MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.01X FallTerm 2002

Solutions to Problem Set #9

Problem 1: Oscillating Plate (Y&F 13-53)

This is a situation where neither the initial position x_0 nor velocity v_0 are zero. Assume the motion is described by

$$x(t) = A\cos(\omega t - \phi) \quad (1)$$
$$v(t) = -A\omega\sin(\omega t - \phi) \quad (2)$$
$$a(t) = -A\omega^2\cos(\omega t - \phi) \quad (3)$$

From these, we can find

$$\tan\phi = v_0/(x_0\omega) \quad (4)$$
$$A^2 = x_0^2 + (v_0/\omega)^2 \quad (5)$$

We are given $x_0 = 0.060\,m$, $v_0 = 0.300\,m/s$, and $A = 0.100\,m$. Substituting these values into (5) we find

$$\frac{1}{\omega^2} = \frac{(0.100)^2 - (0.060)^2}{(0.300)^2} = 0.07111$$

or $\omega = 3.75\,rad/s$. From (4) $\tan\phi = 1.33$, and $\phi = 53.1°$.

These results can be used to find the following answers:

(a) The period is $T = 2\pi/\omega = 1.68\,s$.
(b) From (2) we have $0.160 = -0.100 \times 0.375\sin(\omega t - \phi)$ so that $\sin(\omega t - \phi) = -0.160/0.375$, $\omega t - \phi = -25.26°$, and the displacement from (1) will be $0.0904\,m$. This could be $x = \pm 0.0904\,m$, as both give a *speed* of $0.160\,m/s$.
(c) The coefficient of friction is $\mu_S = a_{max}/g$, where a_{max} is the acceleration at which the carrot starts to slide. From (3), you see the acceleration at the end point is equal to $0.100 \times (3.75)^2 = 1.41\,m\,s^{-2}$, so $\mu_S = 1.41/9.80 = 0.144$.

Problem 2: Tennis Ball Hit (Y&F 8-58)

In this problem, we are only asked to find the impulse and final momentum of the tennis ball, so we do not need to use the kinematic equations for constant acceleration that we used at the beginning of the course. Because the force on the ball is constant, the impulse $\Delta\vec{p}$ is just the force multiplied by the time it acts.
(a)

$$\Delta\vec{p} = -380 \times 3.00 \times 10^{-3}\,\hat{\mathbf{i}} + 110 \times 3.00 \times 10^{-3}\,\hat{\mathbf{j}} = -1.14\,\hat{\mathbf{i}} + 0.330\,\hat{\mathbf{j}}$$

(b) The final momentum is the vector sum of the initial momentum and the impulse. If the ball weighs $0.560\,N$, its mass is $0.560/9.80 = 0.0571\,kg$.

$$\begin{aligned}\vec{p}_f &= \vec{p}_i + \Delta\vec{p} \\ &= 0.0571 \times 20\,\hat{\mathbf{i}} - 0.0571 \times 4.0\,\hat{\mathbf{j}} + -1.14\,\hat{\mathbf{i}} + 0.330\,\hat{\mathbf{j}} \\ &= 0.00\,\hat{\mathbf{i}} + 1.02\,\hat{\mathbf{j}}\end{aligned}$$

Dividing by the mass gives the final velocity

$$\vec{v}_f = 0.00\,\hat{\mathbf{i}} + 1.79\,\hat{\mathbf{j}} \;\; m/s \;\; \text{(not a very good hit).}$$

Problem 3: Stuntman (Y&F 8-70)

This is a collision in which the maximum amount of mechanical energy is lost; it cannot all be lost because momentum must be conserved. First, use conservation of mechanical energy to find the stuntman's velocity v_0 at the start of the collision. This will give $v_0 = \sqrt{2gh}$ where $h = 5.00\,m$. The stuntman's initial momentum will be $\vec{p}_i = m_s v_0\,\hat{\mathbf{i}}$, all in the horizontal direction, where m_s is the mass of the stuntman. The momentum of the entwined bodies just after the collision will be $\vec{p}_f = (m_s + m_v)v_f\,\hat{\mathbf{i}} = \vec{p}_i$, where m_v is the mass of the villain and v_f is their speed just after the collision.

(a) Putting this together

$$v_f = \frac{m_s}{m_s + m_v}\,v_0 = \frac{m_s}{m_s + m_v}\,\sqrt{2gh} = \frac{80.0}{80.0 + 70.0}\sqrt{2 \times 9.80 \times 5.00} = 5.28\;m/s$$

(b) Use the work–kinetic energy theorem

$$\frac{1}{2}(m_s + m_v)v_2^2 - \frac{1}{2}(m_s + m_v)v_1^2 = W_{NC} = -\mu_k(m_s + m_v)g\,\Delta x$$

where $v_1 = v_f$, $v_2 = 0$, and Δx is the distance they slide. From this you find

$$\Delta x = \left(\frac{m_s}{m_s + m_v}\right)^2 \frac{h}{\mu_k} = 5.70\,m$$

Problem 4: Exploding Projectile (Y&F 8-84)

The mechanical energy is increased when the projectile explodes, but the only external force is gravity. Assume the explosion is so rapid that the impulse of gravity during the explosion can be ignored. Let the mass of the unexploded projectile be $2m = 20.0\,kg$. We must also ignore the mass of the explosive in order to be able to solve the problem, so each fragment has mass m after the explosion. Conservation of energy gives us the initial conditions just before the explosion. All of the initial kinetic energy in the vertical direction will have become gravitational potential energy at the height $h = v_{0y}^2/(2g) = (v_0 \sin 60°)^2/(2g)$, where $v_0 = 80.0\,m/s$. It will take $t_1 = \sqrt{2h/g} = (v_0/g)\sin 60°$ seconds to reach this height. So, just before the explosion $\vec{p}_i = 2mv_0 \cos 60°\,\hat{\mathbf{i}}$.

This will also be the total momentum of the two fragments after the explosion. One fragment has zero speed after the collision, and hence zero momentum. The second fragment must have $v_{y2} = 0$ after the explosion and $mv_{x2} = 2mv_0 \cos 60°$ in order to conserve momentum. The explosion does not change the motion of the *center of mass*.

(a) The fragment starts at height h, with $v_{0y} = 0$ and $v_{0x} = v_{x2} = 2v_0 \cos 60°$ It will take $t_2 = \sqrt{2h/g} = t_1\,s$ to reach the ground. During this time it will travel horizontally $v_{x2}\,t_2\;m$. The total distance traveled from the firing point will be

$$\Delta x = t_1\,v_0 \cos 60° + t_2\,2v_0 \cos 60° = 3(v_0^2/g)\sin 60° \cos 60° = 848\,m$$

(b) The vertical mechanical energy does not change in the explosion. The horizontal kinetic energy was $mv_0^2 \cos^2 60°$ before the explosion and $(1/2)mv_{x2}^2$ after. The energy released is

$$\frac{1}{2}m(2v_0 \cos 60°)^2 - m(v_0 \cos 60°)^2 = m(v_0 \cos 60°)^2 = 1.60 \times 10^4\,J$$