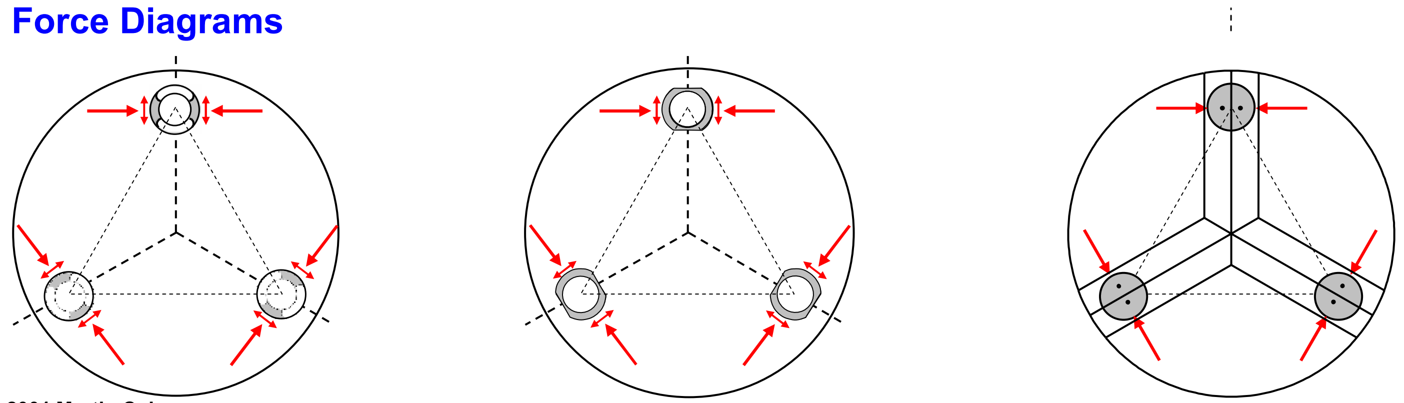


QKC Methods vs Kinematic Method

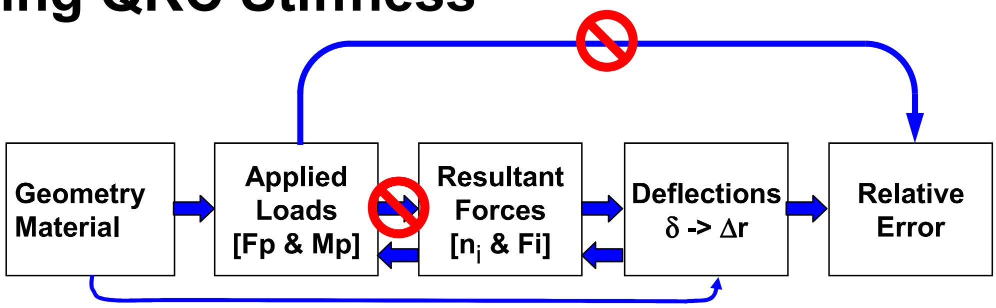
Components and Definitions



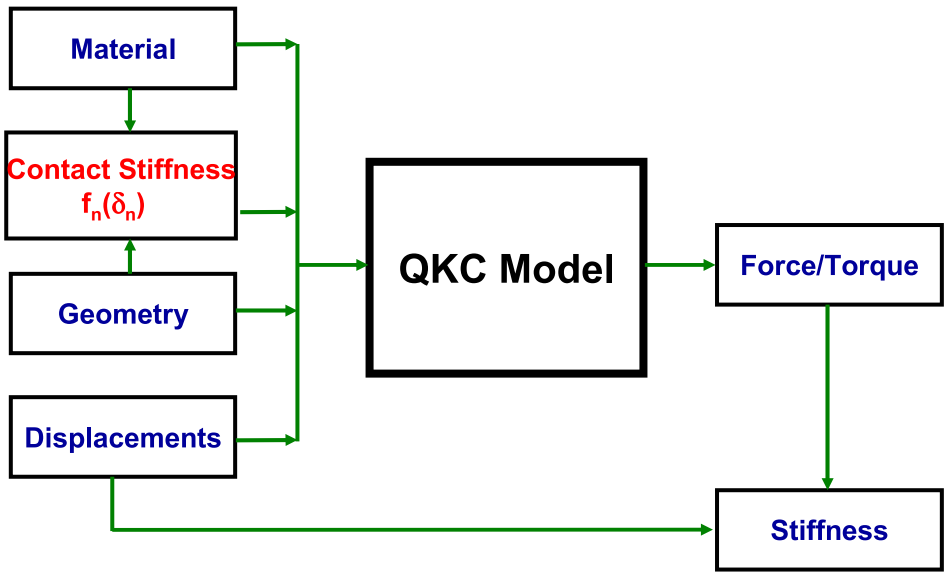
Force Diagrams



Modeling QKC Stiffness



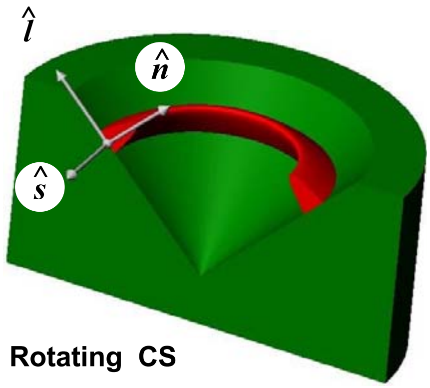
24 Unknowns & 7 Eqxns.



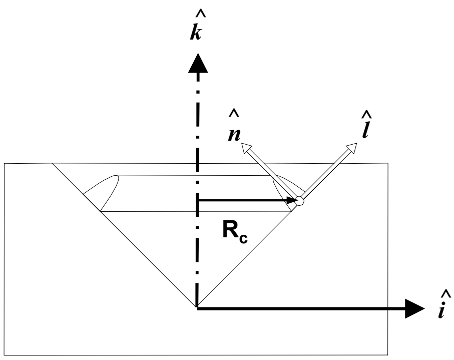
Contact Mechanics

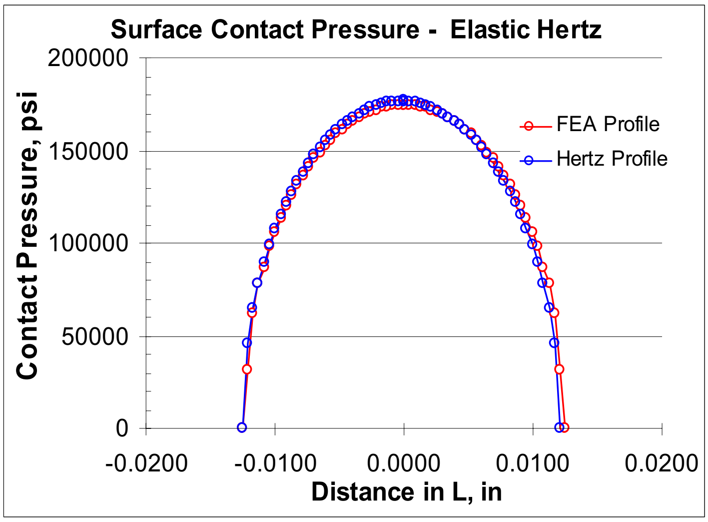
MECHANICS:

- **Use Rotating Coordinate System**
- **Assume Sinusoidal Normal Distance of Approach**
- **Obtain Contact Stress Profile as Function of Above**
- **Integrate Stress Profile in Rotating CS thru contact**



Rotating CS

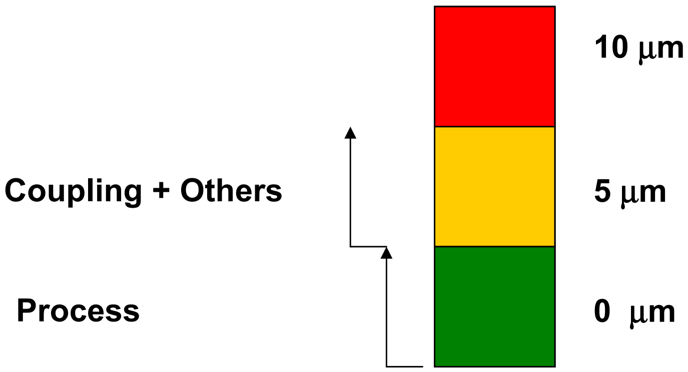




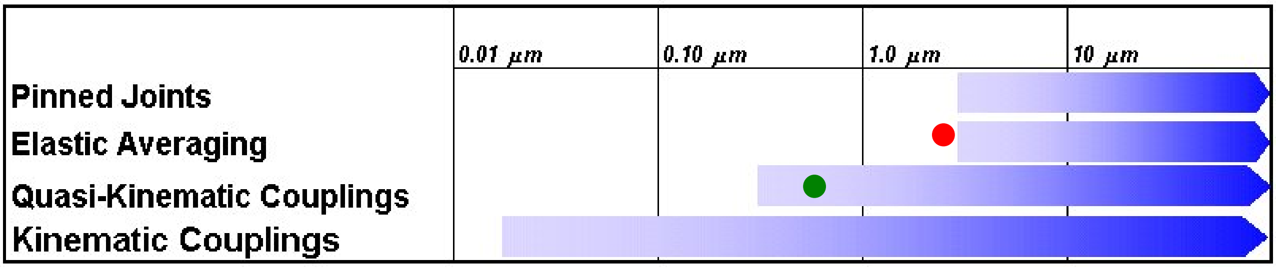
Example: Duratec Assembly

Characteristics:

- Ford 2.5 & 3.0 L V6
- > 300,000 Units / Year
- Cycle Time: < 30 s



Rough Error Budget

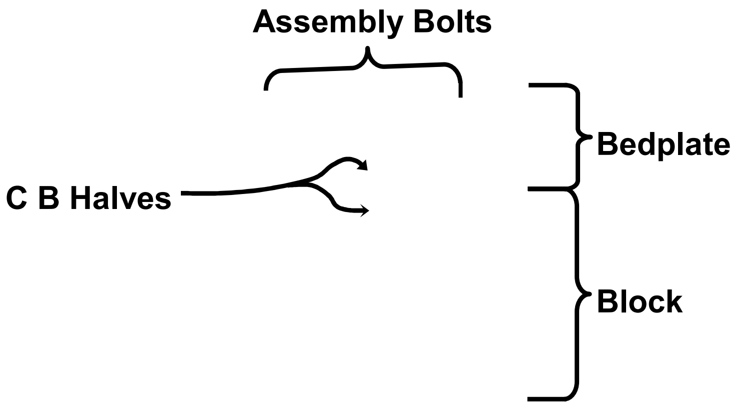


Example: Assembly of Duratec Block and Bedplate

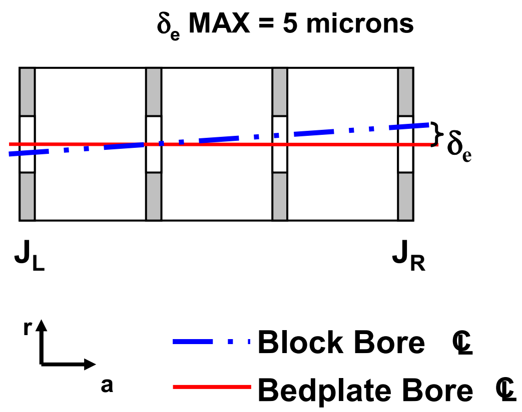
COMPONENTS

Block | Bedplate

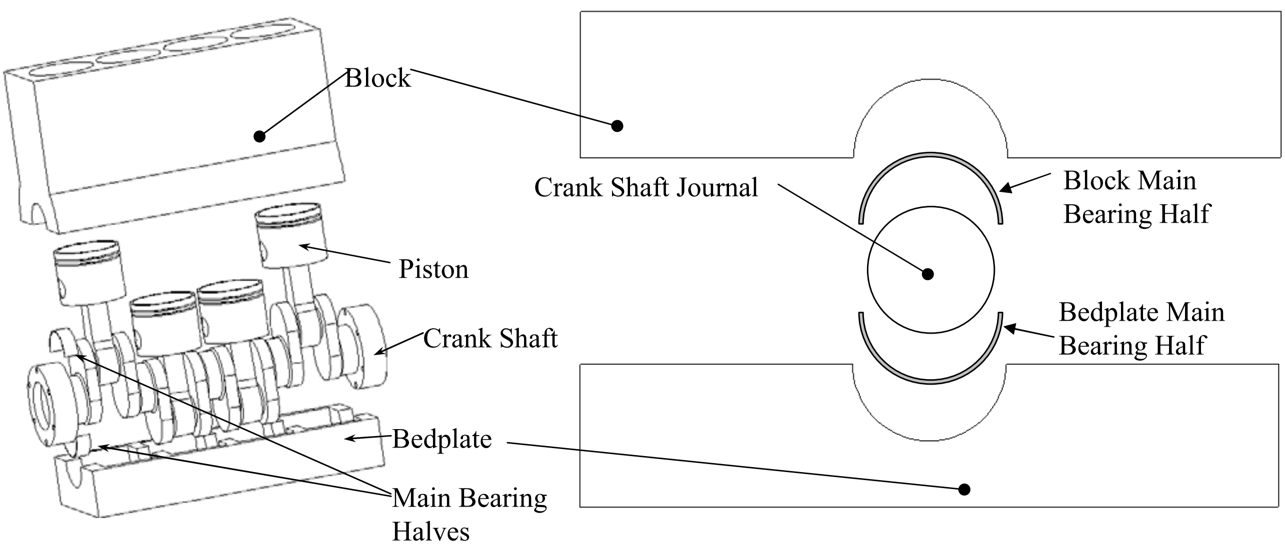
ASSEMBLY



ERROR



Bearing Assemblies in Engines



Results of Duratec QKC Research

MANUFACTURING:

Engine Manufacturing Process With Pinned Joint

Op. #10		Op. #30		Op. #50		Op. #100
• Mill Joint Face • Drill/Bore **16 Holes** • Drill Bolt Holes	→	• Drill Bolt Holes	→	• Press in **8 Dowels** • Assemble • Load Bolts • Torque Bolts	→	• Semi-finish crank bores • Finish crank bores

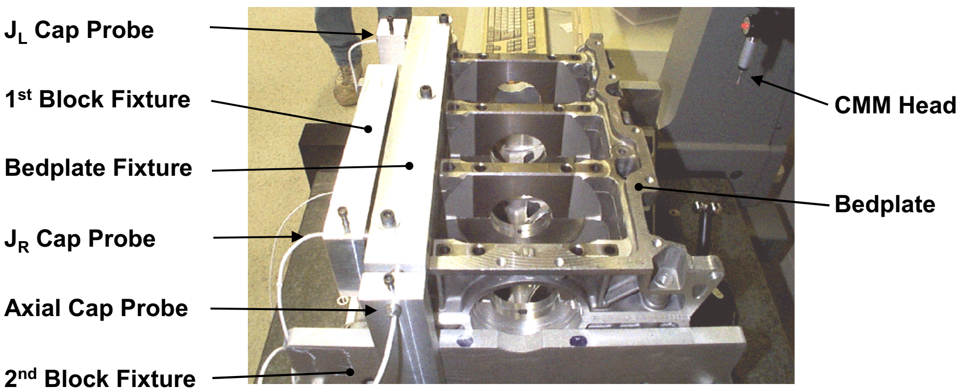
Modified Engine Manufacturing Process Using Kinni-Mate Coupling

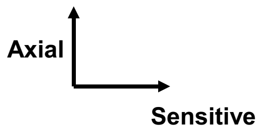
Op. #10		Op. #30		Op. #50		Op. #100
• Mill Joint Face • Drill/Bore **3 Peg** Holes • Drill Bolt Holes & Form **3 Conical Grooves**	→	• Drill Bolt Holes	→	• Press **3 Pegs in BP** • Assemble • Load Bolts • Torque Bolts	→	• Semi-finish crank bores • Finish crank bores

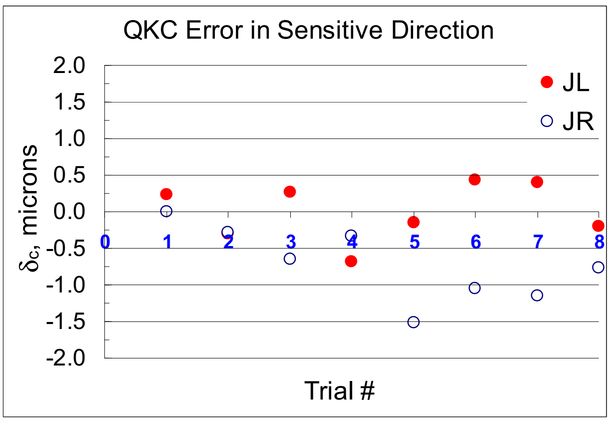
DESIGN:

ITEM	QKC	Pinned Joints
# Precision Pieces	3	8
# Precision Features	3	16
Feature Placement Tolerance	+/- 0.08mm	+/- 0.04mm
Average Centerline Repeatability	0.65 µm	4.85 µm
Normalized $/Engine	0.64	1

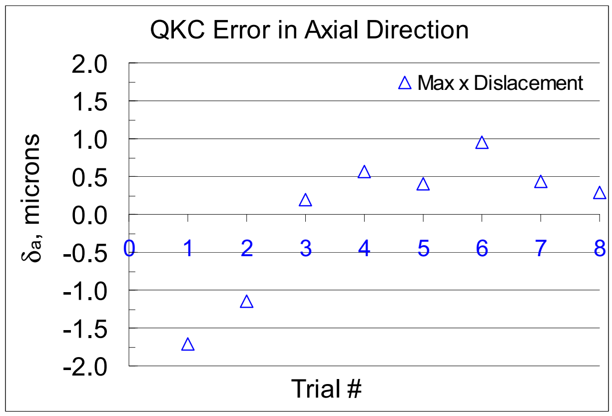
Engine Assembly Performance







$({}^{Range}/_{2})|_{AVG} = 0.65\ \mu m$



$({}^{Range}/_{2}) = 1.35\ \mu m$