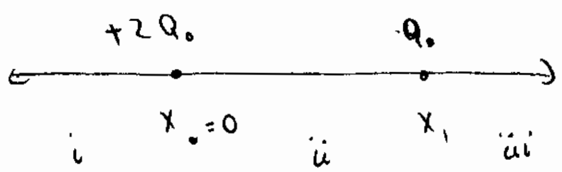
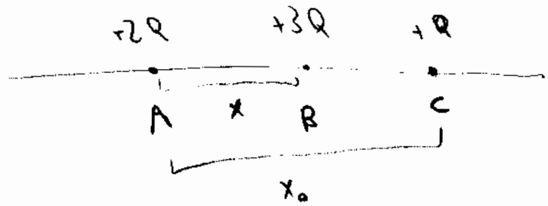
Practice Quiz #16

1.



a) In region i and iii, both charges push test charge away. So solution must be in region ii.



$$\vec{F}_{AB} = k\,\frac{2Q\cdot 3Q}{x^2}\,\hat{x} \quad , \quad \vec{F}_{CB} = k\,\frac{Q\cdot 3Q}{(x_0-x)^2}\,(-\hat{x})$$

$$\vec{F}_{Total} = kQ^2\hat{x}\left(\frac{6}{x^2} - \frac{3}{(x_0-x)^2}\right) = 0$$

$$6(x_0-x)^2 = 3x^2$$

$$\sqrt{2}\,(x_0-x) = x$$

$$x = \frac{\sqrt{2}}{1+\sqrt{2}}\,x_0$$

b) If +3Q goes closer to +Q, it is pushed to the left more strongly. If +3Q goes closer to +2Q, it is pushed to the right more strongly. So the charge oscillates around its equilibrium point. If energy is conserved, this oscilation goes on forever.

c) if +2Q charge is moved to the right, the equilibrium for +3Q charge shifts to the right. An oscilation as in b) will start.

Problem 2 (25 points)

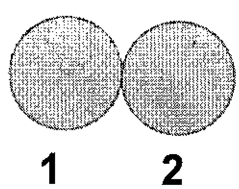
Consider the sequence of events (1) to (4) shown below. We start with 2 neutral conducting spheres that touch each other, but are insulated from the rest of the world (1). Then a positively charged rod is brought close to sphere 1, while sphere 1 and sphere 2 still touch. The rod does not touch the spheres(2). In the next step, the spheres are separated (3). Finally, the charged rod is removed (4).

(a) For each of the 4 steps, sketch the charge distribution on the spheres on the pictures below.

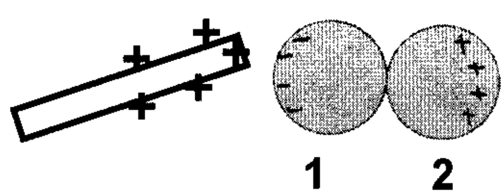
(b) After the rod has been removed (step (4)), consider the charges Q_1 on sphere 1 and Q_2 on sphere 2. Which of the following statements is true:

- (i) $Q_1 = Q_2 = 0$
- (ii) $|Q_1| > |Q_2|$
- (iii) $|Q_2| < |Q_1|$
- →(iv) $Q_1 = -Q_2$, $Q_1 < 0$
- (v) $Q_1 = -Q_2$, $Q_1 > 0$

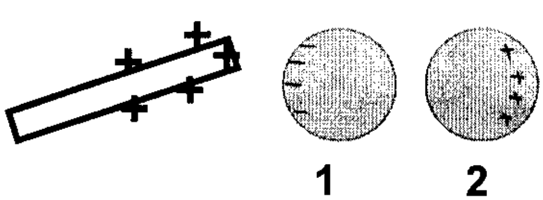
(1)



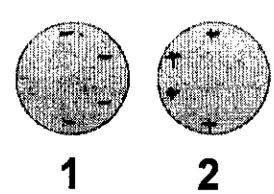
(2)



(3)



(4)

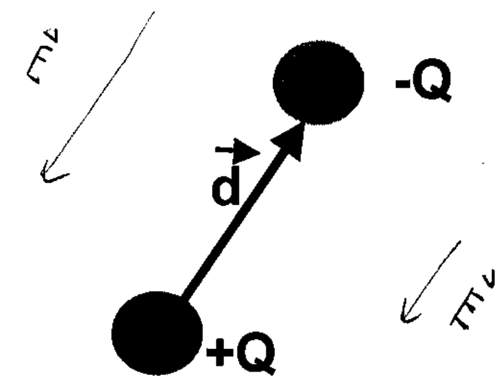


Problem 3 (25 points)

Shown below is an electric dipole with equal charges +Q and –Q separated by a distance d. The dipole is free to rotate or move. Consider the following information: The dipole sits inside an electric field with |E| > 0. The dipole does not feel a net torque. The dipole does not feel a net force. When rotated from its original orientation and released, the dipole moves back towards the original orientation.

(a) On the picture below, sketch field lines corresponding to an electric field that is compatible with this description.
(b) Qualitatively, describe what would happen if the positive charge +Q was doubled, while keeping the negative charge the same (two sentences max.)?

a)

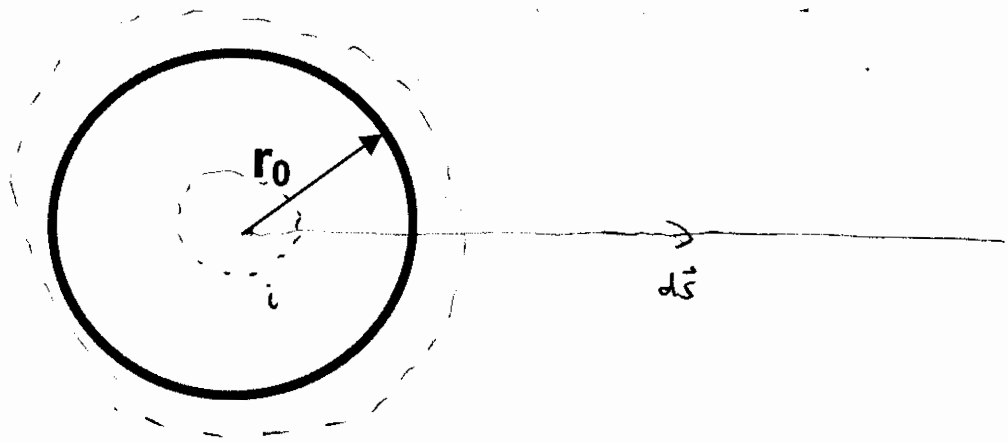


b) Net torque around center of mass remains 0 but the net force is in the $-\vec{d}$ direction. The dipole accelerates in the $-\vec{d}$ direction keeping its orientation.

Problem 4 (25 points)

Shown below is a thin conducting spherical shell of radius r_0, carrying a total charge $Q > 0$ (the thickness of the shell can be neglected).

- **(a) Find the electric field E(r) created by the charged shell as a function of r, where r is the distance to the center of the shell. Determine E(r) both for $r < r_0$ and for $r > r_0$**
- **(b) Determine the corresponding electric potential V(r) as a function of r, relative to V(r=0) = 0.**
- **(c) On the graph below, sketch the potential energy U(r) for a negative point-charge $q_0 < 0$ in the field created by the shell.**

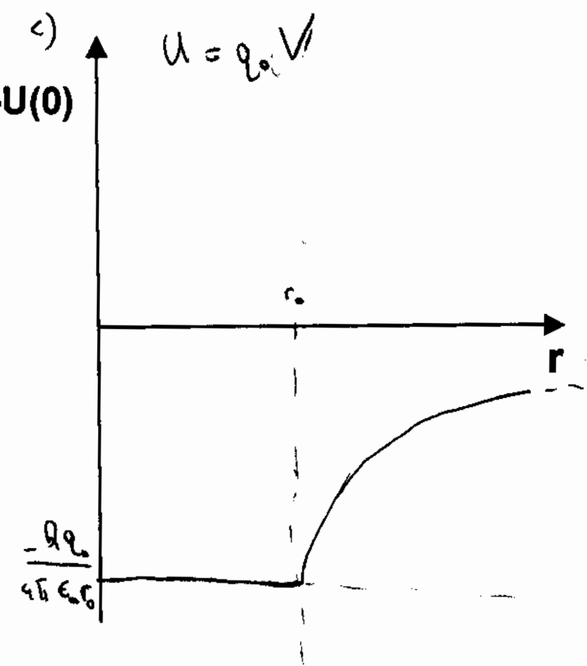


a) Because of the symmetry of the problem, we know that E must point in the $\hat{r}$ direction and is uniform on the surface of a sphere. So we can use Gauss's law.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0} = 4\pi r^2 E$$

$$\vec{E} = \frac{Q}{\epsilon_0 4\pi r^2}\hat{r} \text{ outside}, \qquad \vec{E} = 0 \text{ inside}$$

c) $U = q_0 V$



b) $V = -\int \vec{E} \cdot d\vec{s} = \int_r^\infty E\, dr$

outside $\quad V = \int_r^\infty \frac{Q}{4\pi\epsilon_0 r^2}\, dr = (-)\frac{Q}{4\pi\epsilon_0 r}\Big|_r^\infty = \frac{Q}{4\pi\epsilon_0 r}$

inside $\quad V = \int_{r_0}^\infty E \cdot dr = \frac{Q}{4\pi\epsilon_0 r}$