

When we used our energy principle, suppose we have an object that starts at a height h_0 .

And this object is dropped, and when it gets to the ground, it has some final velocity.

And when we applied our energy principle, assuming that there was no air resistance, what we saw was that the change in kinetic energy plus the change of potential energy was 0, so we had 0 was equal to ΔK plus ΔU .

And we saw that the kinetic energy changed by $\frac{1}{2} m v^2$, and the potential energy changed by $-m g h_0$.

So we can compute the velocity of the object as it's falling given by square root of $2g h_0$.

Now that we're considering kinetic energy of rotation, recall that we show that the kinetic energy of a pure rotation about a fixed axis was $\frac{1}{2}$ the moment of inertia about that axis times the angular speed squared.

We now would like to apply our energy principle to include rotational kinetic energy along with the translational kinetic energy.

And the example that we want to look at is something very simple.

Suppose we have a pulley.

Now our pulley has a mass p , and we'll say it has a moment of inertia of the pulley about the fixed axis passing through the center of the pulley.

And we have a mass 1 and another block 2.

And let's suppose that we release this system.

And for the moment let's make this surface frictionless.

And suppose that block 2 falls down a certain distance.

So in the final state, block 2 will have dropped a distance each final from its initial position.

And what we like to consider is find the velocity final of block 2.

So we begin in the same way that we've done this before, by considering our energy diagrams.

And so we'll have an initial state.

And in our initial state, what we'll do is we'll just have the initial 1, 2.

And I'm going to choose u equals 0, here's my pulley.

And everything is at rest.

And so in the initial state, the initial energy, i initial, k initial, plus u initial is 0.

And in our final state, we have the pulley is rotating with ω final.

Block 1 is moving with a velocity v final 1.

And block 2 is also moving with v_2 final.

And let's just suppose that this was our u equals 0 position.

And although it's not so clear in the diagram, u final, it has moved down to height h final.

So what is the energy in our final state?

Well, we have to consider all the different pieces.

We have block 1, $\frac{1}{2} m_1 v_1$ final squared.

We have the motion of block 2, $\frac{1}{2} m_2 v_2$ final squared.

And we also have the kinetic energy of the pulley, which is given by $\frac{1}{2} I$ about the pulley ω final squared.

And what about our final potential energy?

Well, we've dropped the height, h final.

So we have block 2 has moved minus $m_2 g h$ final.

And now we have our two energy states.

And what we'd like to consider is apply the energy principle just like we applied it for this simple case.

But before we do that, there is a constraint condition that because the rope is fixed in length, as block 1 moves, the pulley is rotating and block 2 is moving.

What we'll have is fixed and not slipping.

So as the rope moves around the pulley, the pulley is moving with the same motion as the rope, and the rope is moving with the speeds of block 1 and 2, so we see that v_1 final is equal to v_2 final.

And now what about the pulley?

If this is radius r , we know that the velocity of a point on the rim of a disk is moving with the speed of the rope, which is the speed of block 1 and block 2, So that's our ω final.

And so that makes our final energy, let's now gather terms.

The velocities are the same.

So we have $\frac{1}{2}$ and 1 plus m_2 .

And we'll just call this the final for simplicity.

$\frac{1}{2} m_2$ times v final squared.

That accounts for these two terms.

And we have the moment of inertia, kinetic energy associated with the wheel, which is $\frac{1}{2} I \omega$ final squared, which is v final squared over r squared minus $m_2 g h$ final.

And so now we can solve our energy principle, which is because we're assuming everything's frictionless, we have 0 equals E final minus E initial, implies that E final equals E initial.

And we chose our initial potential energy to be 0 .

So of course that's just a choice of constant.

So I can now solve this equation by setting E final equal to 0 .

That's the same statement.

And then you can see algebraically I can solve for V final.

And what I get is I'm just going to write all these terms over.

I get $m_2 g h$ final.

I'm going to divide by this common coefficient, $\frac{1}{2}$ and 1 plus m_2 plus the moment of inertia divided by radius squared.

And I now have to take the square root of the whole thing.

And that's how I can find the velocity of block 2 when it's dropped down a certain distance to h final.

So here we've generalized our energy approach to include rotational kinetic energy.