TYPES OF UNCERTAINTY

Aleatory, due to the variation in outcomes of random trials. Is from "Alea jacta est," or "the die are cast." Attributed to Julius Ceasar upon crossing the Rubicon, thereby invading the territory of Rome when he siezed power as dictator.

Epistimic, due to uncertainty in our state of knowledge about phenomena and data. Is from Greek term for knowledge, "epistemikos." Is the root of epistimic and epistemological.

Consider events

$\Theta = \theta_i$, $E = e_j$.

$$P\left[\Theta = \theta_i, E = e_j\right] = P\left(\Theta = \theta_i | E = e_j\right) P\left(E = e_j\right) = P\left(E = e_j | \Theta = \theta_i\right) P\left(\Theta = \theta_i\right)$$

$$\Rightarrow P\left(\Theta = \theta_i | E = e_j\right) = \frac{P\left(E = e_j | \Theta = \theta_i\right) P\left(\Theta = \theta_i\right)}{P\left(E = e_j\right)}$$

OR

$$L'\left(\Theta = \theta_i | E = e_j\right) = \frac{P\left(E = e_j | \Theta = \theta_i\right) L\left(\Theta = \theta_i\right)}{P\left(E = e_j\right)}$$

Bayes' Theorem

Now $E = e_j$ is possible for various values of θ_i, as

$$P\left[\left(E = e_j\right)\right] = \sum_{i=1}^{n} P\left(E = e_j \middle| \Theta = \theta_i\right) P\left(\Theta = \theta_i\right)$$

$$\boxed{L'\left(\Theta = \theta_i \middle| E = e_j\right) = \frac{P\left(E = e_j \middle| \Theta = \theta_i\right) L\left(\Theta = \theta_i\right)}{\sum\limits_{i=1} P\left(E = e_j \middle| \Theta = \theta_i\right) P\left(\Theta = \theta_i\right)}}$$

Let the evidence, E, following a random trial be equal to e_i. We wish to know the change in our knowledge of $P(\Theta)$ due to new evidence, $P(\Theta_i = \theta_i | E = e)$.

Consider, prior to the trial, that Θ may take several values, as $\Theta = [\theta_1, \theta_2, ..., \theta_n]$. Also consider that the outcome of a random trial, E, is governed by a “model of the world,” where Θ is a model parameter that takes a specific value, Θ_i.

BAYESIAN UPDATING

Which value of Θ actually obtains is unknown, but we (an observer) will have a "model of belief" regarding the likelihood of the alternative possible values, $L(\Theta)$, where

$$L(\Theta = \theta_i) = \Pr(\Theta = \theta_i).$$

Let the alternative possible outcomes from the random trial be designated $E = [e_1, e_2, ..., e_k]$.

Let the specific outcome of the next random trial be e_i, i.e., $E = e_i$ is the "evidence" of the next trial.

Then, we wish to know the effect of this result upon our "model of belief," concerning the likelihood of the possible values of Θ, $L'(\Theta)$.