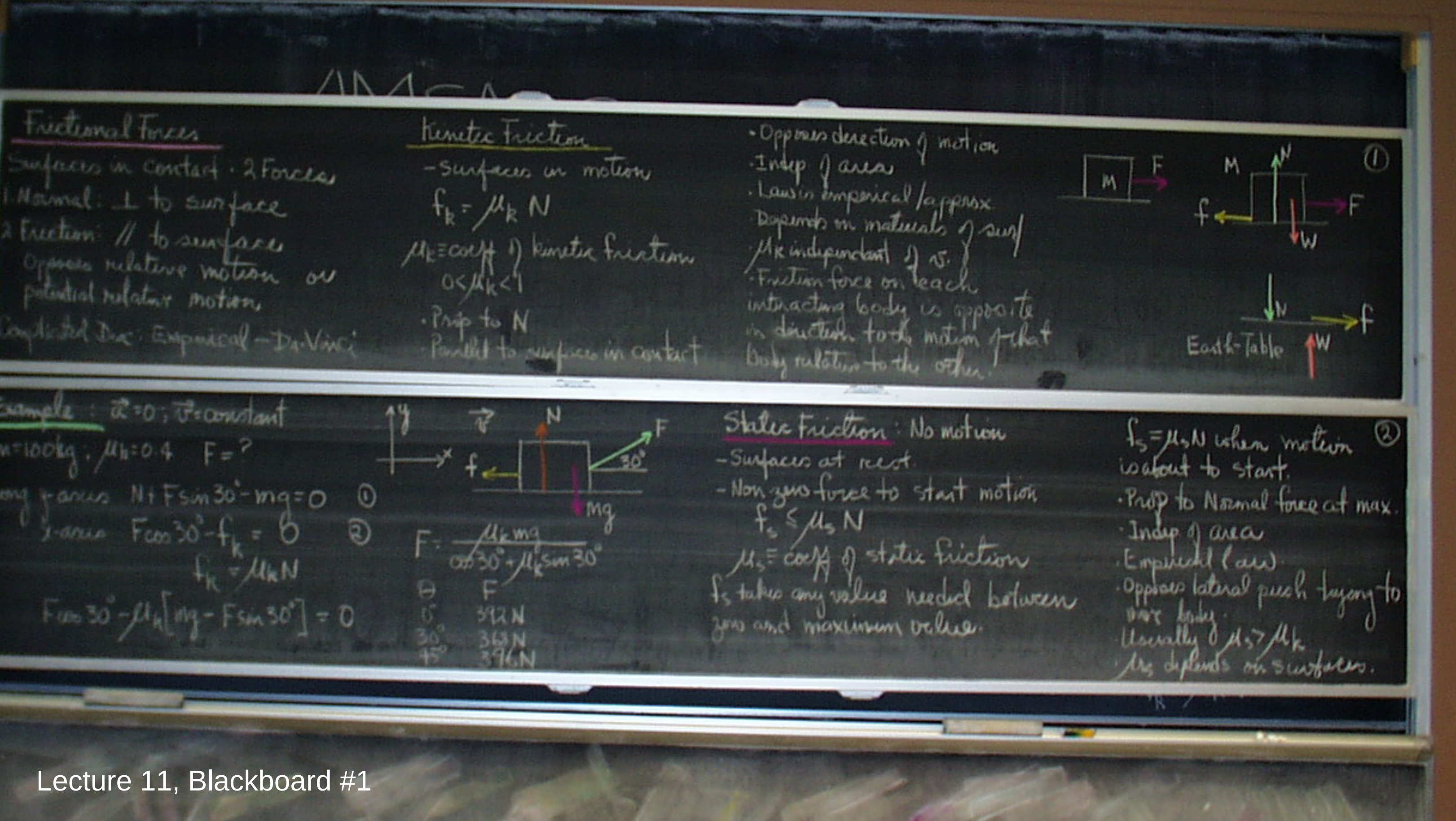


Lecture 11, Blackboard #1

Example: $\vec{a}=0$; $\vec{v}$ = constant

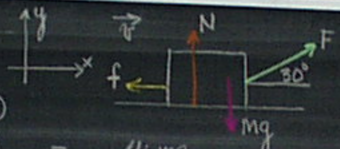
$m = 100\,kg$; $\mu_k = 0.4$ $F = ?$

Along y-axis $N + F\sin 30° - mg = 0$ ①

x-axis $F\cos 30° - f_k = 0$ ②

$f_k = \mu_k N$

$F\cos 30° - \mu_k[mg - F\sin 30°] = 0$



$$F = \frac{\mu_k mg}{\cos 30° + \mu_k \sin 30°}$$

θ	F
0°	392 N
30°	363 N
45°	396 N

Static Friction: No motion

- Surfaces at rest
- Non zero force to start motion

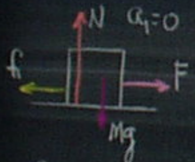
$f_s \le \mu_s N$

$\mu_s \equiv$ coeff of static friction

f_s takes any value needed between zero and maximum value.

$f_s = \mu_s N$ when motion is about to start.   ②

- Prop to Normal force at max.
- Indep of area
- Empirical law
- Opposes lateral push trying to move body.
- Usually $\mu_s > \mu_k$
- μ_s depends on surfaces.

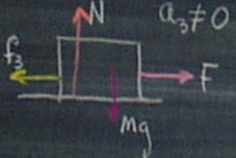


$f_1 < \mu_s$ No motion

$f_1 = F$

$f_2 = \mu_s N$ Almost

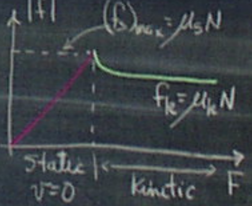
$f_2 = F$



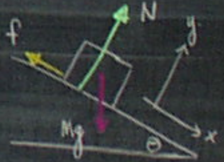
$f_3 = \mu_k N$

$F_3 - \mu_k N = m a_3$

Moving



Example: Block-on-Plane   ③



Arrange so: $\vec{a} = 0$

$v = 0$ or constant speed.

y-axis:

$N - mg\cos\theta = 0$ ①

x-axis:

$mg\sin\theta - f = 0$ ②

$$\frac{f}{N} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

$f_s \le \mu_s N$

$f_k = \mu_k N$

Lecture 11, Blackboard #2

④

Case 1 Static: Just starts to slip as θ is increased.

$$\left(\frac{f}{N}\right)_{max} = \mu_s = \tan\theta$$

$\theta \equiv$ Angle of Repose.

Case 2: Block moves at constant speed down plane.

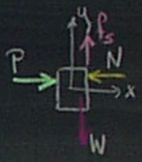
$\Rightarrow$ Kinetic Friction

$$\left(\frac{f}{N}\right) = \mu_k = \tan\theta$$

$\Rightarrow$ Measures μ_k.

Example

Push object against wall.



$\Sigma F_x = P - N = 0$ ①

$\Sigma F_y = f_s - W = 0$ ②

From ② $f_s = W$

① $P = N$

$f_s \le \mu_s P \le \mu_s N$

For no slipping $f_s \ge W$

$\mu_s P \ge W$

$P \ge \frac{W}{\mu_s}$

$P = W/\mu_s$ minimum push needed.

$\mu_s < 1$ so $P > W$

⑤

Example Block up a Plane

$m = 5kg$ $\quad \mu_k = 0.42$

$a = ?$ $\quad F = 20N$

Assume $\vec{a}$ is up $\rightarrow +x$.

$\theta = 60°$

ΣF_x: $F - f - mg\sin60° = m a_x$

ΣF_y: $N - mg\cos\theta = 0$ (no accel)

$\therefore N = mg\cos\theta$

$f = \mu_k N = \mu_k mg\cos 60°$

$$a_x = \frac{F - mg\sin60 - \mu_k mg\cos60}{m}$$

$a_x = \frac{F}{m} - g\sin60 - \mu_k g\cos60$

$= \frac{20}{5} - 9.81\times0.866 - 0.42\times9.81\times0.5$

$= -6.55\,m/s^2$ [Block moves down plane] $\boxed{\text{NG!!}}$

Change direction of f!!

$$\left.\begin{array}{l} F - mg\sin60° + f = m a_x \\ N - mg\cos60° = 0 \end{array}\right\}$$ Solve $a = -2.43\,m/s^2$

Dis. cons. with assumption!!

Example Block up a Plane

$m = 5\,kg$ $\quad \mu_k = 0.42$

$a = ?$ $\quad F = 20\,N$

Assume $\vec{a}$ is up $\rightarrow +x$.

ΣF_x: $F - f - mg\sin 60° = m a_x$

ΣF_y: $N - mg\cos\theta = 0$ (no accel)

$\therefore N = mg\cos\theta$

$f = \mu_k N = \mu_k mg\cos 60°$

$$a_x = \frac{F - mg\sin 60 - \mu_k mg\cos 60}{m}$$

$a_x = \frac{F}{m} - g\sin 60 - \mu_k g\cos 60$

$= \frac{20}{5} - 9.81 \times 0.866 - 0.42 \times 9.81 \times 0.5$

$= -6.55\,m/s^2$ [Block moves down plane] $\boxed{\text{NG!!}}$

Change direction of f!!

$$\left.\begin{array}{l} F - mg\sin 60° + f = m a_x \\ N - mg\cos 60° = 0 \end{array}\right\}$$ Solve $a = -2.43\,m/s^2$

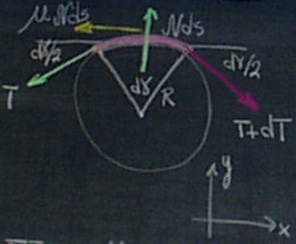
In. cons. with assumption!!

⑤

Ropes and Posts

- Assume no motion
- Static friction μ
- No slipping
- Rope wrapped around post
- Angle $d\gamma$ at center
- Assume a normal force $\mathcal{N}$ per unit of length.

$N = \mathcal{N}\,ds$



$\Sigma F_y = 0$ $\quad \mathcal{N}ds - (T+dT)\sin\frac{d\gamma}{2} - T\sin\frac{d\gamma}{2} = 0$ ①

$\Sigma F_x = 0$ $\quad (T+dT)\cos\left(\frac{d\gamma}{2}\right) - T\cos\left(\frac{d\gamma}{2}\right) - \mu\mathcal{N}ds = 0$ ②

Small angles.

$\cos\left(\frac{d\gamma}{2}\right) \approx 1$

$\sin\left(\frac{d\gamma}{2}\right) \sim \left(\frac{d\gamma}{2}\right)$

$dT \ll T$

$T\,d\gamma = \mathcal{N}\,ds$ From ①

$T\left(\frac{d\gamma}{ds}\right) = \mathcal{N}$ ③

$dT = \mu\mathcal{N}\,ds$ From ②

$\therefore \frac{dT}{ds} = \mu\mathcal{N}$ ④

④/③ $\quad \frac{1}{T}\frac{dT}{d\gamma} = \mu$

$$\int_{T'=T_0}^{T} \frac{dT'}{T'} = \mu \int_{\gamma'=0}^{\gamma} d\gamma'$$

Solve

$T = T_0 e^{\mu\gamma}$

Angle of rope around post in radians.

Suppose $\mu = 0.40$

$\gamma = 2\pi \Rightarrow$ 1-turn.

$T = T_0 e^{0.4 \times 2\pi}$

$= 12.3\,T_0$ Tremendous Advantage.

⑥

Lecture 11, Blackboard #4

Drag Force and Terminal Speed

- Objects in fluids (air, water, etc)
  - Drag forces retard motion
- Two types of Flow:

1. Laminar Flow
   - Smooth Flow
   - Small particles in fluid.

   $F_D \sim$ velocity.

2. Turbulent Flow
   - disturbed motion
   - baseball in air
   - parachute

   $F_D \sim (\text{velocity})^2$

   - motion leaves large wake.

Resistive Force $\sim$ Velocity

$\vec{D} = -b\vec{v}$ Resistive Force

$\vec{v}$: velocity

b : drag coeff. Depends on shape

Sphere: $b \sim R$

mg : weight (corrected for buoyancy.

$D = -bv$ drag

$\Sigma F_y = ma_y$

$mg - bv = ma = m\frac{dv}{dt}$.

$a = \frac{dv}{dt} = g - \frac{b}{m}v$ differential equation

⑦

At $t=0$, $v=0$, so $D=0$

$\therefore a(t=0) = \frac{dv}{dt} = g$.

As t increases, v increases and drag increases

$\therefore a$ decreases.

When $D = mg$, $a = 0$.

Body moves at terminal velocity, v_t.

When $a = \frac{dv}{dt} = 0$

$g - \frac{b}{m}v_t = 0$

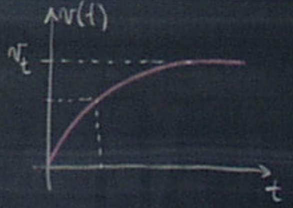
$$\boxed{v_t = mg/b}$$

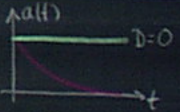
$\frac{m}{b}\frac{dv}{dt} = \frac{mg}{b} - v$

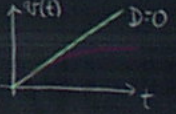
$\frac{dv}{v - v_t} = -\frac{b}{m}\,dt$

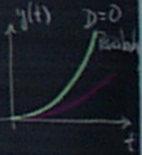
Solve: $v(t) = v_t\left(1 - e^{-\frac{b}{m}t}\right)$

Can show: $y(t) = v_t\left[t - \frac{m}{b}\left(1 - e^{-b/m\,t}\right)\right]$









⑧

Lecture 11, Blackboard #5

At $t=0$, $v=0$, so $D=0$

$a(t=0) = \frac{dv}{dt} = g.$

As t increases, v increases and drag increases

$\therefore a$ decreases.

When $D = mg$; $a = 0$.

Body moves at terminal velocity, v_t.

When $a = \frac{dv}{dt} = 0$

$g - \frac{b}{m} v_t = 0$

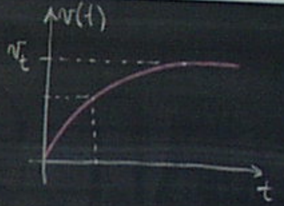
$$\boxed{v_t = mg/b}$$

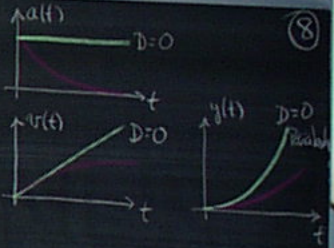
$$\frac{m}{b}\frac{dv}{dt} = \frac{mg}{b} - v$$

$$\frac{dv}{v - v_t} = -\frac{b}{m}\,dt$$

Solve: $v(t) = v_t\left(1 - e^{-\frac{b}{m}t}\right)$

Can show: $y(t) = v_t\left[t - \frac{m}{b}\left(1 - e^{-b/m\,t}\right)\right]$





8

Turbulent Flow

$D = -\frac{1}{2} C \rho A v^2$

$A \equiv$ Eff area

ρ : density of air

v : speed of fall

C : drag coeff : (0.5 – 1.0)

Body released

$v = 0 \;\therefore D = 0$

v increase, D increases

When $D = mg$, $a \equiv 0$.

$v = v_t$ terminal speed.

$$\frac{1}{2} C \rho A v_t^2 = mg$$

$$v_t = \sqrt{\frac{2mg}{C\rho A}} \text{ m/s}$$

9

Lecture 11, Blackboard #6