8.01 Exam #3 Solutions

11.21.03

Name: _____

Problem 1: Collision (15 Points)

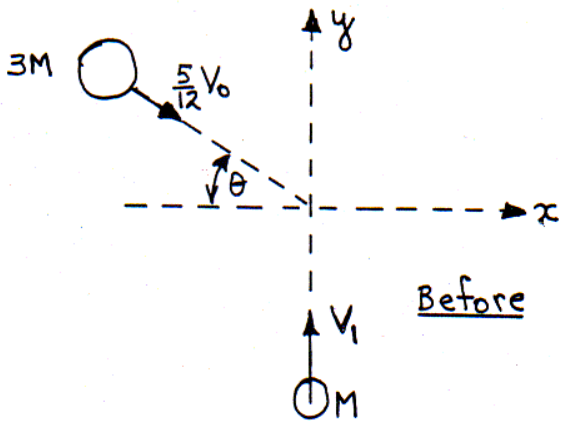
Two masses M and 3M collide on a horizontal frictionless surface as shown. Before the collision the mass M has a velocity V_1 in the y-direction. The mass 3M has a velocity $\frac{5}{12}V_0$ making an angle θ to the x-axis as shown. Assume $\sin\theta = 3/5$ and $\cos\theta = 4/5$. After the collision the mass 3M is at rest. The mass M moves along the x-axis with the velocity V_1'. Neglect gravity. Give all your answers to parts b), c), and d) in terms of M and V_0. Be careful do not confuse your symbols.

a) What are the x and y-components of the net linear momentum before the collision in terms of M, V_1, V_0 and θ?

b) What is the speed V_1 of the mass M before the collision?

c) What is the speed V_1' of the mass M after the collision?

d) What is the velocity of the center-of-mass?



a) $P_x^{tot} = 3M\frac{5}{12}V_0\cos\theta$

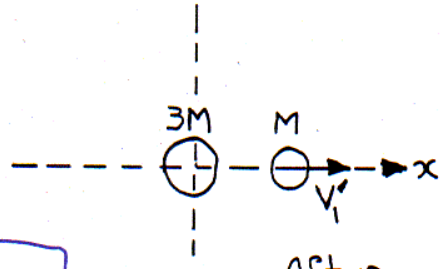
$= \boxed{MV_0}$

$P_y^{tot} = MV_1 - 3M\frac{5}{12}V_0\sin\theta$

$= \boxed{MV_1 = \frac{3}{4}MV_0}$

b) $P_y^{tot} = 0 \Rightarrow \boxed{V_1 = \frac{3}{4}V_0}$

c) $V_1'M = P_x^{tot} \Rightarrow \boxed{V_1' = V_0}$



d) $\boxed{V_x^c = \frac{P_x^{tot}}{3M+M} = \frac{1}{4}V_0}$, $\boxed{V_y^c = \frac{P_y^{tot}}{4M} = 0}$

Name: _____

Problem 2: Rotational Dynamics (15 points)

A spool of wire of mass M and radius R is unwound along a horizontal surface under a constant force $\vec{F}$. Assume the spool is a uniform solid cylinder that does not slip. The coefficient of static friction is μ_s. Assume that the radius of the spool does not decrease significantly while the spool is rolling. Give all you answers in terms of F, M, R, μ_s, g and L.

a) State the moment of inertia, I, of the cylinder about its central axis.

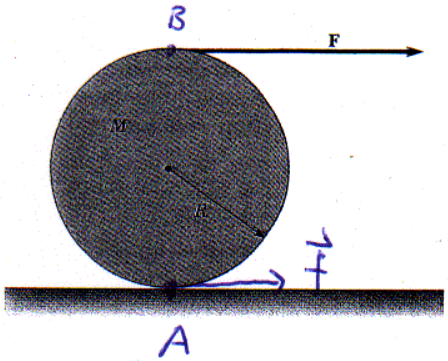
b) What is the force of friction, $\vec{f}$ (magnitude AND direction) acting on the spool? Show the direction of f on the diagram.

c) What is the acceleration of the center-of-mass?

d) What is the angular acceleration?

e) What is the total kinetic energy of the spool when it has rolled through a distance L?

a) $I = \frac{1}{2}MR^2$



c, d) $\tau = I\alpha$ about A

$\tau = 2FR$ $\quad I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

$\boxed{\alpha = \frac{\tau}{I} = \frac{4}{3}\frac{F}{MR}}$ $\quad \boxed{a = \alpha R = \frac{4}{3}\frac{F}{M}}$

b) Note $a > \frac{F}{M}$, so $\vec{f}$ and $\vec{F}$ point to the same direction. $f + F = Ma = \frac{4}{3}F \Rightarrow \boxed{f = \frac{1}{3}F}$

e) When the center moves a distance L, B points moves a distance 2L (or the string is pulled by a distance 2L). So $\boxed{K = 2LF}$

Name: _____

Problem 3: Rotational Collision (15 points)

A uniform cylindrical shell (hoop) sits on one of its flat sides on a frictionless surface. The hoop has mass M, radius R and height H. A bullet of mass μ moving horizontally with velocity V_0 strikes the hoop with impact parameter R at mid-height (H/2 from the surface). After the collision the bullet continues with velocity $V_0/2$ in its original direction. Ignore any hole the bullet creates. Give all your answers in terms of M, R, H, V_0, and μ.

a) What was the angular momentum, $\vec{L}$, of the system about the center of the hoop before the collision?

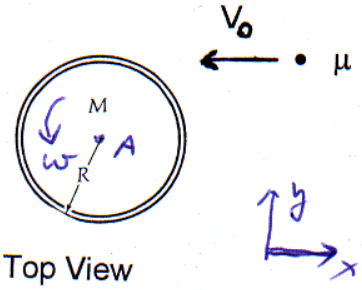
b) What is the linear velocity, $\vec{V}$, of the center of the hoop after the collision?

c) What is the angular velocity, $\vec{\omega}$, of the hoop after the collision?

a) $\boxed{L = \mu V_0 R}$ pointing up (out of paper) relative to A.

b) $M\vec{V} + \mu\frac{\vec{V}_0}{2} = \vec{V}_0\mu$

$\Rightarrow \boxed{V_x = -\frac{\mu/2}{M}V_0 \qquad V_y = 0}$



Top View

c) $I\omega + \mu\frac{V_0}{2}R = \mu V_0 R$

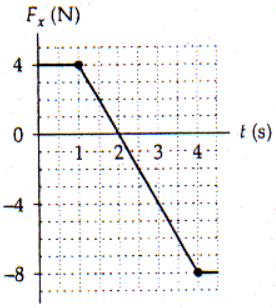
$\omega = \frac{\mu V_0 R}{2I} = \boxed{\frac{\mu V_0}{2MR}}$

Name: _____

Problem 4: Multiple Choice (15 Points)

a)

The only force acting on a 2.0-kg object moving along the x axis is shown. If the velocity v_x is –2.0 m/s at $t = 0$, what is the velocity at $t = 4.0$ s?



a. –2.0 m/s
b. –4.0 m/s
c. –3.0 m/s
d. +1.0 m/s
e. +5.0 m/s

$\int dt\, F = 4 + 2 - 8 = -2\ N \cdot s$

$P(t=4) = -4 - 2 = -6$

$v = \frac{p}{m} = -3\ \frac{m}{s}$

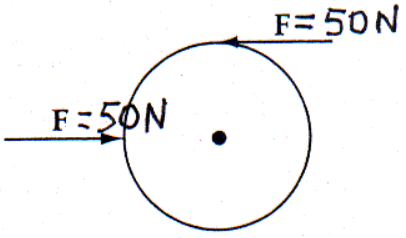
b)

A car of mass m_1 traveling at velocity v passes a car of mass m_2 parked at the side of the road. The momentum of the system of two cars is

a. 0.
b. $m_1 v$.
c. $(m_1 - m_2)v$.
d. $\dfrac{m_1 v}{m_1 + m_2}$
e. $(m_1 + m_2)v$.

c)

Two forces of magnitude 50 N, as shown in the figure below, act on a cylinder of radius 4 m and mass 6.25 kg. The cylinder, which is initially at rest, sits on a frictionless surface. After 1 second, the velocity and angular velocity of the cylinder in m/s and rad/s are respectively



a. $v = 0; \omega = 0.$
b. $v = 0; \omega = 4.$
c. $v = 0; \omega = 8.$
d. $v = 8; \omega = 8.$
e. $v = 16; \omega = 8.$

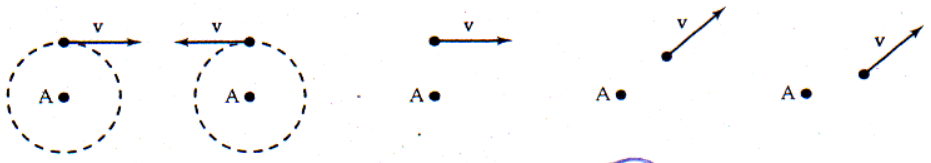
$I = \frac{1}{2} M R^2 = 50$

$T = 50 \times 4 = 200$

$\alpha = 4 \qquad \omega = 4/s$

d)

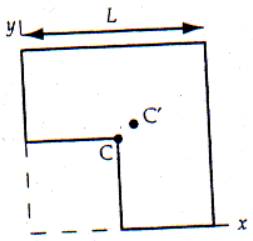
Five objects of mass m move at velocity v at a distance r from an axis of rotation perpendicular to the page through point A, as shown below. The one that has zero angular momentum about that axis is



(a) (b) (c) (d) (e)

e)

A square of side $\frac{L}{2}$ is removed from one corner of a square sandwich that has sides of length L. The center of mass of the remainder of the sandwich moves from C to C′. The displacement of the x coordinate of the center of mass (from C to C′) is



a. $\frac{1}{12}L$.

b. $\frac{\sqrt{2}}{12}L$.

c. $\frac{1}{6}L$.

d. $\frac{1}{8}L$.

e. $\frac{\sqrt{2}}{8}L$.

$3 \cdot x = 1 \cdot \frac{L}{4}$

$\Rightarrow x = \frac{L}{12}$