

Today, we'd like to discuss tension in a rope.

Under what conditions is the tension uniform?

Recall that if we had a rope and say we were pulling-- here's our one end.

We'll call that we're pulling at this end with the force a .

And we pull on this end b with a force b .

Then what we meant by tension at any point in the rope was that we took an imaginary slice of the rope p and we looked at the two sides.

And here-- if we call this the left side and this is the right side-- then on the left side, we have that the right side of the rope is pulling the left side of the rope by some force f .

And this is the left side.

And the right side is being pulled by the left side.

So that's the right side being pulled by the left.

And what we define the tension at the point p was to be the magnitude of this action-reaction pair.

And that's how we define tension at a point inside a rope.

Now we'd like to address the question, under what conditions can we say that the tension is uniform?

Which means that at all points p , the tension is the same.

In order to analyze this, we'll apply Newton's second law.

So let's begin with the same situation as before.

Let's consider a rope where we're pulling this side with a force at the end.

By the way, we can call the tension at that end T of a .

And on this side, we can refer to this as the tension on side b .

Now suppose that our mass of the rope is non-zero.

And we now want to apply Newton's second law.

But let's also apply the condition that the acceleration of the rope is non-zero.

So you're holding the rope under some tension by pulling both sides.

But the rope is not accelerating.

And so what does that mean when we apply Newton's second law to the rope?

Well, let's just arbitrarily call that our positive direction.

We see by vector decomposition that we have T_b minus T_a equals mass of the rope.

But because the acceleration of the rope is zero, this side is zero.

And we can use Newton's second law to conclude that the tension in the rope is uniform when the rope is not accelerating.

And this is what we'll call our case one.

A zero tension in the rope is uniform.

Now let's suppose case two, that a is not zero.

And so now we have the rope.

This is point b.

Here's point a.

We'll choose a uniform direction a .

And now we can see when we apply F equals Ma , that T_b minus T_a equals mass of the rope times the acceleration of the rope.

So you're pulling such that T_b is greater than T_a by precisely the quantity Mr times a .

So the tension at the ends are not the same.

And therefore, the tension in the rope is not uniform.

Now there's a special case here that we want to consider.

So even though a is nonzero, let's assume that the mass of the rope is very small.

And this is often-- people call this assume the rope is massless.

Well, that's a little strange.

We can say that the very, very light string is often the same type of modeling.

And under those circumstances with that assumption, you see that there is no contribution here.

And with that assumption, that the tension is approximately uniform.

Ropes are not massless.

There's a small difference.

But under those circumstances when we say a very light rope that-- even if it is accelerating-- implies that T is uniform in the rope.

And we can say the rope is under uniform tension.

And we will often refer to this case when we analyze problems where we're pulling objects, for instance, in a typical pulley problem.