Solutions for 8.01x Problem Set 10

8-79: Following the suggestions in the book, one can first show that the angle between the two outgoing pucks is 90°. Momentum conservation yields two equations:

$$\begin{aligned} mv_{A,i} &= mv_{A,f}\cos\alpha + mv_{b,f}\cos\beta && (1)\\ 0 &= mv_{A,f}\sin\alpha + mv_{b,f}\sin\beta && (2) \end{aligned}$$

If we square and add the two first equations and eliminate the mass, we obtain

$$\begin{aligned} v_{A,i}^2 &= v_{A,f}^2 + v_{B,f}^2 + 2v_{A,f}v_{B,f}(\cos\alpha\cos\beta - \sin\alpha\sin\beta) && (3)\\ &= v_{A,f}^2 + v_{B,f}^2 + 2v_{A,f}v_{B,f}\cos(\alpha+\beta) && (4)\\ & && (5) \end{aligned}$$

Energy conservation gives

$$v_{A,i}^2 = v_{A,f}^2 + v_{B,f}^2 \qquad (6)$$

Subtracting the two equations we obtain

$$\begin{aligned} 0 = 2v_{A,f}v_{B,f}\cos(\alpha+\beta) \text{ and therefore} && (7)\\ \alpha+\beta = \pi/2. && (8) \end{aligned}$$

Knowing this, we see that the angle for puck B is 65°. Momentum conservation in y direction gives $v_{B,f} = 0.466 v_{A,f}$. Momentum conservation in x gives the final speed of puck A as $v_{A,f} = v_{A,i}/(\cos(25°) + 0.466\cos(65°)) = 13.6$ m/s. For puck B we get $v_{B,f} = 6.34$ m/s.

9-6: By differentiating $\Theta(t)$, we obtain

$$\begin{aligned} \omega(t) = (250 rad/s) - (40.0 rad/s^2)t - (4.50 rad/s^3)t^2 \text{ and}\\ \alpha(t) = (40.0 rad/s^2) - (9.00 rad/s^3)t. \end{aligned}$$

for the angular velocity ω and the angular acceleration α.
a) $\omega(t) = 0$ for $t = 4.23$s.
b) At $t = 4.23$, $\alpha(t) = -78.1$ rad/s^2.

c) At $t = 4.23$, $\omega(t) = 586$ rad, corresponding to $586/(2\pi) = 93.3$ revolutions.
d) At $t = 0$, $\omega(t) = 250$ rad/s.
e) The average ω is $\omega_{ave} = 586$ rad / 4.23s = 138 rad/s.

9-36: For the slender rod, the moment of inertia is $I = \frac{1}{12}mL^2$. The kinetic energy is therefore

$$K_{rot} = \frac{1}{2}\frac{1}{12}mL^2\omega^2 = 1.3 \times 10^6\,\mathrm{J}.$$

To gain the same kinetic energy in a free fall (neglecting air resistance), the object needs to drop a height

$$h = K/(mg) = 1160 \text{ m}.$$

16-25:
a) Average kinetic energy $K_{ave} = 3/2kT = 6.21 \times 10^{-21}$ J.
b) Average speed squared: $v_{ave}^2 = 2K_{ave}/m = 2.34 \times 10^5 \ \mathrm{m^2/s^2}$.
c) RMS speed : $v_{rms} = \sqrt{\frac{3RT}{M}} = 4.84 \times 10^2$ m/s.
d) Momentum : p = m $v_{rms} = 2.57 \times 10^{-23}$ kg m/s.
e) Average force is obtained from the change in momentum of the molecules per collision, divided by the time between collisions, as $J = F_{ave} \cdot \Delta t$. Average momentum change is $2 \times p$, average time is $2 \times L/v$, where L is the length of the container. This gives

$$F_{ave} = \frac{2mv_{rms}}{2L/v_{rms}} = mv_{rms}^2/L = 1.24 \times 10^{-19}\mathrm{N}.$$

f) Pressure per molecule: $P_{ave} = F_{ave}/L^2 = 1.24 \times 10^{-17}$ Pa.
g) $N = P/P_{ave} = 8.15 \times 10^{21}$ molecules.
h) Number of molecules $N = nN_A = N_A PV/(RT) = 2.45 \times 10^{22}$.
i) The discrepancy arises from assuming in g) that all molecules move in the same direction (one dimension), whereas in reality their motion is three-dimensional.