I'd like to now calculate the angular momentum of a two particle system.

So here's one particle.

Call that 1.

And here's the other particle 2.

And particle 1 has a momentum in the downward direction.

And particle 2 has a momentum in the upper direction.

Now, in this case, I want to make some of the momentum-- so this momentum of the system is 0.

And now I want to calculate the angular momentum about two different points.

So let's just draw an axis here.

And let's choose one point A. And we'll call this distance r, and this distance r.

And let's choose another point B, and we'll call that distance d.

Now, in order to calculate angular momentum of the system, I need to draw my vectors.

Let's start with the angular momentum about the point A. That's going to be a vector from A to object 1, cross the momentum of 1, plus the vector from A to object 2, cross the momentum of 2.

Now what's crucial here, is at this point, things are a little bit of a abstract, but we want to draw these vectors, rA1 and rA2.

So we're now constructing our angular momentum diagrams.

So there's rA1, and there rA2.

but again, in order to calculate this, we could do it geometrically with right hand rules, but I now want to introduce unit vectors, so I can do vector decomposition at every point.

So I'm going to choose a unit vector, i-hat, j-hat, and k-hat.

And now I can decompose all of these vectors in terms of my unit vectors.

This is the angular momentum of the system.

So now it's not hard at all to write these out.

rA1 is minus r in the i-hat direction.

Cross P1 is down.

That's minus P1 in the j-hat direction, where P1 is the magnitude .

And rA2 is in the plus i-hat direction.

So that's r i-hat.

And I'm crossing that with P2, which I'll write right now as P2 in the j-hat direction.

Now remember, P2 has the same magnitude as P1, but they're pointing in opposite directions.

So when I take the cross project, i-hat cross j-hat is k-hat minus sign minus sign.

So I get rP1 k-hat.

And now over here, P2 is equal to P1.

i-hat cross j-hat is k-hat.

And I get another rP1 k-hat.

And so I have 2rP1 k-hat.

And that's the angular momentum of the system about point A.

Now I'd like to calculate the angular momentum about B So let's draw the same diagram, 1, 2.

Here is P2.

Here's P1.

Here is B. And now I'll draw my vectors.

This is rB2 and this is rB1.

This distance was 2r.

This distance was d.

I'm going to use the same unit vectors.

And so I get l for the system about B. Again, I'll just write everything out.

P1 plus rB2 cross P2.

So I'm going to write P2 as, in this case, it's equal to minus P1.

And magnitudes P2 equals P1.

In magnitude directions are opposite.

So rB1 is equal 2r plus D i-hat, and it's pointing in the minus i-hat direction.

Cross P1 is in the minus j-hat direction.

rB2 is also in the minus i-hat direction, so that's minus D i-hat.

And P2 is in the plus j direction, so that is plus-- the magnitudes are the same-- P1 j-hat.

We don't need the plus sign there.

And now I take the cross products.

i-hat cross j-hat is k-hat.

A minus sign minus sign.

That makes plus.

So I have 2r plus D P1 k-hat.

Now notice here I have i-hat cross j-hat, which is k-hat, but there's a minus sign.

So I have minus D P1 k-hat.

Here I have plus D P1 k-hat minus D1 P k-hat.

And so I get 2rP1 k-hat.

And I have the same result. L system A equals L system B.

Now that's not a coincidence in this problem.

And the reason is that whenever-- so we can say it this way.

Whenever the momentum of the system is 0, then LA is independent of the choice of the point A.

So coming back to our example, no matter where I picked our points A and B-- any point I could pick, anywhere I want-- I could pick a point C up there-- I make this cross product calculation.

I would get exactly the same answer, 2rP1k.