Let's consider a system in which there is no non-conservative work and there's a potential energy associated with forces in the system.

And for an example, let's model some system by the following function b x-cubed.

And we know that our definition of force is that it's minus the change in potential energy.

So if we differentiate this, our force is a function of x.

And because of that minus sign, you always have to be careful when you take that type of derivative, we get as far as the force goes, 2ax minus 3b x-squared.

Now we ask ourselves, where does this force vanish?

So if we look at the 0 points for the force, we can see immediately that there is one x equals 0.

The other one, if we said 2ax equals a 3b x-squared tells us that we have another 0 point at 2/3 over 2/3 a over b.

Now let's also for example, just put some numbers in.

For some simplicity, let's write this as 4 joules per meter squared for a.

Why do we have meter squared?

Well, the units of potential energy are joules, and we're multiplying by x-squared.

And so you can see that d also we're going to write as 2 joules metered to minus 3.

And if we put those numbers in, we can see that we'll get 8 over 6.

And 8 over 6 is just 4/3, and that will be in meters where the other 0 point is.

So that's nice, but we'd like to do some properties of these 0 points.

If we had a particle that we started with some energy at one 0 point, where would it go?

What would it do at the other point?

So in order to understand a little bit more detail about these 0 points, let's make a plot of our potential energy U of x verse x.

Now to begin with, we want to ask ourselves how does this function behave?

And for big values of x, the x cube piece will dominate.

And if we're positive x, we dominate with a negative value for potential energy.

So our potential energy function goes off into negative infinity as x goes to infinity.

Now on the other side of the scale, the x-cubed term will dominate for negative values of x.

But this term is negative, and that's negative, so up here it will go off to infinity in the positive direction.

Now, where are the zeroes of the potential function?

There's obviously one at x equals 0.

And with these values, it's not 100% obvious immediately, but 4x squared minus 2b x-cubed-- if you set x equal to 2, 4 times 2 is 8, b 2 times 2, 4 times 4 is 16.

If b is 2 and x is 2, that's 16.

So we get a cancellation at x equals 2.

And so we have a point up here.

I'm sorry, that's our 0 point at x equals 2.

Now, we said that the force at x equals 4/3 had a 0.

And so if we plot this function, it's going to look something like that.

Now, let's focus on these two points right here where our force is 0.

Remember, that this is minus the slope.

And so we can see immediately that these two points have 0 slope and so the force is 0 there.

Notice, potential energy always depends on a reference point.

So whether the potential energy is 0 or not is not crucial.

But the physical fact that the force 0 there has physical mean.

So here this is one of our points F of x is 0.

Now, what is the property of this point?

Well, suppose we ask ourselves if the particle were just displaced slightly from this position where the force is 0, which way would the force point?

Well in order to answer a question like that, we can look at the slopes on either side of the 0.

So over here the slope is negative, but the force is minus the slope.

So F of x is positive on this side.

And so if a particle is displaced a little bit from the 0 point, it moves off to the right, and it will continue to feel a force in this direction, and so it will move away from the 0 point of the force.

Now on the other side, we do the same type of argument.

And over here the slope is positive, but the force is negative.

So again, what we see on this side, the particle will move away from this 0 point.

And so if our particle just started there and it had a little bit of kinetic energy, it would move one way or the other away from that point.

And so we call this an unstable equilibrium point.

And that's an abbreviation for equilibrium.

Now, what about this 0 point over here?

Well, we already know that if a particle is on this side of the 0 it feels the force this way.

But what if our particle were on this side?

Well, once again, we analyze the slope.

The slope is negative, force is minus the slope, so it's positive.

So everywhere on this side, the particle is getting a restoring force back to equilibrium.

And so if we displace this particle just on either side, if it doesn't have enough energy to get over this point, then the particle will stay around this area.

And that is why we give this point the name of stable equilibrium point.