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PROFESSOR: OK, so happy to see you again. Welcome back to 8.03. So in the beginning of the class, I will give you a reminder about what we discussed last time. Then the main topic we are going to discuss today is about how to take good photos. So let's get started.

So last time we have been working together trying to understand how to produce electromagnetic waves which can travel to a distance which is actually very, very far away, a place which is very, very far away from the source. And what we actually figured out is that in order to do that, you have to create a kink in the electric field line such that you will propagate and produce radiation.

And this is actually what we have done last time. And what we concluded is that if you want to produce electromagnetic waves, you have to introduce acceleration of that charge, such that you will be able to produce electromagnetic waves. And also, we have derived, based on geometrical arguments, what would be the magnitude and the direction of the radiated electric field and magnetic field.

And it's actually showing here. The radiated electric field is going to be in the opposite direction of the projection of the acceleration, a perp. The magnitude is proportional to a perp, but only evaluated at retarded time, t prime equal to t minus r/c, where r is actually the distance between the observer and the radiating charge.

And the reason why we have this t prime, the retarded time, is because of the speed of propagation of information. So that's actually what we discussed last time. And you cannot actually instantaneously send the information about the acceleration of this charge to somewhere which is very, very far away. Therefore, the acceleration, a perp, is evaluated at t prime, which is t minus r/c, the amount of time for the speed of the information traveling at the speed of light.

And also, the corresponding magnetic field can be also evaluated, whether it's through Maxwell's equation or using this formula, this t here. And finally, we will be able to evaluate what would be the pointing vector, the energy flux direction or energy flux through exactly the same equation which we used before.

That's actually what we have learned last time. And today, what we are going to do is to learn how to take photos. So we have prepared ourself. We know how to produce electromagnetic waves. We also know about polarization of electromagnetic waves. And we also know how the polarizer actually works.

So that means we will be able to make very good photos, theoretically. So yeah, that's what we all care about. Theoretically, we can actually make very good photos.

So the first thing which we would like to discuss is how to make very good contrast when you actually take a photo of the sky. So as you can see, the left-hand side is a photo taken without a polarizer. And the right-hand side is the photo taken with a polarizer.

You can see-- aha! -- the contrast, or say the sky is actually darker, therefore you can see the cloud much more clearly. And also on the same graph, you can see there is a photo at the beach. And you can see exactly the same phenomenon.

And now we are in the position to understand what is going on. So this is actually why we can use polarizer to make such a good photo. So now we know what is actually happening at the sun, right?

So at the sun, there are something which is oscillating-- OK, some kind of emission from the sun. And those emissions are not correlated to each other. And that produces unpolarized sunlight. So if you're looking to the sun, you are looking at unpolarized light.

On the other hand, if you are looking at the sky, roughly like 45 or 90 degree-- OK, 45 degree from on the sun, what is going to happen is that what you are actually seeing, all this light from the sky, actually, the sunlight after scattering between sunlight and the dust in the air. So basically, on this guide, in our air, there are many, many little dust, right? And when light-- as you shine on this dust, they change direction, this so-called scattering. And those light are collected by your camera.

The interesting thing is that if you have a molecule which is actually here, and you have some unpolarized sunlight shining on this molecule, and it changed direction by 90 degrees, what is going to happen is that all the things-- originally you have an unpolarized sunlight. Therefore, you have all kinds of different polarizations, if you look at the electromagnetic wave.

However, if you only choose the light which are scattered and are going toward this direction, apparently, the electromagnetic wave, or the polarization, or let's say the direction of the electric field, has to be perpendicular to the direction of propagation. Therefore, what is going to happen is that only this direction, only the polarization in this direction, which is perpendicular to the direction of propagation will survive. All the other components, like the one which is pointing upward or pointing downwards, or coming from the original sunlight will not survive.

Therefore, what is going to happen is that when you do get 45 degrees-- the sky 45 degrees from the sun, the sunlight is actually what kind of sunlight? Is polarized sunlight. Therefore, if you tune your filter to be aligned with the polarization, you will be able to filter a large amount of the scattered sunlight. And that is actually how this works.

And then you can see that, indeed, the sky becomes darker after you apply this polarizer in front of your camera. So that's essentially the first thing which we learn. The second thing is that, OK, we also found that the polarizer is particularly usefull for the filtering of the reflected light on the window.

And of course, we can also use exactly the same technique to filter out the reflected light from the water for dipole. And how does this work? And it turns out that this actually much more complicated than what we thought. And we have to actually derive this. And that is actually related to the electromagnetic wave propagation inside the material, and also related to Brewster's angle. And that is actually the main topic which we are going to talk about today.

So let's immediately get started. So now what we are interested is, how does this actually work? And why is this happening? And why is the reflected light become polarized? So that's the question we are trying to answer.

So we have talked about electromagnetic waves in vacuum. And we also know how to generate electromagnetic, and now we are interested in electromagnetic wave in dielectrics. So we have talked about two kinds of materials already. The first one is a perfect conductor. And the second one which we are going to talk about is a dielectrics material.

In case of perfect conductor, it costs nothing to move all the charges inside this conductor around. And basically, that will give you a zero electric field inside the conductor. And also, we have a limited supply of charges. And therefore, inside this kind of perfect conductor, there will be no electric field.

On the other hand, if you have a dielectrics material, what is going to happen is that there are a lot of charges inside the material, but all those charges are attached to a specific atom or molecule. They cannot be moving freely all over the place. And that is actually so-called bound to the atom or the location of the molecule.

And that introduces a little bit of complication. So this kind of material, they also respond to the external magnetic field or electric field. For example, in this case, I have electron cloud which is around the nuclei in this-- around the positive charge nucleus in this figure. And you can see that before we apply an external electric field, it's symmetric around zero.

After we introduce this electric field, external electric field, there can be some kind of polarization produced, because the electrons around this nucleus can be moved slightly, such that this material is actually trying to compensate a little bit the effect of the external force. So that is actually leading to a modification of the electric field. But as I mentioned, you are not going to cancel all the effect of the external field.

So how do we understand this? The idea is the following. So since this system is complicated, you have a free charge. It could have free charge as we really kick out or add some electrons into this system. It can have bounce charge. And if becomes a rather complicated description.

So the idea is the followiong. So in order to actually-- for our convenience, to be similar to what we have been doing in vacuum case, our goal is to define a field which is actually with the material itself subtracted. In this case, what we can do is to define a D field, which is related-- we hope that this is actually only related to the free charge inside the material. And then we can actually do all the tricks which is actually similar to what we have already learned from the vacuum-- Maxwell's equation to solve the problems inside material. So that's essentially our goal.

In order to do that, we have to classify the total charge density, rho, into two components. The first component is the free charge, which essentially is the charges which can travel freely inside the media. And the bound charge, which is actually, as I mentioned, for example, those electrons-- the electron cloud. And essentially, they are bound around central location in the media.

So the idea is the following. So I can now define a D field. This D field is a so-called electric displacement field. It's defined as epsilon0 times E field plus a P vector, which is the polarization vector. Where this P vector is defined in the following way-- minus divergence of the P factor is actually equal to the bound charge density, which is rho.

If we have this definition and we actually continue and write down the Gauss' law-- and we can see that this is the Gauss' law, epsilon0-- divergence of E will be equal to rho. And we also-- under our classification, essentially, there are two components, rho f and rho p. The bound charge and the free charge.

And according to our definition, this can be written as rho f minus delta P vector. And now what we can do is that we can use the definition of the electric displacement field and collect all the terms except the rho f to the left-hand side. Then you will conclude that the divergence of D will be equal to rho f.

So after this calculation, you can see that what we have achieved is that we have defined a field, a displacement field D, which is totally related to the effect of the free charge. In this case, what we derived is actually delta D equal to rho f. And this actually looks pretty similar to the situation in vacuum, right? Because what we actually have is epsilon0 delta E equal to rho. And after we actually remove the contribution of the bound charge, this becomes this expression.

Any questions so far? All right, so this is actually a purely definition. And we can also do a very similar thing to the current. So the total current, J, will have the following three components.

The first component is the free current, which is the current related to free charge moving around inside this dielectrics method. There can be contribution from the bound current. The bound current is actually the current which is only moving around some specific location.

And finally, the polarization contribution. So changing polarization also introduced a current, because polarization is actually defined as q dot times D, which is the distance between charges. And that if I have a changing polarization, that means also there are some charges floating around.

And that actually gives you the third contribution, which is, let's say, P. Once we have classified the current into three pieces, and basically, we can define H field, which is actually defined as B over mu0, and the minus M, where M is a magnetic dipole moment. And the M field is actually defined as the curl of M defined as the magnitude and direction of the bound current.

And once we have finished this definition, we can actually plug that into Ampere's law. Ampere's law, just a reminder, is curl B will be equal to mu0 times J. J is actually the total current. Plus a component which is actually added by Maxwell. So that actually results in the electromagnetic waves, which is epsilon0 partial E partial T.

By using those definitions and classification, we can immediately write down-- so if we divide both sides by mu0, basically, you conclude that left-hand side is 1 over mu0 curl B. And right-hand side, you have the contribution of free charge current. And you have the contribution of bound current. And you have the third contribution which is related to a change in polarization.

And finally, you have the term which is added by Maxwell epsilon 0 partial E partial T. By using the definition which we defined here for the D displacement field, and H, which is actually defined here, you can see that now we conclude that the curl of H will be equal to Jf plus partial D partial P.

So remember why we are doing this. The reason is the following. We would like to classify the effect coming from free charge or free current inside the material by subtracting the effect from the bound current and the bound charge. So that's actually what we have been doing.

Then once we define new fields, which is actually showing here, D, which is the displacement field, is only related to-- which is the field related to the free charge. After all, those are actually just definition. And the edge field, the magnetic field which is actually only related to the displacement field and the free current, we actually arrive something really, really similar to the vacuum case, Maxwell equation.

So that's, essentially, the excitement. And you will see that in 8.03, we will use immediately those conclusions. And also, we would limit ourselves in the discussion of linear homogeneous and isotopic materials. And only work on this kind of material. And that is, you would need to highly simplify the solution for the electromagnetic field or waves inside material. Any questions so far? I hope those are just different issues. Yes.

STUDENT: [INAUDIBLE]

PROFESSOR: The bound current is actually inside the whole dielectric material. But of course, you can have many, many small loops and they will cancel. Because if you are looping-- for example, you can have many, many bound current, which is actually surrounding the atom. But you can see that all those things-- all the nearby little bound current will cancel each other.

So therefore, you could do an integration, and it becomes a total bound current, which is happening around the surface of the dielectrics material. So it depends on what you mean by how this bound current actually moves. So you do have little ones. And then if you do integral, then it becomes a surface bound current.

So if we step these two conclusions we arrive here, basically, we can immediately write down what would be the Maxwell's equation in matter. So now, instead of electric field and E and the-- E field and the magnetic field, B field, we also have-- oh, I think there's a typo there probably. Ah, there's a typo in the lower left.

So the lower left equation, which is, unfortunately, propagated to many places, should be like this. So the lower left equation should be del cross H. That would be equal to Jf plus partial D partial t. So somehow, this is actually propagated to many places.

So basically, we have Maxwell's equation in matter, which is actually really similar to what we have in the vacuum case. So very similar to a discussion we had in the vacuum case, what we actually do is to set the Jf and rho f equal to 0. If I set the Jf equal to 0, the last equation will become curl of H. And that would be equal to partial D partial t.

So you can see that this is very similar to what we had before, but the problem is that we have the H field and the D field. The question is, how do we actually relate H field and the D field with the electric field? So as we mentioned before, the D field is as we defined as epsilon0 D field plus P, which is actually the induced polarization of the material.

And in the case of very small electric field, and there is more magnetic field, and also a linear and homogeneous isotropic material, this induced polarizaiton can be proportional to the size of the electric field. So if this polarization is proportional to the electric field, I can immediately write down that the D field is actually some kind of constant epsilon times E.

On the other hand, I can also discuss the H field. H feel is defined as 1 over mu0 B minus M, which is the magnetic dipole moment. In the case of very small magnetic field and the linear material, basically, this and M vector can be proportional to the B vector.

Therefore, I can quickly rewrite this saying since the M field is also proportional to the B field in this linear material, therefore, I can rewrite that H will be equal to B divided by mu. What is epsilon and the mu?

Epsilon is the permittivity inside the material. Permittivity is actually-- they tell you the resistance, the resistance of forming electric field in some place. So if you have a large epsilon, that means there'll be large resistance coming from the material. So this makes sense, right?

Because that means you can easily introduce, or say induce, larger amount of polarization in your material. Then that means this material is not happy. It's going to try to cancel your electrical field. Therefore, if you have a large P, that will give you a large epsilon. And therefore, that means you have a lot of resistance of forming electric field, which is still the E field here. in matter.

On the other hand, this mu is actually permeability of the, material, which is the resistance of forming-- it's related to the resistance of forming a magnetic field. And basically, these two quantities are actually telling you the D field and the H field are related to the electric field and the magnetic field.

So if we assume a linear relation between D and E, and elsewhere at H and B, then we can actually immediately rewrite our Maxwell's equation in matter in the following way. So you can see that the resulting Maxwell's equation in matter considering a linear material is really remarkably similar to what we have in the vacuum case. Where is that difference? Can you see that?

The only difference is in the last three equations, which essentially, in the lower right part of the equation, instead of mu0 epsilon0, you now get mu times epsilon. So what does that mean? This means that the speed of the propagation of the electromagnetic field is changed.

It's now changed to-- instead of 1 over-- in the vacuum case, you have c equal to 1 over the square root of mu0 epsilon0. Instead of c, we are going to get c over n, which n is the refractive index. And that is actually equal to 1 over the square root of mu and epsilon. It's as simple as that.

So based on what we have prepared in the last few lectures, we can now immediately make sense of this equation. And now that we know that almost everything is the same, what is different is that now we have a difference speed of propagation, which is actually related to the refractive index we discussed last time. And now we understand what is actually this refractive index. n is actually equal to square root of mu epsilon divided by square root of mu0 epsilon0. Any questions? Yeah.

AUDIENCE: What are the D fields and H field? Are they just coefficients?

PROFESSOR: Yes. So the D field and the H field are the fields which is actually related to only the free charge. So you can see that now the gradients-- sorry, now the divergence of D field is actually only equal to the density of the free charge.

So basically, in short, the D field absorbs the effect of the bound charge into this field. And when you try to actually understand what would be the field associated or induced by the free charge, it is actually the D field. But the D field is not the full story of the electric field.

How is that related to the electric field in the exact form? This is actually defined here. And from the D field, you will be able to evaluate what would be the corresponding E field. And in the linear material, there's a spatial relation between D field and the E field, because the induced polarization is proportional to electric field in the linear material, which is a special case.

And in that sense, D field is proportional to E field. And this factor is actually so-called epsilon, which is permittivity of the material. Describing how large is the resistance of forming an electric field inside a material. OK? Is that clear? OK, good. And the H field is similar argument.

All right, so now we have made sense of the electric field and magnetic field and the Maxwell's equations in matter. And in 8.03, we will only discuss linear material. You can imagine that the relation between the polarization and the external electric field can be very complicated.

So for example, it depends on how large is the wavelengths of the external force. So if you have very slowly varying electric field, you can imagine that you will not be able to create a polarization, which is the inner electron ground level. Because the variation of the electric field is too slow.

But instead, you will be able to excite the polarization related to ions inside this material, or inside the plasma. So on the other hand, if you have a very, very fast oscillating electric field, then you will be able to measure the shift or create a polarization which is actually related to the displacement of the electron cloud inside that. So it really depends on many, many factors. But what we have been discussing here and elsewhere is highly idealized linear materials. And a lot of interesting things to explore in the future beyond 8.03.

So once we have this relation, we will be able to get Maxwell's equation in matter. And we would like to understand, just to remind you, why are we doing this?

So we have a physical question. We have a question about a phenomenon which we see in this slide. So when we use the polarizer, very strange, you will be able to filter out the reflected light from the sun and through-- when the sunlight hit the window and got reflected, this kind of contribution can be filtered out almost completely by polarizer. So that means the reflected light is somehow also polarized.

And then why is that the case is the question we were trying to answer here. So therefore, what I am going to do is to, again, take a look at the boundary between two materials. The one side is actually in the air, which we call it number 1. And the other side is actually the glass, which I call it material number 2.

And now I have, again, some incident prime wave, which is coming from the sun. And actually, this incident prime wave actually goes into this surface which is the boundary between air and glass in the discussion which I am trying to get into. I assume that this glass is actually very wide. It's actually fielding the whole universe. Therefore, this is actually just a simple plane, a boundary which actually divides our world into air and glass.

We know from the discussion of previous lecture, so there must be a reflected light and there must be transmission into the glass. Because this is actually the general property of waves. So OK, it has nothing to do with electromagnetic wave yet, but if you have wave, and you have an incident wave, you are going to have a reflected wave and the transmitted wave.

We learned from the two laws of geometrical optics, we know that the incident angle, theta i, will be equal to the reflective angle, theta r, with respect to the normal direction of this surface. And we also know that the KI, which is the wave number of vector of the incident wave, KR, which is the wave number vector, the K vector, of the reflected wave, and the K vector for the transmitting wave, KT. And this is actually theta T.

These three K vectors must have a fixed relation so that the electromagnetic field, or say all those three wave equations-- the sum of these three wave equations in the left-hand side world and right-hand side world, they are connected to each other. They don't break. There's no discontinuity between the sum of the left-hand side plane waves and the sum of the right-hand side plane wave.

So that means it has to obey Snell's law. And there will be a fixed-- the size of the projection of the K vector onto the direction of this surface will be the same. Otherwise, you will have different wavelengths in the vertical direction. Then the electromagnetic wave will break, right? Because you can-- as you always change your position when evaluating the total contribution of the incident, reflected, and transmitted waves.

So once we have all this information in hand, we can actually write down the expression for the incident wave, EI, which is actually a function of r and t. And of course, I would like to define my coordinate system first. So this is the x direction. And this is y direction. OK, sorry. This is usually the z direction actually.

The x direction is going up. y direction is actually pointing to you. And the z direction is pointing to the right-hand side of the board. And this is actually at z equal to 0, this interface between glass and air.

And now I can actually write down what will be the incident wave electric field. And that is actually equal to E0I, which is a vector, tell you about the polarization of the incident wave. Cosine-- by now this should look pretty familiar to you now. This is actually just a KI dot r, which describes the direction of the propagation. Minus omega t.

So this is actually describing the incident wave, which I call it EI. And I will assume that since I know what would be incident wave, that E0I is a known quantity. So I say that this is actually a known quantity.

So now I can also do the same thing and write down what would be the reflected wave. ER as a function of r and t. And this will be, very similarly, E0R, which is actually telling you the magnitude and the polarization of the reflected wave, cosine KR dot r minus omega t.

All right, and finally, you have the ET, which is a function of r and t again. And this will be E0T cosine KT times r minus omega t. Of course, they will have three electric fields, therefore, you must have three what field?

AUDIENCE: Magnetic.

PROFESSOR: Magnetic field, yes. So you must have the corresponding magnetic field. So you, for example, BI. It's a function of r and t. Will be equal to 1 over V1, which is the velocity in the air. V1 is actually equal to c.

KI hat, the direction of the propagation cross-- OK, so I'm not going to write down BR and BT, because this is actually a very similar expression as the corresponding associated magnetic field for the incident wave. All right, so basically, we have actually translated the physical situation into mathematics using the coordinate system, which we defined here.

So one thing which is actually very interesting is that we have solved half of the question in the previous lecture. So in the previous lecture when we discussed two-dimensional or three-dimensional waves, we have concluded that if you have a wave, and it's continuous at the surface, z equal to 0, you can immediately conclude that basically KI dot r will be equal to KR dot r. This will be equal to KT dot r.

This is the first thing which we actually conclude. That actually leads to what? What law? Snell's law. Yes, very good. Snell's law. If this expression doesn't hold, then your left-hand side and right-hand side total electric field doesn't match at the surface of z equal to 0.

The second thing which we learned is that theta I will be equal to theta R. OK, so basically, that's actually what we have learned from the previous lecture. Therefore, this match in between left-hand side electric field and right-hand side electric field becomes much simpler.

Since that they are actually just some E0 times cosine, a functional formula showing here. Since these three products are identical, therefore, what is going to happen is that the location dependence and the time dependence of this relation completely cancels. Because you just have the individual vector, E0I, E0R, E0T, multiplied by a cosine function which is identical for those three incident transmitted and the reflected waves. Therefore, in the discussion of this kind of thing, you can actually ignore the location dependence and the time dependence, since they always cancel. So that's actually a pretty useful thing to have.

And don't forget what is our goal, right? So our goal is to know, OK, so if I have a given incident prime wave, what would be the resulting reflected wave, what would be the resulting transmitted wave? And I would like to tell you the conclusion first.

The relation between the reflected wave, transmitted wave, and incident wave depends on the Maxwell simulation. You can see that this relation have nothing to do with Maxwell's equation so far. It's actually really related to the wave description we are using. And also, the the match in between the left-hand side and right-hand side wave equation.

So you know, now we have learned this is actually only related to a generic property wave equation. So now we need to get the help from the Maxwell's equation in matter so that I can make sense about the relation between E0R, E0T with respect to what is given, E0I. So how do we do that?

So the first thing which I would like to do is that, as you see from here, I divided the whole universe into two parts. The left-hand side wall, I call it 1, is air. The right-hand side wall is glass, which is I call it 2. Therefore, I can now calculate what will be the sum of the electric field in the world number 1, which is actually defined as E0I plus E0R.

So here you can see that I already dropped the cosine, because they always cancel. Therefore, I just write down E0I and E0R without the cosine. And of course, I can also calculate what will be the total electric field in the right-hand side world, inside the glass. And that is actually equal to E0T.

The question is how to relate E1 and E2. And for that, as I mentioned, I need the help of Maxwell's equation in matter. How do we actually do that? So what we could do is that since we have Maxwell's equation, the first thing which we can do is that we can look at the perpendicular direction.

The perpendicular direction means the projection of the electric field in the perpendicular direction to the surface. That's what I mean by perpendicular direction. So for that, I can make use of the first Maxwell's equation in matter, which is actually del dot D equal to 0. We are considering the case without any free charge.

So that means I have Gauss' law, which is actually D times da. I do a surface integral. And that will be equal to 0. So this means that I can have a pillbox again, like this. I arrange this pillbox like this.

And this is actually the surface. The size of one side perpendicular to the direction of the surface is actually called D here. And I have this box here. And the left-hand side, D1, will be equal to epsilon1 E perp-- D1 perp, actually, will be equal to epsilon1 E perp 1. Epsilon1 is the permittivity of the material number 1, which is actually equal to epsilon0 in this case, because it is actually air.

And the right-hand side, you have D perp 2. This is actually equal to epsilon2 E perp 2. So now I have figured out what would be the field going into this pillbox and the field going out of this pillbox.

What I can do now is the following. I can now shrink the size of the pillbox, having d goes to 0. And it becomes smaller, smaller, and smaller. So what is going to happen is that when this d goes to 0, this surface integral will go to what? Go to also 0.

Therefore, I can immediately conclude that epsilon1 E perp 1-- OK, so when I have this d goes to 0, the contribution from the side goes to 0. I know already the sum of all the contribution of the surface integral will be equal to 0, but I don't know what is the contribution from the side. But if I have that d goes to 0, then the area of the four-- 1, 2, 3, 4-- sides, it's actually going to go to 0. Therefore, you have zero contribution to this surface integral.

Therefore, I can conclude that epsilon1 E perp 1 will be equal to epsilon2 E perp 2, based on this discussion. All right, any questions so far? I hope everybody is following.

Now, I can also use the third equation, which is actually del cross E-- the fourth equation-- which is actually the-- oh, sorry for that. I can actually use the second equation, which is the curl of E would be equal to minus partial B partial t. So that means I can have an integral E dot dl. And that is going to be equal to minus d dt, B dot dx.

All right, so if I used the third equation in Maxwell's equation, basically, I have an integral over a loop, over some closed surface, and that would be equal to minus d dt, B dot-- this the actually the integral looking at the flux going through this little area.

So what I can do is now I, again, zoom in to the surface which connects the two worlds. And I can now define a loop which is like this. With width equal to d. And there can be a contribution from the magnetic field in the right-hand side integral contributing to this equation.

I can immediately write down the left-hand side will be E parallel 1. Why? Because right now we are actually looking at the component of the electric field parallel to the surface. And I can also immediately write down the right-hand side of this loop integral. You are going to have E parallel 2.

And now I can do exactly the same trick, having this d goes to 0. The effect of this is the following. So if I have this rectangular loop, and now if I have this side d goes to 0, that means you will have no area to integrate the flux for the B field. So therefore, if I have d goes to 0, this will be equal-- this also goes to 0.

Therefore, I can immediately conclude that this means that E parallel 1 will have to be equal to

E parallel 2, based on this discussion. So this means that, well, we can immediately conclude that first of all, in order to figure out the relation between the incident wave, reflected wave, and transmitted wave, we need the help not only from the general property of the wave equation, and also some matching boundary conditions, we also need the help to provide additional boundary conditions to relay the electric field in the left-hand side and right-hand side. That is actually coming from our understanding of Maxwell's equation in matter.

So you can see that the second two boundary conditions, epsilon1 E perp 1, perpendicular, will be equal to epsilon2 E 2 perpendicular. And that's essentially related to this surface integral D. And the second condition which tells us, will relate the field in the parallel direction is actually also coming from Maxwell's' equation. And we conclude that E parallel in the left-hand side will be equal to E parallel in the right-hand side.

So we are almost there to solve the puzzle. So now what I have to do is the following. So now I would like to assume that we have some kind of polarization for the incident and the transmitted waves.

So I assume that the polarization, it should be the following. The polarization is actually in parallel. I assume that the polarization is in parallel to the xz plane. So that's actually my assumption.

So now I introduce a new assumption, which is that the incident wave is actually having a polarization. The electric field is actually oscillating up and down in this direction, which is actually parallel to the xz plane. So I can write down the corresponding E0R vector. And finally-- and this will be perpendicular to the direction of propagation.

And finally, I will be able to also write down what will be the corresponding polarization for the transmitted wave, which is also, again, perpendicular to the direction of propagation So therefore I can now make use of the boundary condition one and boundary condition two to actually figure out what would be the relation between E0I, E0R, and E0T.

The first thing which we consider is to consider the perpendicular direction. For equation number one, I will be able to conclude that epsilon1 minus E0I sine theta I. So basically, that is actually the contribution from the first vector, E0I. You can see that now I am trying to project everything to the direction perpendicular to the surface.

And you can see that I have a minus sign. And that this angle is actually theta. So therefore, I have-- So this angle is actually what? This angle is actually I minus theta. So therefore, I have the cosine theta I. And I have this minus sign, because it's pointing to the left-hand side of the board.

So that's actually the contribution of the incident wave. And I have a second term, which is the contribution of the reflected wave, E0R sine theta I. So E0I, E0R with all vector is actually just the length of the vector in my definition. And based on the first boundary condition, I have the left-hand side looks like this. On the right-hand side, I will have minus epsilon2 E0T sine theta T.

Again, I am looking at the projection of this E0T vector in the direction which is actually perpendicular to the surface. So that's the first expression I can get. And then from the expression number two, I can now-- taking the projection, which is parallel to the surface, so now this is actually the parallel direction. And basically, what I'm going to get is E0I cosine theta I plus E0R cosine theta I-- oh sorry, theta R. Theta R is actually equal to theta I, so therefore, I actually replaced that by the I already.

And that will be equal to E0T cosine theta T. Any questions so Far so I was pretty fast. So here, I already immediately write down this is actually theta R. Theta R is equal to theta I, so therefore, I already replaced that by theta I. Do you have any questions?

So the painful period is going to end in like 3 minutes, OK? So we are almost there. And look what we have been doing. We basically figured out the boundary condition from Maxwell's equation. Then we are plugging in that.

So we assume that the polarization is actually parallel on the plane of xz plane. And now we are actually just evaluating the parallel component and the perpendicular component. So that's actually what we've got.

The goal, as a reminder, is to write E0R and E0T in terms of the known part, which is the E0I. I would like to relate these three-- the magnitude of these three vectors. So from equation number one, I can actually rewrite that. Basically, I can actually conclude that I can divide everything by epsilon1 sine theta I.

So what I'm going to get is E0I minus E0R. And this will be equal to epsilon2 sine theta T divided by epsilon1 sine theta R. And this is actually E0T. So basically, I'm dividing everything by minus epsilon1 sine theta I.

Then basically, what you are going to get is this expression. And this actually can be related to another expression, which is epsilon2 n1 divided by epsilon1 n2. Because I can use Snell's law. n1 sine theta I will be equal to n2 sine theta T. Therefore, I can replace this ratio of sine angle by refractive index. Everybody's following? OK, very good.

And this will be multiplied by E0T. I can now define this. This is actually defined as theta E0T to make our life easier. The same thing can be done for the second expression. What I'm going to get is E0I plus E0R.

Basically, what I'm doing is to divide everything by cosine theta I. So what is going to happen is that you are going to get cosine theta T divided by cosine theta I E0T. And that is actually defined as alpha E0T. So alpha is defined as cosine theta T divided by cosine theta I.

Therefore, I can already immediately, based on these two expressions, to solve what would be the E0R. I can write down the solution. So basically, you can actually quickly derive what would be the E0R. And that is actually going to be my alpha minus beta divided by alpha plus beta, E0I.

And you can also solve based on these two equations what would be the E0T. And now we will conclude that this will be equal to 2 divided by alpha plus beta, E0I. So this means that the refractive index will be equal to alpha minus beta. Sorry, refraction coefficient will be equal to alpha minus beta divided by alpha plus beta. And the transmission coefficient is actually 2 over alpha plus beta.

So we will take maybe a three minute break for you to be able to ask some questions. But you can see that we have solved the relation between E0I, E0R, and E0T. And what we have to do in the rest of the time is to enjoy what we actually already derived, and what actually that means, after the break. So we come back at 45.

OK, so very good. So we have survived this. And now it's time to enjoy what we have learned from this equation. All right, so welcome back everybody. So we have actually saw what would be the refraction coefficient and the transmission coefficient tau. And those are the functions of alpha and beta.

So that's considered three different interesting cases. So if I have normal incidence. And that means alpha will be equal to cosine theta T divided by cosine theta I. This is the definition. If I have no more incidence, that means both theta T, theta I, and theta R will be equal to 90 degrees.

And in this case, basically, you will have the same cosine theta-- I mean cosine I. Theta T and cosine theta I. And that means your alpha will be equal to 1 in this case. So in normal material, mu1 is roughly equal to mu2 and roughly equal to mu0. So therefore, this means that if mu1, mu2, and mu0 are very close to each other, then actually, the refractive index based on that equation which is showing there, will be basically, roughly equal to square root of epsilon divided by epsilon0.

So therefore, beta would be equal to epsilon2 divided by epsilon1, n1 divided by n2. This is actually the definition. And this will be basically equal to-- since this is actually proportional to epsilon2 divided by epsilon1. And n is actually proportional to epsilon, square root of epsilon. Therefore, you can conclude that this actually will be equal to n2 squared divided by n1 squared, n1 divided by n2.

And that will give you n2 divided by n1. You can actually cancel one of the n2 and one of the n1. So beta will be equal to n2 divided by n1. And therefore, you can conclude that R will be a function just related to the refractive index, which means that you are going to get n1 minus n2 divided by n1 plus n2. Which means that the amount of reflected light is related to the difference in the refractive index.

Then the amount of the transmitted light will be equal two n1 divided by n1 plus n2. So what does that mean? This means that if you have some material which is essentially like diamond, diamond have an n2 equal to something like 2.6, that means a lot of light will get reflected, even if you have no more incidence.

So that's actually why the diamonds are so beautiful, because a lot of light, pretty bright, and a lot of things are actually reflected. The transmitted fraction is actually pretty small. I can also assume that there can be a grazing incidence. That happens-- this means that I am going to have theta I. This theta I should be here.

This theta I is going to be roughly 90 degrees. In the case of no more incidence, theta I should be equal to zero. Maybe I misspoke in the beginning. So what does that mean? This means that alpha will go to infinity, because theta I is going to 90 degrees. Therefore, you have R roughly equal to 1, because R is actually alpha minus beta divided by alpha plus beta.

If alpha goes to infinity, then R will go to 1. And the tau will be-- actually, roughly goes to 0. So that means if you have a grazing incidence, then basically, most of the light are reflected. So that's actually why when we see, for example, reflected light from the sun, which is actually on the road, we see that a lot of light are reflected. When we see a lake which is actually very far away from me and the sun in front of it, you see a huge amount of lite got reflected and going to your eyes. Looks really bright.

So finally, there is a very interesting angle, which is actually considering a situation when alpha is equal to beta. This is a very interesting angle, theta B. If we choose this theta B such that alpha is equal to beta, what is going to happen? Somebody can tell me. If I make--

STUDENT: [INAUDIBLE]

PROFESSOR: Exactly, right? So if you choose an angle such that alpha is equal to beta, then R will be equal to 0. There will be no reflection. And everything goes through the material. So that is actually so-called Brewster's angle.

And this happens when theta B-- which theta B is actually the incident angle-- plus theta T is equal to pi over 2. There's a proof of this Brewster's angle in the lecture notes. But we are kind of running out of time.

But the conclusion is that you will need to have theta B, which is actually equal to theta I, the incident angle, plus the transmission light angle, theta D. If that is equal to 90 degrees, then you can make alpha equal to 0. And what is going to happen is that there will be no reflected light.

So when this happens, when the reflected light and the transmitted light have an angle of 90 degrees, then the amplitude goes to 0. This is actually a very interesting property and it only works for electromagnetic waves, because this is actually coming from, really, the effect of Maxwell's equation. So now what I'm going to say is that basically, we look at this demonstration.

So if we have an incident light, and the transmitted light. Originally, the incident light is unpolarized. So you can have all kinds of different polarization. So we can become post polarization into a component which is actually pointing to you, which is the dot. And the component, which is actually the parallel to my slide, which is actually what we have been working on, that situation.

So what is going to happen is that the component which is pointing you is actually never gets suppressed, because there will be no perpendicular component. So therefore, even if you add Brewster's angle, it should get reflected. On the other hand, all the components which essentially heavy polarization parallel to this slide is eliminated because of this relation.

So that means the reflected light will be highly polarized. Do you believe me? Maybe not. We can do an experiment and really show you that's the case. So we are almost there.

So now I need to turn off the light and hide the image. And you can see that there is a setup here which I produce unpolarized light. And there's a glass here, which actually I reflect the unpolarized light. So now you can see that if I have some random angle, and I have a polarizer here-- I hope you can see it-- you can see that the polarizer cannot eliminate all the light.

So basically, no matter what kind of direction, it will not be able to eliminate all the reflected light. This means there's some mixture of all kinds of different polarization. But now, if I change the direction to Brewster's angle, it's roughly here, so you can see that now, indeed, I can actually eliminate all the contribution of the reflected light. Because the reflected light is highly polarized.

As you can see from the slide, all the component which is actually parallel to the slides is actually eliminated due to Brewster's angle. And that produces a polarized light. And that can be filtered out by the polarizer.

So coming back to the question which we had before, so why can we take such a good photo? That is because of Brewster's angle. So once the sunlight gets reflected by the window, it becomes linearly polarized, and therefore, you can actually filter out the majority of the contribution by using polarization filter.

OK, thank you very much. And I hope you enjoyed the lecture today. And hope this will improve your technique, your skill, for taking good photos. If you have any questions, please let me know.

Hello, everybody. So today I'm going to show you a demonstration of Brewster's angle. So during the class, we were discussing about how to make very good photos, how to use polarizer to filter out the reflected light from the sun. Usually, when you actually take a photo of water or a car, there are refracted light from the sun on the window or on the water. And then you can actually use polarizer to filter them out.

And that has to do with the property of the electromagnetic wave and the Brewster's angle. So here I have an experimental setup here, which consists of three components. The first component is a polarized light source. And it meets unpolarized light. And those light are getting refracted by glass here.

And the unrefracted light will actually be shown on the screen as a spot there. So at first, if I have my glass, which essentially-- the position of the glass is in a way such that it's actually not on Brewster's angle. And now I can actually check if this light is actually polarized by using a polarizer here. And if I put this polarizer between the screen and the glass, you can see that, huh, as a function of the angle which I am rotating this polarizer, you can see that no angle can actually completely eliminate the reflected light.

So that means that the reflected light is actually not perfectly polarized. But on the other hand, you can also see that in some angles, you can actually significantly lower the intensity. And that, essentially, is also pretty good for photo taking, because that means all the reflected light, although now the angle is actually not at Brewster's angle, you still have the reflected light slightly polarized.

So that actually your polarizer in front of the camera will still do some work. Now what I'm going to do is to change the angle so that it matches with Brewster's angle. So now you can see that if I insert a polarizer between the glass and the screen, you can see that at some angle, for example, now we can actually filter out or completely eliminate the spot on the screen.

So that means at Brewster's angle, basically, the reflected light is actually completely polarized, as we actually predicted from the lecture. And the reason is the following. There's only one direction of the polarized light from the unpolarized source can get reflected due to the boundary condition of the electromagnetic wave. And therefore, we see this very unique phenomena which we can only see in electromagnetic waves.