When we're looking at polar coordinates, one of the important issues is to understand the unit vectors.

Let's describe our coordinate system, again.

We have a point where there is an object.

And at this point, we have a pair of unit vectors, r hat and theta hat.

Now those unit vectors will change, depending on where you are in space.

We now want to address the question, how do I compare polar coordinates and Cartesian coordinates.

In Cartesian coordinates, at this point here, we would have-- let's say we choose a Cartesian plus x and plus y, then we would have a unit vector i hat and j hat.

And how are these vectors related?

Remember our angle theta, so we have the angle theta here and the angle theta there.

And now I would like to apply a simple vector decomposition to r hat and theta hat and express each of these unit vectors in terms of the unit vectors i hat and j hat.

Let's begin with r hat.

As you can see in the diagram, r hat has a horizontal component and a vertical component.

So what we have is r hat.

It's a unit vector, so its length is 1.

Its horizontal component is adjacent to the angle, so that's cosine of theta i hat plus sine of theta j hat.

The vertical component is opposite the angle.

In a similar fashion, theta hat-- well, theta hat has a component in the negative i hat direction, which is opposite the angle.

And it has a component in the positive j hat direction, which is adjacent to the angle.

So we have minus sine theta i hat plus cosine theta j hat.

And that's how we can decompose our unit vectors r hat and theta hat in terms of Cartesian coordinates.

Now why is this significant?

Because if we're describing the motion of an object, for instance, an object that's going around a circle, then our polar coordinate theta is a function of time.

And so these unit vectors are actually changing in direction.

You saw that before.

Over here, r hat and theta hat point in different directions.

So what we actually have as functions of time is r hat of t equals cosine theta of t i hat.

Now the unit vectors don't change in Cartesian coordinates.

At every single point, you have the same Cartesian unit vectors.

And so this vector is time dependent.

Now the significance of that is our first important vector in kinematics is the position vector.

The position vector is a vector that goes from the origin to where the object is.

We'll call that r of t.

So this position vector r of t can be expressed as a length r.

And its direction is in the r hat direction, which is a function of time.

So we have r cosine theta of t i hat plus r sine theta of t j hat.

Now we can now define the velocity of this object where the velocity is the derivative of the position vector.

When you differentiate, remember, r is a constant, so we get r.

Now what is the derivative with respect to time of cosine theta t.

Because the argument of theta is a function of t, we need to use the chain rule.

So the derivative is minus sine theta of t, d theta dt i hat.

And the derivative of the sine is cosine theta of t d theta dt j hat.

Now notice that I can pull out the common term d theta dt.

So I have r d theta dt.

And I have minus sine theta of t i hat plus cosine theta of t j hat.

And if you'll notice, this is exactly the unit vector theta hat.

So we can write our velocity vector for this object that's moving in a circle as r d theta dt theta hat.

When we write a vector like this, it's pointing tangentially, the theta hat direction, and this part is the component.

So often we can use a notation v theta theta hat, where v theta is the component r d theta dt.

Now this component can be positive or negative or 0.

For example, if d theta dt is positive, what does that mean?

That means that our angle theta is increasing so the object is moving the way I indicate with my finger.

If d theta dt is 0, then the angle is not changing, so the object is at rest.

And finally, if d theta dt is negative, then the angle theta is decreasing, and so the object is moving in this direction.

So this is our velocity for a circular motion expressed in polar coordinates.