Massachusetts Institute of Technology
Department of Physics
Physics 8.022 – Fall 2002
Quiz#3

- Total points in the quiz are 100. **ALL** problems receive **equal** points (20 each). Work on all problems.
- This is a closed book and closed notes exam. An equations table is given to you below.
- No programmable, plotting, integration/differentiation capable calculators are allowed.

Circuits and Waves Formulae for Quiz #3

Impedance: $V = ZI, Z_R = R, Z_C = -\frac{i}{\omega C}, Z_L = i\omega L$

Admittance: $Y = 1/Z$

Complex notation: $e^{i\theta} = cos\theta + isin\theta, z = \alpha + ib = |z|e^{i\theta}, |z| = \sqrt{\alpha^2 + b^2}, tan\theta = b/\alpha$

Displacement: current $I_d = \frac{1}{4\pi}\frac{d\Phi_E}{dt}$, density $\vec{J}_D = \frac{1}{4\pi}\frac{\partial \vec{E}}{\partial t}$

Maxwell's equations: $\vec{\nabla} \cdot \vec{E} = 4\pi\rho, \vec{\nabla} \cdot \vec{B} = 0, \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$

Electromagnetic waves: $\vec{E} = \vec{E}_0 f(\vec{k}\vec{r} - \omega t), \vec{B} = \vec{B}_0 f(\vec{k}\vec{r} - \omega t), k = 2\pi/\lambda, \omega = 2\pi/T, c = \omega/k, \hat{k} = \vec{E} \times \vec{B}$

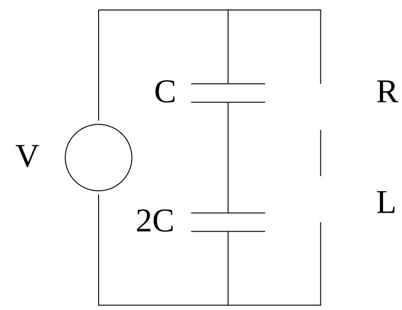
Wave "mechanics": Poynting vector $\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B}$, momentum density $\vec{g} = \frac{1}{4\pi c}\vec{E} \times \vec{B}$, energy density $u_T = \frac{E^2}{8\pi} + \frac{B^2}{8\pi}$, radiation pressure $\frac{F}{A} = u_T$

Waveguides: $\frac{\partial I}{\partial x} = -C_0\frac{\partial V}{\partial t}, \frac{\partial V}{\partial x} = -L_0\frac{\partial I}{\partial t}, \frac{\partial^2 I}{\partial^2 x} - L_0 C_0 \frac{\partial^2 I}{\partial^2 t} = 0, \frac{\partial^2 V}{\partial^2 x} - L_0 C_0 \frac{\partial^2 V}{\partial^2 t} = 0$

1. **RLC curcuit.**

   An *RLC* circuit is shown in the figure. It is driven by a sinusoidal voltage $V = V_0 sin(\omega t)$. Consider all quantities on figure as given and express all your answers in terms of them.

   - Find the total *admittance* presented to the sinusoidally varying voltage.
   - At what frequency is the above admittance purely real?
   - Determine the current as a function of time (including amplitude and phase) flowing through each of the circuit elements (i.e., I_{2C}, I_C, I_R, I_L).



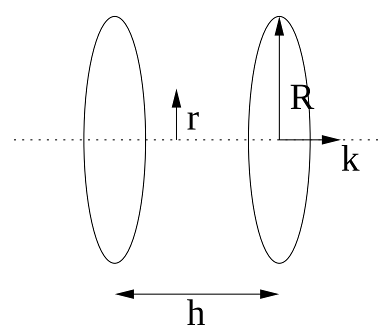
- What is the total current leaving the source (give amplitude and phase)?

2. **Displacement current in cylindrical region.**

   A cylindrical region of space of radius R and length h contains a non-uniform time-varying electric field $\vec{E}$ given by

$$\vec{E} = E_0(1 - \frac{r}{R})sin(\omega t)\hat{k}$$

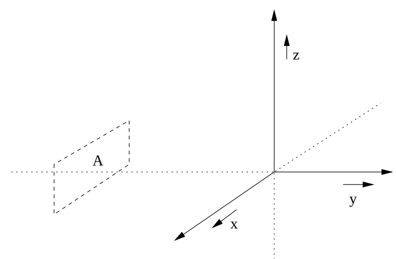
   where $\hat{k}$ is the unit vector along the axis of the cylinder, E_0 is a positive definite constant and r is the radial distance from the axis of the cylindrical region.



   - Find the displacement current density J_d in the cylindrical region ($r < R$).
   - Find the magnetic field induced in the same region.
   - Indicate on the figure the direction of $\vec{B}$ at radial distance r at $t = 0$.

3. **Electromagnetic wave.**

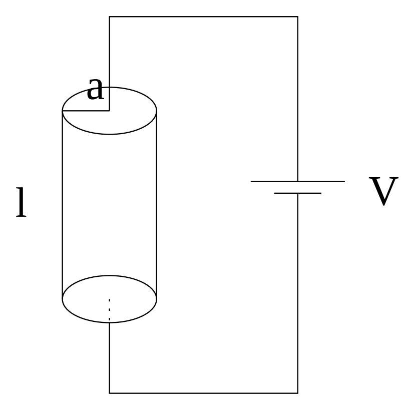
A plane polarized electromagnetic wave propagates in the vacuum in the negative y direction. The magnetic field vector $\vec{B}$ is sinusoidal (or cosinusoidal) with amplitude B_0 and it lives on the z direction. The speed of the wave is $c = \omega/k$. At $y = 0, t = 0$ $\vec{B} = B_0\hat{z}$ where $\hat{z}$ is the unit vector in the z axis.



- Write down expressions for the vectors $\vec{E}$ an $\vec{B}$.
- Sketch one wavelegth of the electric and magnetic waves on the xyz axes. Show at least one sample of the $\vec{E}$ and $\vec{B}$ vectors.
- Suppose this wave impinges on a totally absorbing plane surface of area A that lies perpendicularly to the direction of propagation. An observer monitoring this area A measures the energy U absorbed by it in time $t = 10(2\pi/\omega)$. Find an expression for U in terms of B_0, A, ω and c (the speed of light in vacuum).

4. **Poynting vector.**

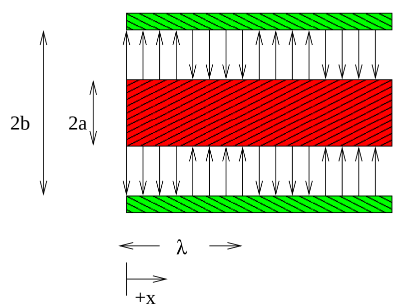
A cylindrical resistor of length l, radius a and total resistance R is connected to an Emf of strength V.



- Find the Poynting vector over the cylindrical surface of the resistor in terms of the given quantities. Draw a picture showing the direction it points to. Will this direction change if we invert the polarity of the Emf?
- Integrate the Poynting vector over the surface of the resistor and show that it is equal to the ohmic losses on it.

5. **Coaxial line.**

A transverse electromagnetic wave propagates in the annular region between two coaxial cylindrical conductors (coaxial line). Assume the line has infinite conductivity and the space between the conductors is in vacuum. We also assume that there is *no reflected* wave traveling in the opposite direction. The figure shows a longitudinal cross section of the wave guide and the electric field lines at one instant in time. The electric field $\vec{E}$ is radial and given by $\vec{E} = \frac{K}{r} cos(\frac{\omega}{c}x - \omega t)\hat{r}$ while the magnetic field is given by $\vec{B} = \frac{K}{r} cos(\frac{\omega}{c}x - \omega t)\hat{\phi}$ where K is a constant, r is the radial distance from the axis of the line ($a < r < b$) and x is the distance along the wire. Assume all constants appearing in the above two expressions as given and express all your answers in terms of them.



- Draw the $\vec{B}$ field and the Poynting vector $\vec{S}$ on the figure at the instance shown.
- Find the line voltage V, i.e., the potential of the inner conductor with respect to the outer conductor.
- Find the line current I, i.e., the current flowing along the surface of the inner conductor (and "returning" along the inner surface of the outer conductor).
- Show that the average power transmitted, i.e., the integral of the average Poynting vector over the annular area between the two conductors is equal to $\frac{I_{max} V_{max}}{2}$.