Example

- No friction
- Sliding Object
- What is $\vec{v}$ at bottom?

$\vec{a} \neq$ constant.

Difficult problem using forces!!

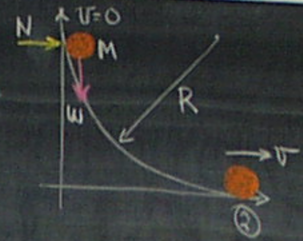
$W_N = 0$ $N \perp$ displ.

$\therefore K_2 + U_2 = K_1 + U_1$ 1 ≡ start, 2 ≡ finish

$\frac{1}{2} m v_2^2 + 0 = 0 + mgR$

$v_2 = \sqrt{2gR}$

If there is friction, v_2 would be less!



②

Example

Pull swing to side with force $\vec{P}$.

Slowly: KE = 0

$$\left.\begin{aligned} P &= T\sin\theta \\ w &= T\cos\theta \end{aligned}\right\} \quad P = w\tan\theta = mg\tan\theta$$

$$W = \int_0^{\theta_0} \vec{P}\cdot d\vec{l} = \int_0^{\theta_0} P\cos\theta \, dl$$

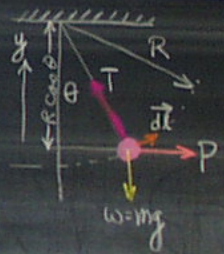
$dl = R\,d\theta$ = arc length

$$W = \int_0^{\theta_0} R w \tan\theta \cos\theta \, d\theta$$

$$= wR\int_0^{\theta_0} \sin\theta \, d\theta$$

$$= wR(1-\cos\theta_0)$$

← Increase in Height



$W_{other} = W_T + W_P = \Delta K + \Delta U = \Delta E$

$W_T = 0$ $T \perp$ displacement

$\Delta K = 0$ Swing moves slowly.

$\therefore W_P = \Delta U = w\,\Delta y$

$\Delta y = R(1-\cos\theta)$

$W_P = wR(1-\cos\theta)$

①

Lecture 14, Blackboard #1

(2)

Example

- No friction
- Sliding Object
- What is $\vec{v}$ at bottom?

$\vec{a} \neq$ constant.

Difficult problem using forces!!

$W_N = 0$ $N \perp$ displ.

$\therefore K_2 + U_2 = K_1 + U_1$ 1 ≡ start, 2 ≡ finish

$\frac{1}{2} m v_2^2 + 0 = 0 + mgR$

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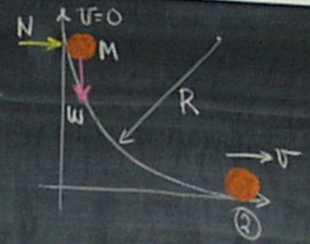
If there is friction, v_2 would be less!



(3)

Block on Incline

- Move a distance D

$v = 0$ at $t = 0$

Find v at end

$W_g = (-mg\sin\theta) D$
$= -mgH$

$W_F = \int \vec{F} \cdot d\vec{r} = FD$

$f = \mu N$

$N = mg\cos\theta$

$W_f = \int \vec{f} \cdot d\vec{s} = -\mu N D = -\mu m g D \cos\theta$

$W_{Net} = W_g + W_F + W_f$
$= -mgD\sin\theta + FD - \mu mgD\cos\theta$

$K_f = \frac{1}{2} m v^2$ $K_i = 0$

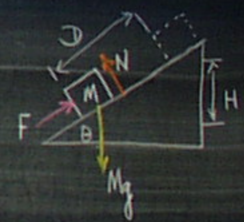
$K_f - K_i = W_{Net}$

$\frac{1}{2} m v^2 - 0 = FD - mgD(\sin\theta + \mu\cos\theta)$

$v = \sqrt{\frac{2FD}{m} - 2gD(\sin\theta + \mu\cos\theta)}$

If $\left[\frac{2FD}{m} - 2gD(\sin\theta + \mu\cos\theta)\right] = 0$ $\Rightarrow v = 0$

If $[\quad] < 0$???



Lecture 14, Blackboard #2

Conservative Forces

Gravity } Spring } Kinetic + Potential Energy $\equiv$ Const.

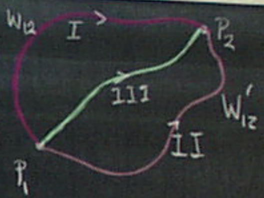
If $E = K + U$ [Constant $\Rightarrow$ Conserv. Force!

What are general requirements for a force to be conservative? Assume $\vec{F}$ is a function only of position.

$W_{12} = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{r}$ from $P_1 \rightarrow P_2$ along Path - I

$W'_{12} = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{r}$ from $P_1 \rightarrow P_2$ along Path - II

Definition: $\vec{F}$ is conservative if $W_{12} = W'_{12}$ for any two paths.



⑤

Example

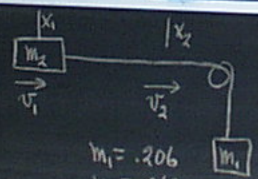
$E = K + U$

$E_1 = 0$

$E_2 = \frac{1}{2}(m_1+m_2)v_2^2 - m_1 g(x_2 - x_1)$

$E_1 = E_2$?? $\quad x_2 - x_1 = 0.750\,m$

$$v_2^2 = \frac{2m_1 g(x_2-x_1)}{m_1+m_2} = \frac{2\times .010 \times 0.750}{.010 + .206} =$$



$m_1 = .206$

$m_2 = .010$

$v_2 = \frac{10}{t_2} =$

Conservation of Energy

. Cases where $K + U \neq$ Constant!

$\rightarrow$ Dissipative forces.

Forms of Energy

. thermal energy

. electrical

. chemical

. nuclear

mechanical

General Principle: Changes in all forms of energy:

$\Delta KE + \Delta U + \Delta(\text{All others}) \equiv 0$

Law of Conservation of Energy!

Holds in all domains. Even where Newton's laws fail!

④

Lecture 14, Blackboard #3

Conservative Forces

Gravity, Spring } Kinetic + Potential Energy ≡ Const

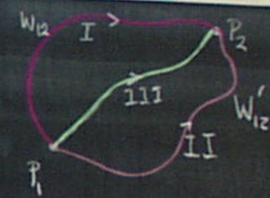
If $E = K + U$ [Constant ⇒ Conserv. Force!

What are general requirements for a force to be conservative? Assume $\vec{F}$ is a function only of position.

$W_{12} = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$ from $P_1 \to P_2$ along Path - I

$W'_{12} = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$ from $P_1 \to P_2$ along Path - II

Definition: $\vec{F}$ is conservative if $W_{12} = W'_{12}$ for any two paths.



⑤

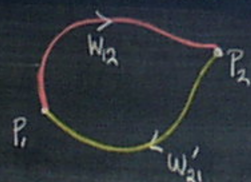
Round Trip

$P_1 \to P_2 \to P_1$

If F is conservative total $W = 0$

$$W_{12} + W'_{21} = W_{12} - W'_{12} = 0$$

$$\int_{P_1}^{P_2} \vec{F} \cdot d\vec{r} + \int_{P_2}^{P_1} \vec{F} \cdot d\vec{r} = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r} - \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r} \equiv 0$$



$\oint \vec{F} \cdot d\vec{r}$ [Line Integral] around a closed loop.

Gravity ≡ conserved

Spring ≡ Conserved

Friction : NO!!

$\oint \vec{F} \cdot d\vec{r} \equiv 0$

Always opposes motion.

Work → Heat / Dissipated.

⑥

Lecture 14, Blackboard #4

Mechanical Energy

ΔKE = Work by Force.

$K_2 - K_1 = W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$

$K_2 - K_1 = \mathcal{U}(P_1) - \mathcal{U}(P_2)$

$K_2 + \mathcal{U}(P_2) = K_1 + \mathcal{U}(P_1) = K + \mathcal{U}$

Total Mech Eng = Constant.

Pot. Energy: Gravity (Near Earth)

$\vec{F} = -mg\,\hat{z}$

$\mathcal{U}(P) = -\int_{P_0}^{P} \vec{F} \cdot d\vec{r} + \mathcal{U}(P_0)$

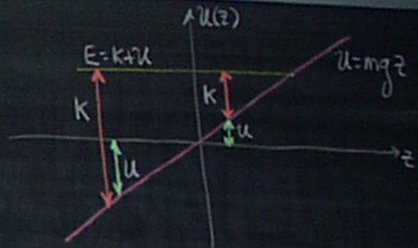
Take $\mathcal{U}(P_0) = 0$

$\mathcal{U}(P) = -\cancel{\int_0^x F_x dx} - \cancel{\int_0^y F_y dy} - \int_0^z F_z dz$

$= -\int_0^z -mg\, dz$

$\boxed{\mathcal{U}(z) = mgz}$

$\mathcal{U}(0) = 0$



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Potential Energy of Conservative Forces

Let P_0 = Reference Point

$\mathcal{U}(P_0)$ = Pot. Energy of Reference Point

P = General Point

$\mathcal{U}(P) = -\int_{P_0}^{P} \vec{F} \cdot d\vec{r} + \mathcal{U}(P_0)$

$\mathcal{U}(P_2) - \mathcal{U}(P_1) = -\int_{P_0}^{P_2} \vec{F} \cdot d\vec{r} + \mathcal{U}(P_0) - \left[-\int_{P_0}^{P_1} \vec{F} \cdot d\vec{r} + \mathcal{U}(P_0)\right]$

$= -\int_{P_0}^{P_2} \vec{F} \cdot d\vec{r} - \int_{P_1}^{P_0} \vec{F} \cdot d\vec{r}$

$\Delta\mathcal{U} = \mathcal{U}(P_2) - \mathcal{U}(P_1) = -\int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$

$\Delta\mathcal{U} \Rightarrow$ Negative of work done by the force between P_1 and P_2!

$\mathcal{U}(P_0)$: Drops Out!! Always!

Choose $\mathcal{U}(P_0) \equiv 0$!

8

Lecture 14, Blackboard #5