Methods for Project Evaluation

March 8, 2004

Alternative Methods

- Present worth (PW) method
- Future worth (FW) method
- Annual worth (AW) method
- Benefit-cost ratio (BC) method
- Internal rate of return (IRR) method

Assumptions

- Future cash flows are known with certainty
- Analysis is in constant dollars
- Cost of capital is known
- Capital is always available for profitable projects (i.e, access to capital is not restricted.)

PW Method: $NPV = \sum_{n=1}^{N} \frac{r_n - c_n}{(1+i)^n}$

Decision criterion: Accept if NPV>0; reject if NPV<0

FW Method: $FV = \sum_{n=0}^{N} (r_n - c_n)(1+i)^{N-n}$

Decision criterion: Accept if FV>0, etc.

Example of PW Method: Pricing a Bond

- At what price should a buyer purchase a 10-year bond paying 6% per year (payable semi-annually) that is redeemable at par value if the buyer is seeking a 10% per year yield? The face value of the bond is $1000.

  N = 10 x 2 = 20 periods

  r = 6%/2 = 3% per period

  $i = [1.1^{1/2}-1]100 = 4.9\%$ per compounding period

  C = Z = $1000

$$V^{(N)} = \$1000(P/F, 0.049, 20) + \$1000(0.03)(P/A, 0.049, 20)$$
$$= 384.1 + 377.06 = \mathbf{\$761.16}$$

Note: The yield typically increases for longer-term bonds.

Example: Influence of interest rates on bond prices

- A 10-year U.S. treasury bond that matures in eight years has a face value of $10,000. The bond pays 8% per year (payable quarterly). A prospective buyer of the bond wants to earn 10% per year on her investment (compounded quarterly) because interest rates have risen since the bond was issued. How much should the buyer pay for the bond?

$$V^{(N)} = \$10,000(0.02)(P/A, 0.025, 32) + \$10,000(P/F, 0.025, 32)$$
$$= \$8,907$$

. I.e., an increase in interest rates causes bond prices to decline.

Example: Pricing stock

- Stock in a company represents a share of ownership, as opposed to a bond, which is essentially a promissory note.
- Common stock is more difficult to value than bonds because dividends and prices of common stocks are not constant; investors hope that they will increase over time.
- **If** reliable forecasts of future earnings, dividends, and stock prices could be made, stock valuation would result from discounting the forecast cash flow.

Example (from Riggs and West):

An investor is investigating the stock performance of two companies: A and B. Company A has consistently paid dividends that increase 10 cents per year while the selling price of the stock has averaged a 2% annual rise. Company B is a fast growing new company that has paid no dividends because all earnings are retained for expansion, but its market price is expected to increase by $10 per year. Current data about the two companies are summarized below:

	Company A	Company B
Dividend	$2.25 (10 cent/yr increase)	0 (2% of market price after 5 years)
Market Price	$28 (2% annual increase)	$65 ($10/yr increase)
Risk-adjusted discount rate for stock valuation	9%	12%

Disregarding tax effects and brokerage commissions to buy or sell, which stock is more attractively priced?

Annual Worth (AW) Method

Example: An investment company is considering building a 25-unit apartment complex in a growing town. Because of the long-term growth potential of the town, it is felt that the company could average 90% of full occupancy for the complex each year. If the following items are reasonably accurate estimates, use the AW method to determine the minimum monthly rent that should be charged if a 12% rate of return per year is desired.

Land investment cost = $50,000
Building investment cost = $225,000
Study period, N = 20 years
Rent per unit per month = ???
Upkeep expense per unit per month = $35
Property taxes and insurance per year = 10% of total initial investment
Assume: Land cost can be recovered at the end of the 20 year period

Solution: First determine the equivalent AW of all costs at an interest rate of 12%/yr. To earn 12% on this project, the annual rental income must equal the AW of the costs:

Initial investment cost = $50,000 + $225,000
Taxes and insurance/yr = 0.1 x 275000 = $27,500
Upkeep/yr = $35 (12 x 25)(0.9) = $9450
Annual worth of capital costs = $275,000 (A/P,0.12,20) - $50,000 (A/F, 0.12, 20)
= $36,123

Equivalent annual worth of costs = -$27000 - $9450 - $36123 = -$73073

Therefore, the minimum annual rental required equals $73,073 to achieve a 12% rate of return, and with annual compounding, the monthly rental amount is given by:

$$\frac{73{,}073}{(12)(25)(0.9)} = \$270.64$$

Internal Rate of Return (IRR) Method

For a project with net cash flows, F_j the IRR, i*, is given by

$$PV\left(i^*\right) = \sum_{j=0}^{N} \frac{F_j}{\left(1+i^*\right)^j} = 0$$

Decision criterion:

If the minimum required rate of return < i^*, accept the project.
If the minimum required rate of return > i^*, reject the project.

IRR Method (contd)

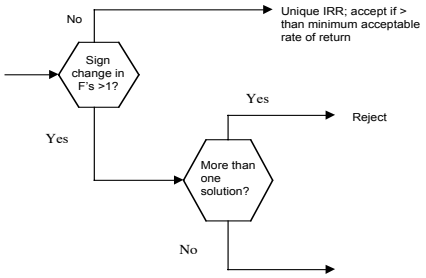
The equation for the IRR is an N^{th} order polynomial in i*. There will in general be more than one root. If more than one of the roots is real and positive, how do we interpret the results?

Question: When is there a unique solution to the IRR problem?

> Descartes' Rule of Signs:
> For an N-th degree polynomial with real coefficients, the number of real, positive roots is never greater than the number of sign changes in the sequence of coefficients.

If we write 1/(1+i*) = X, we can rewrite the IRR equation as

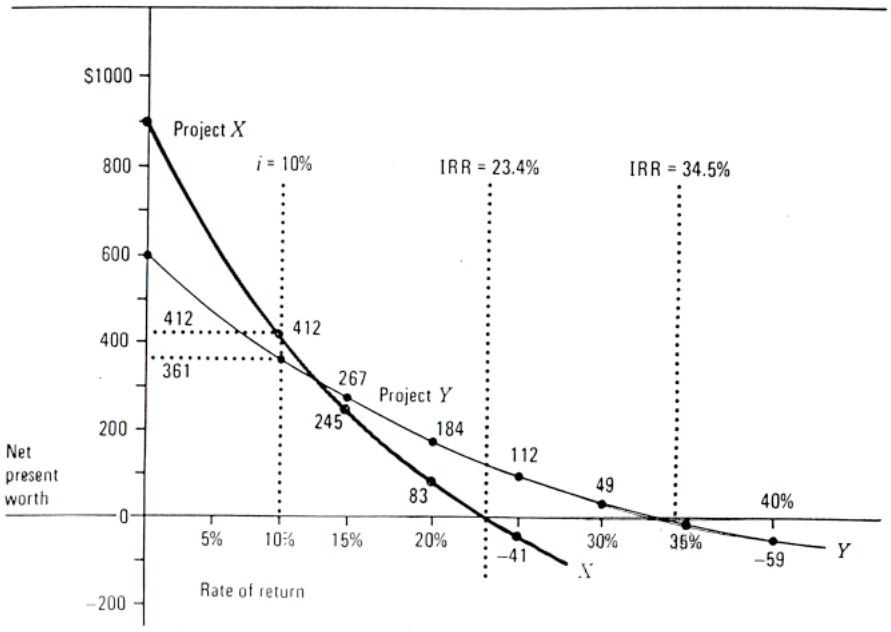
$$F_0 + F_1X + F_2X^2 + \ldots\ldots + F_N X^N = 0$$



The Project Balance, PB_n
(the amount of money committed to a project at time n)

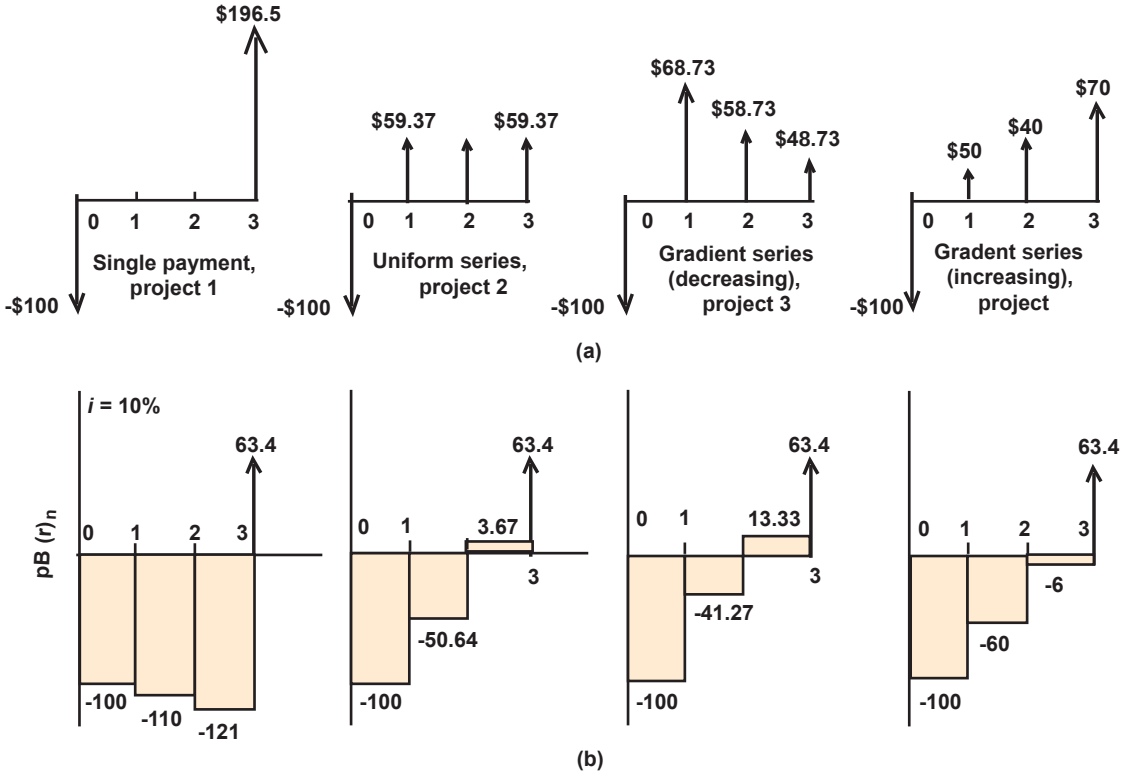
<u>An important distinction</u>:

Projects for which $PB(i^*)_n \neq 0$ for all $n < N$	"PURE INVESTMENT" PROJECTS
Projects for which $PB(i^*)_n > 0$ for some n	"MIXED INVESTMENT" PROJECTS



Investment flexibility as a decision criterion

Nuclear Energy Economics and
Policy Analysis



Project balance for four cash flow patterns.

Summary

1. The PV, FW, and AW criteria always yield the same decision for a project

2. Only for pure investment projects is there a true IRR for the project.

3. For pure investments, the IRR and PV criteria yield identical acceptance/rejection decisions.

4. For mixed investments, the return on invested capital varies with the external cost of capital, and the IRR criterion isn't meaningful. (The phenomenon of multiple IRRs can occur only with mixed investments, but even if there is only a single positive solution, it doesn't necessarily provide useful information.)

5. The aggregate B/C ratio criterion will always agree with the PV criterion.

6. The payback period is not an acceptable criterion taken on its own. In general it will not agree with the PV criterion. However, it may serve a useful purpose as a supplementary consideration.