YEN-JIE LEE: So welcome back, everybody. This is the final exam checklist. For the single oscillator, we need to make sure that you know how to write down the Equation of Motion. We have discussed about damped, under-damped, critically damped, and over-damped. We did that. Oscillators, and we have tried to drive oscillators. We observed transient behavior in steady state solution. Resonance, right, so which we actually demonstrated that by breaking the glass. And then we moved on and tried to couple multiple objects together. And that brings us to the coupled system. What are the normal modes? And how to actually solve M minus 1 K matrix, the eigenvalue problem.

What is actually the full solution for the description of coupled systems. And can we actually drive the coupled system, and we found out we can. So the system would respond as well similar to what we have seen in the single oscillator case. We see resonance as well. We can excite one of the normal modes by driving the coupled system. Then we put more and more objects until at some point, we have infinite number of coupled objects. What is actually the solution of refraction and the transmission-- refraction and the translation symmetric system. That is actually the discussion of symmetry.

We go to the continuum then, and we actually found wave and wave equations. So we found that finally, we made the phase transition from single object vibration to waves, and that is actually an achievement we have done in 8.03. We have discussed about different systems, massive string, massive spring, sound wave, electromagnetic waves, and we have discussed a progressive wave and standing waves. For the bound system, we have also normal modes.

We discussed about how to actually do Fourier decomposition, and what is actually the physical meaning of Fourier decomposition in 8.03. For the infinite system, we also learned about Fourier transform and uncertainty principles. And we learned to apply boundary conditions so that we constrain the possible wavelengths of the normal modes. Therefore, we also learned about how to put a system all together. Finally, how to determine the dispersion relation, which is omega as a function of K, the wave number.

Until now, we discussed idealized systems, and we also moved on to discuss dispersive medium. We have learned some more, even more about dispersion relation for the dispersive medium and signal transmission, how to send signal through a highly dispersive medium. The solution we were proposing is to use an amplitude modulation radio and also the pattern of dispersion. The group velocity and phase velocity, we covered that. As I mentioned before, the uncertainty principle. A 2D/3D system.

We have bound system, which we have normal modes for two-dimensional and three-dimensional systems, as well. Because we're all over the place, so just make sure that you know how to actually dewrap all those standing waves for different dimensions of systems. We showed and approved geometrical optics, which essentially is the direct consequence of waves. Wave function, a continuation of the wave function and boundary condition. We learned about the refraction rule and also Snell's law.

We talked about polarized waves, linear, circularly polarized, elliptically polarized, and the polarizer and quarter-wave plate. At the end of the discussion of 2D/3D systems, we discussed about how to generate electromagnetic waves by accelerated charge. Finally, we went on and talked about how those EM waves propagate in dielectrics and again, boundary conditions, which leads to interesting phenomena, which belongs only to electromagnetic waves. For example, Brewster's angle. So the refraction amplitude-- refraction-- the wave amplitude is governed by the property of the electromagnetic waves, which is coming from the laws which governs electromagnetic waves, which is Maxwell's equations in matter.

We were trying to also manipulate those waves by adding them together, and we see constructive and destructive interference and diffraction phenomena. Then we connect that to quantum mechanics by showing you a single electron interference experiment. That connects us to the beginning of the quantum mechanics, which is the probability waves, which behave very different from other waves we have been discussing. But you are going to learn a lot more in 8.04. OK, so don't worry. All right. So that is the checklist. You can see that I can write it in two pages, so it's not that bad, probably. I hope that there was nothing really sounds like new to you by now. If you find anything is new, you have to review that part. That means you missed a class.

All right. So what I'm going to do now is to go through all the material faster than the speed of light. All right. So that you will get nauseated. No, you are going to get a list of the topics. You just have to feel it. If you feel good, like when you are having a cupcake, right, then you are good for the final. If you don't feel good, what is Professor Lee talking about? He's talking about nonsense now. Then you are in trouble, and you have to review that part. All right? OK. So that's what we'll do. So let's start.

All right. Why 8.03? We started a discussion-- welcome, We started a discussion of 8.03 and it's vibrations and wave systems, is name of this, 8.03. And the motivation is really simple, because we cannot even recognize the universe without using waves and vibration. You cannot see me, and you cannot hear anything, and you cannot feel the vibrations-- sorry, the rotation of a black hole by your body anymore, then it's not very cool. Therefore, we study 8.03 to understand the basic ideas about waves and vibrations. And we found that waves and vibrations are interesting phenomena. Waves are connected to vibrations. Because if you look at only, for example, a single object on these waves, you see that it is actually a single object which is oscillating up and down, oscillating up and down, and this is your vibration. So there's a close connection between single particle vibration and the waves. And that is the first thing that you learned.

Therefore, we need to first understand the evolution of a single particle system. And we make use of this opportunity to start the discussion of scientific matter. So using this opportunity, basically, what we have been doing for the whole class is the following. So the first step is always to translate the physical situation which we are interested into mathematics, right? Because mathematics is the only language which we know which describes the nature. If you come out with a new language, and that is going to be a super duper breakthrough, it cannot be estimated by Nobel Prize. But the problem we are facing now is that this is the only language we know which works. Therefore, we really follow this recipe, which is similar to many, many other physics classes.

And we have a physical situation, we use laws of nature or models, and we have a mathematical description, which is the Equation of Motion. And this is actually the hardest part, because you need to first define a coordinate system so that we can express everything in a system in that system, then you can make use of the physical laws you have learned from the previous 8.01 and 8.02 to write down the Equation of Motion. And most of the mistakes, and also most of the problems or difficulties you are facing is always in this step. Then we can solve the Equation of Motion, which is, strictly speaking, not my problem. It's the math department's problem. Yeah, that's their problem.

Then we solve the Equation of Motion, and you will be given the formula. Then we use initial conditions and then make predictions. And then we would like to compare that to experimental results. And that is the general thing which we have been doing for physics. So let's take a look at those examples. Those are examples of simple harmonics motions. And you can see that these, all these systems have one object, which is oscillating. And you can see that their Equations of Motion are really similar to each other. It's theta double dot plus omega, zero squared, theta equal to zero for those idealized simple harmonic motions. And we learned that the solution of those equations are the same, which is a cosine function.

Then we went ahead and added more craziness to the system. So basically, what we tried to do is to add a drag force into the game. And we were wondering if this more realistic description can match with experimental data. So this is the Equation of Motion, and the additional turn is the one in the middle, the gamma theta dot turn. And after entering these turns, not only is this an interesting model to describe the physical system we are talking about, but the mathematical solution is far more richer than what we talked about in the single harmonic oscillator case. Basically, you see that a general solution depends on the size of the gamma compared to omega dot zero, which is the oscillation-- the natural frequency of the system. OK. And then you can see there are three distinct different kinds of solutions. They have different mathematical forms. And we call them under damped solution, critically damped solution, and over damped solution. So those equations will be given to you.

And the excitement is the following. So you can see that those solutions, if you plot the solution as a function of time, they look completely different as a function of time. So in the case of no damping, the amplitude is actually the constant, it's not actually reducing as a function of time. But when the damped system, the damping is turned on, then in the under damped situation, you can see that they end up reducing as a function of time. And if you have too much damping, you put the whole oscillator into some liquid, for example, and you see that oscillator disappear. The cool thing is the following. The excitement from-- as a physicist is that all of those crazy mathematical solutions actually match with experimental results. Wow. That is really cool. Because there is nobody saying that these should match and how, naturally, I should learn that OK, when should I change the behavior of the system. So this is really a miracle that this complicated mathematical description is useful and that it is super useful to describe the nature.

Once we have learned that, we can now add a driving force into ligand. From the equation here, we can see that there is a natural frequency, omega dot zero, of this system, and there is a drag force turn, which is actually to quantify how much drag we have, we have a gamma there. We are driving it at a driving frequency omega t. So what we have learned from here is that if you are driving this system, you are-- for example, I am shaking that student, shaking you. OK, in the beginning, this student is going to resist. No, don't shake me. Come on. But at some point, he knows that Professor Lee is really determined. Therefore, he is going to be shaked at the frequency I like. OK. So that is actually what is happening here.

This is so-called transient behavior. So in the beginning, the system doesn't like it. So this is making use of the superposition principle. So you can solve that homogeneous solution, which is on the right-hand side. It depends on the physical situation you are talking about. You choose the corresponding homogeneous solution. And lamba and psi is the driving force from E and G, right, and that is going to win at the end of the experiment, because I'm going to shake it forever, until the end of the universe. So you can see that at the end, you-- what is left over is really the steady state solution. And it has this structure, A omega d, depends on omega. And you get resonance behavior. Don't forget to review that. So you have a delay in phase because when I shake the student, the student needs some time to respond. Therefore, the delta is non-zero if the student is damped. All right.

So now we have learned all the secrets about a single object system. Then we can now go ahead and study coupled oscillators. There are a few examples here, which is coupled pendulums or coupled spring-mass systems. And we found that a very useful description of this kind of system is to make use of the matrix language. So originally, if you have n objects in a system, you have n Equations of Motion, and that looks horrible. But what is done in 8.03 is that we introduce a notation with a matrix. Basically, if you write everything in terms of matrix, then it looks really friendly, and it looks really like a single oscillator. OK? Although solving this equation is still a little bit more work. And basically, you can see that from this example, we can actually derive M minus 1 K matrix, and the whole equation won't be-- the Equation of Motion problem solving problem becomes an M minus 1 K matrix eigenvalue problem. What is an M minus 1 K matrix? This is describing how each component in the system interacts with each other.

Once we have this, we can solve the eigenvalue problem, and we are going to be able to figure out the normal modes of those systems. So what is a normal mode? Normal modes is a situation where all the components in the system are oscillating at the same frequency and they are also at the same phase. So that is the definition of normal mode. And those are what is used in a deviation, also, which leads us to the eigenvalue problem. We define Z equal to X 1 H or I omega t plus 5. Everybody is oscillating at omega and also at phase 5, right? So that is what we actually learned.

And what is actually the physical meaning of those normal modes? So if we plot the locus of the two coupled pendulum problem, what we see is the following. So basically, you will see that the locus looks like really complicated as a function of time if you plot X1 X2 versus time. But if we rotate this system a bit, then we find that there's a really interesting projection, which is the principal coordinate. You see that all those crazy strange phenomena we see with coupled systems are just illusions. Actually, you can understand then by really using the right projection. To one-- to the right coordinate system. Then you will see that actually the system is doing still simple harmonic motion. So that is actually the core thing which we learned from coupled systems. So we learned about how to solve the coupled system, and we also learned about going to an infinite number of coupled systems.

So then this is an example here. So for example, I can have pendulum and springs, and we connect them all together, and I need to hire many, many students so that they plays it, plays until it fills up the whole universe. So this is the idea of an infinite system. You can see that that means my M minus 1 K matrix is going to be an infinite times infinite long matrix. It's two dimensional. And the A is infinitely long. And that sounds really scary. And in general, we don't know how to deal with this, really. And it can be as arbitrarily crazy as you can imagine.

What we discuss 8.03 is a special case. Basically, we are discussing about systems which are having a spatial kind of symmetry, which is translation symmetry, as you can see from all those figures. And you can see that all those figures will have to all have the same normal modes because of this base translation symmetry. What we discussed about is that we introduce an S matrix, which is used to describe the kind of symmetry that this system satisfies. And if we calculate the commutator S and M minus 1 K matrix, if this commutator shows that the evaluate-- if you evaluate this commutator and you get zero, now it means they commute. And the consequence is that the S matrix and the S M minus 1 K matrix will share the same eigenvectors.

So you don't really need to know how to derive this-- to arrive at this conclusion, but it is a very useful conclusion. So that means instead of solving M minus 1 K matrix eigenvalue problem, I can now go ahead to solve the S matrix eigenvalue problem. And usually, that's much easier.

So for the exam, you need to know how to write down S matrix. You need to know how to solve eigenvalue problems, including M minus 1 K matrix and the S matrix. And then we can get to normal mode frequency, omega squared, and we can also solve the corresponding normal modes.

And here is telling you what would be the solution for space translation of the matrix system. And basically what we will see is that making use of the S matrix should be-- brings you to the conclusion that A, j must be proportional to exponential i, j, k, a, where this A is the length scale of this system, the distance between all those little mass. And the j is a label which tells you which little mass I am talking about. And k is the-- some arbitrary constant. But by now, you should have the idea basically that's-- that's what? That, essentially, is the wave number, right? So that is really cool. So that's all planned in advance.

And basically, you can see that we can also write down the A, k because we know that A, j will be proportional to exponential i, j, k, A, after solving the eigenvalue problem for S matrix. Then we actually went one step forward to make it continuous. So basically, we made the space between particles very, very small. And also, at the same time, we make sure that the string doesn't become supermassive. And we concluded that we get some kind of equation popping out from this exercise. M minus 1 K matrix becomes minus T over rho L partial square, partial x squared. You don't have to really derive this for the exam, but you would need to know the conclusion and that psi j becomes psi as a function of x and t. And the magical function appeared, which is the wave equation. Oh my god, this is the whole craziness we have been dealing with the whole 8.03.

This is actually really remarkable that we can come from single object oscillation, putting it all together, making it continuous, then this equation really popped out. And this equation really describes multiple systems. Then we went ahead to actually discuss the property of the wave equation. It looks like this. Basically, I replaced the t over rho L by v, p squared. By now, you know the meaning of v, p is actually the phase velocity. And we discussed two kinds of solutions, special kinds of solutions. The first kind is normal modes. The second one is progressive wave solution, or traveling wave solution, whatever name you want to call it.

Let's take a look at the normal modes, what have we learned. So if you have a bound system, a bound continuous system, the normal mode is your distending waves for the wave equation we discussed. And basically, the functional form is A, m, sine, k, m, x plus alpha, m and sine, omega, m, t plus beta, m. So what we actually learned from the previous lecture is the following. So basically, you can decide the k, m and alpha, m by just boundary conditions. So before you introduce boundary conditions, which are the conditions allow you to describe multiple nearby systems consistently. So that is the meaning of boundary condition. Before you introduce that, k, m and alpha, m are arbitrary numbers. Whatever number you choose is the-- can satisfy the wave equation.

But after you introduce the boundary condition, you figure that out from the problem you are given, then k, m and alpha, m cannot be arbitrary anymore. And they usually become discrete numbers. OK. So that is what we learned from the previous lectures. And finally, we also see that omega, m is determined by the property of the system, by a so-called dispersion relation. In this case, it's linear, it's proportional to k, m, because we are talking about non-dispersive medium for the moment. And we have this beta, m, which is related to the initial condition. And the a, m, which can be determined by a Fourier decomposition. So if you are not familiar with this, you have to really review how to do Fourier decomposition. I know most of did very well on the midterm, but maybe some of you forgot how to determine a, m and it will be very, very important to review that for the preparation for the final.

Now the second set of solutions is the following. So you have progress-- progressing waves. And the functional form is really interesting. So you can see this can be written as F, F is some arbitrary function, x plus-minus v, p, t. Basically that is that you're describing a wave which is traveling to the positive-- to a negative or positive direction in the x direction. Or you can actually write it down as G function k, x plus-minus omega, t. Actually, they all work for wave equations. Now we went ahead and applied approach which we learned from the general solution of wave equation to massive strings, and we discussed about sound waves.

For the sound waves, it will be important to review what are the boundary conditions for the displacement of the molecules in the sound wave, compare that to the pressure deviation from the room pressure. So I think it's important to make sure that you understand the difference between these two, what are the boundary conditions and basically it should be very similar to the solution-- the boundary condition for the massive strings. And we also talked about electromagnetic waves. And that is another topic which you will really have to review. Several things which are especially interesting is that an electric field cannot be without a magnetic field. They are always together, no matter what.

So if you have trouble with the electric field, then there must be trouble in the magnetic field. And that is governed by the Maxwell's equation. Before we go into the detail of those, we also discussed about dispersive medium. So in the case of dispersive medium, we used a special kind of example, which is strings with stiffness. So basically, what we found is that if you have a certain kind of wave equation, like this one, I am writing this one here. Basically, if I add the additional term to describe the stiffness, then what is going to happen is that the dispersion relation, when I ask you to plot the dispersion relation, you will be-- I am requesting you to find the relation between omega and K. And I'm going over this in more detail because I see so many similar mistakes on the midterm. So basically what I'm asking is omega versus K.

And in the-- if we don't have this turn, then basically, you have a straight line. Straight line means you have a non-dispersive medium. And if you add this turn, you need to know how to evaluate the dispersion relation. The quickest way to evaluate the dispersion relation is to just simply plug in the progresssing wave solution for the G function or harmonic progressing wave solution, find omega-- K, x plus-minus omega t, into this equation, then you will be able to figure out the dispersion relation. And what we figure out is the following. If we include stiffness, then you can see that the dispersion relation is not a line anymore and is actually some kind of curve, and the slope is actually changing.

And there are dramatic consequence from this thing. That means if I have a traveling wave with different wavelengths, that means the phase velocity v, p equal to omega over K is going to be different for waves with different frequencies, or different wavelengths. So that is how you clear the problem. Because if I have initially produced a signal which is a triangle and I let it propagate, what is going to happen is that the slow component will be lagging behind. Those are the slow components. And the fast components will go ahead of the nominal speed. So there will be a spread of the signal. Originally, maybe you have some kind of a square wave, and this thing will become something which is actually smeared out in space, and then you lose the information. And we are going to talk about that later.

And we also learned about group velocity. So what is your group velocity? Group velocity v, p-- oh, sorry, v, g is actually partial omega, partial K, which is the slope of a tangential line here. And where the phase velocity is connecting this point to that point, and the slope of this line is the phase velocity, and the slope of the line cutting through this point, which is giving you the group velocity. And we actually learned the definition of-- the consequence of group velocity and phase velocity by introducing you a bit phenomena. Basically, we add two waves with similar wavelengths, or wave numbers. Basically, what we see is the following. So basically, you see some behavior like this. We see this-- the superposition of these two waves which produce a bit phenomena can be understood by something which is oscillating really fast modulated by a much slower more variating envelope. Basically, you can actually understand the bit phenomena by actually identifying these two interesting structures. And the speed of all those little peaks is traveling at phase velocity. And the speed of the envelope is found to be traveling at group velocity.

So that is what we have learned. And we can have group velocity and the phase velocity traveling in the same direction. And we can also have a negative group velocity. So that is a technique which is really, really very difficult. And I'm still trying to practice and make sure they I can demo that in 8.03. Basically, it's like the whole system, the whole detailed structure moving in a positive direction. But the body, or say the envelope, is actually moving in the negative x direction. So that is also possible. And you can actually construct a system which has a negative group velocity. So once we have done that, we also tried to understand further the description of the solution for the dispersive medium. So basically, what we actually went over during the class is that OK, now, if the f function f of t is describing Yen-Jie's hand, and I'm holding an infinitely long string and I shake it as a function of time, and that essentially, this motion, is actually described by this f function.

What we know is that this oscillation, OK, I can do one, but I won't, but all kinds of f functions can be described as superpositions of many, many, many waves with different angular frequencies. So that's a miracle which we borrowed from the math department again. And you can see that f function can be written as the sum of all kinds of different waves with different angular frequencies with population c omega. This is the weight which makes that become the f function. And we can figure out the c omega by doing a Fourier transform. And finally, what will be the resulting wave function, psi, x, t, which is the wave function generated by the oscillation of my hand. And those are governed by the wave equation, which gives you the relation between omega and the k can be returned in that functional form.

So the good news is that with the help of Fourier transform, we can also describe and predict what is going to happen no matter if this system is dispersive or not dispersive using this approach. OK. So that is really cool. And you can of course can do a cross-check just to-- assuring that this is a non-dispersive medium. And you are also going to get back to what you should expect the solution to non-dispersive medium for the psi, x, t. So that is one thing which is really remarkable. And I think what is needed to know is not a deviation of all those formulas, but how the plotting and the derived c omega by using the formula you are given and how to then put together all the solutions and it becomes the resulting solution for the psi, x, t, which is really the solution we really care. So for that, you need to know how to do the integration. You need to know how to derive the dispersion relation. Then one thing left over is to put the problem into that equation, which is also given to you in a formula. And we will not ask you to do a very, very complicated integration for sure on the final.

So what is the consequence? Basically, one thing which is interesting to know is that if you have a wave in a coordinate space, which is really widely spread out, and you can do a Fourier transform to get the wave population in the frequency space, what we find is that when this wave is really, really wide in the space, then what we find is that the wave population in the frequency space is very narrow by using a Fourier transform. And that just gives you the result. And on the other hand, if you have a really-- a very narrow pulse in the coordinate space, for example, I do this-- shwhew --very, very-- really quickly. I create a very narrow pulse. And then what is actually happening is that I will have to use a very wide range of frequency space to describe this very narrow pulse.

So that leads to-- direct consequences of that is uncertainty principle. And this is closely connected to the uncertainty principle we talk about in quantum mechanics. Delta, p times delta, x greater or equal to h bar over 2. All right. So we have done with the one-dimensional case. And we also talked about a two-dimensional and a three-dimensional case. And this is the example of two-dimensional membranes, and they actually are constrained so that their boundary condition at-- the boundary is equal-- no, the wave function is equal to zero. And you can identify all those normal modes.

And we went ahead also to talk about geometrical optics laws. Basically, how we derive that is to have a plane wave. First, you have a plane wave propagating toward the boundary of two different mediums, and we were wondering well, what is the refracted wave and the transmitting wave. By using the-- by making sure just one point, which is that the membranes don't break, the wave function is continuous at this boundary. That's the only assumption which you use. We went through the mathematics, which you don't really need to remember all of them. But you really need to remember the consequence. The consequence is the following.

Basically, what we see is that if you have incident plane wave with incident angle theta 1, the refractive wave will be having an angle of theta 1 as well. So that's the first law of refraction, refraction law. And then the second one which we learned is that the transmitting wave will satisfy Snell's law, n, 1, sine, theta, 1 equal to n, 2, sine, theta, 2. And that is very interesting because this, Snell's law has also nothing to do with Maxwell's equation. You see? Right? That's actually what you can learn from here. We usually use electromagnetic waves to demonstrate Snell's law. But from 8.03, we learned that it has nothing to do with Maxwell's equation. It applies to all kinds of different systems, which you can-- which can be described by wave functions. So that is actually the very important consequence.

But on the other hand, as we all discussed later, the relative amplitude of the incident wave, refracted wave, and transmitting wave, the relative amplitude is governed by Maxwell's equation. So I would like to make that really crystal clear. So the relative amplitude is governed by really the physical laws, which actually governs the propagation of those plane waves.

OK. So I think we can take a five minute break to have some air. And of course, you can-- you are welcome to continue to use all this juice and coffee. And coming back at 38.

OK. So welcome back, everybody, from the break.

AUDIENCE: [INAUDIBLE]

YEN-JIE LEE: So we are going to continue the discussion. We have learned about the two important laws for the geometrical optics. And we also went ahead to discuss the polarization that's solved in greater detail. So for example, we can have linear depolarized wave. So basically, the wave is essentially moving up and down, up and down. But the direction of the background field doesn't change. It's always, for example, initially, if it's in x direction, then it is x direction forever. And in that case, I call it linearly polarized.

Of course, I can also have the case that I can have a superposition of two waves. One is having the electric field in the x direction. And the other one is in the y direction. And they are off by a phase of pi over 2. If that happens, then basically, you will see that it produces something really interesting. That direction of the electric field is going to be rotating as a function of time-- as a function of the space these waves travel. And we call it circularly polarized waves. And we can also have elliptically polarized wave.

Then we learned about how to do a filtering, which is the polarizer. So suppose I have a perfect conductor here, where I have the easy axis, which is described by the green arrow there. And you can see that easy axis means that if you have electric field parallel to the easy axis, and then since that's the easy axis, so it is supposed to be easy, therefore, this electric field is going to be passing through the polarizer. On the other hand, if the electric field is perpendicular to the direction of the easy axis, that means it's taking the perfect conductor in the hard way. Therefore, when it pass through-- when it is trying to pass through with the perfect conductor, the electrons in those conductors are going to be working like crazy to deflect this wave when the direction of the electric field is perpendicular to the direction of the easy axis. So that is how this works.

For example, in the first example, you can see that in this case, you have an easy axis which is perpendicular to the direction of the electric field, which is the red field, then this wave actually got refracted. There will be no transmission-- sorry, no electromagnetic field passing the perfect conductor. And on the other hand, if you have another perfect conductor, in which you have easy axis which is parallel to the electric field, then you can-- you will see that it will pass through the perfect conductor. So that is the polarizer.

And also, we discussed about quarter-weight plate, which I would suggest you to have a review about the concept which we have learned about polarizer and quarter-wave plate so that you make sure that you understand how to calculate the electric field after passing through a polarizer and quarter-wave plate and how the secondary, or the elliptically depolarized waves are created using all those wave plates, et cetera.

All right. So the next thing which we discussed during the class is how do we produce electromagnetic waves. I think by now, you should know that a stationary charge doesn't produce electromagnetic waves. Even a moving charge at constant speed doesn't create electromagnetic waves. So how do we create an electromagnetic wave which propagates to the edge of the universe? That is-- the trick is to create a kink in the fuel line. So you have to accelerate and stop it. Accelerate and then try to actually stop the acceleration. So then you can create a kink. And this kink is going to be propagating out of the-- as a function of time. And this kink is creating the so-called radiation from this accelerated charge.

So you don't really need to remember all the deviations, but you really need to know the conclusion. So what is the conclusion is the following. The radiated electric field is equal to minus-- very important that there's a minus sign in front of it, which is a common mistake to drop it, and the q is the charge of the oscillating-- the accelerated charge, proportional to the charge. If the particle is more charged, then you have more radiation. Aperp is the acceleration projected to, which is-- the perpendicular projection of the acceleration of the particle with respect to the direction of propagation is so-called the Aperp. And only the perpendicular direction acceleration counts. The one which is parallel to the direction of propagation doesn't really count, as you can see from this equation.

And the t prime what is t prime? t prime is t minus r divided by c. So t prime is the retarded time, so that is telling you that it takes some time for the information to propagate from the origin, which is the position of the moving charge to the observer, which is r, this distance, away from the moving charge. So the information takes some time to propagate, and you cannot know what is really happening, for example, 100 light years away from Earth. You have no idea about what is happening. Maybe a black hole is created there and is going to suck everybody up in a few years. But nobody knows, and we don't care because we cannot control it.

All right so that is very important. And also very important to know the magnetic field must be there. You can see the relation between magnetic field and the electric field. And the Poynting vector is also its joint field. And when we went ahead, given all the knowledge we have learned, we discussed about how to take very beautiful photos using a polarizer filter. And we discussed about how to filter out the scattered light from the sun. And it would be nice to figure out why this is the case, how these polarizer lines, scatter lines are created. It's purely geometrical. And also, we discussed about Brewster's angle and also how it leads to the explanation of the filtering of the light, the refracted light from the, for example, window of a car or from the water.

And this is the demonstration of-- the summary of Brewster's angle. So somebody reminded me that the amplitude should be given. So I think, this is the amplitude formula for Brewster's angle will be given to you. If not, it's asked in the final exam. So don't be worried about it, and you don't have to remember this formula. And I'm not going to ask you to derive that just in such a short time, the three hours in the final exam. But what is very important is to know how this Brewster's angle, why there's no refracted light polarizing in a way that the polarization should be-- why the refracted light is polarized, for example. And also why the transmitting wave is slightly polarized. And I think the conclusions you need to remember, and you need to know how to calculate the angle, at least. Because for this purely polarized light to be produced in a refracted light, you need to have normal angle between the direction of the refracted light and the direction of the transmitted light.

And that, you should be able to remember. And you should be able to derive that also from your mind as well, because that means the direction of the oscillation of the molecule at the boundary will be in the direction of propagation of the refracted wave. Therefore, that cannot be the solution to the progressing electromagnetic wave. Therefore, the refracted waves are polarized. So if you follow this logic, then you don't really need to memorize all those formulas.

All right. So finally, in the last part of the course, we focused on the superposition of many, many electromagnetic waves so you can produce constructive interference. Or that means all those waves are in phase. And you can have destructive interference when they are out of phase. And that is a very important topic, so you should review that for the preparation of the final. And you can see that there are three concrete examples which we used during the class. A laser beam. We talked about a water ripple in a demo. And we also studied how it make use of this phenomena to design a phased radar.

So to detect this unknown object in the sky, what we really need to have is electromagnetic waves pointing to a specific direction. And that can be achieved by using multi-slit interference. And this is the property of the two-slit interference pattern. And you are going to have many, many peaks. They have equal height for two-slit interference If you ignore any effect coming from diffraction. So we've assume that the slit is infinitely small. The slit is super narrow. And then we can ignore the diffraction-- single-slit diffraction. In fact, then all the peaks due to this two-slit interference will have the same height. On the other hand, when we start to increase the number of slits, for example, unequal to 3, unequal to 4, unequal to 5, unequal to 6, as you can see that, the structure of the intensity as a function of delta, which is the phase difference, is actually changing. And you can see that the general structure is the following.

So if you have unequal to 3, then basically, you have 2 of adult, and between them, you have 1 child. And if you have unequal to 6, then basically, you have 2 adults and somehow there are 4 children in this collection. So basically, that is what we learned from the solution of the multi-slit interference. And in this way, we can actually make the width of the principal maxima as narrow as you want. So that is why phased radar works. And then we discussed about diffraction. So that is related, again, to the explanation of laser beams. And we discussed about the design of a Star Trek ship, the gun for the ship. And we also talked about resolution. And what is actually happening here is the following.

A single-slit diffraction essentially can be viewed as an infinite number of source interference. And you just need to integrate over all the point-like sources between the two walls. And all of them are acting like a spherical wave source. So basically, for every point-- continuously, every point between these two walls are a point source of spherical waves. And that is

Huygens' principle. And we can see that the structures is-- of the intensity as a function of position is the following.

So basically, you have a principal maxima, which is a peak in the middle. And at some angle, basically, you have destructive interference such that if you integrate over all the contributions from an infinite number of sources in this window, basically, you would see that they completely cancel each other. So that is the origin of all those deep structure minima. And then, after the minima, actually, you will see another peak, but the height of the peak is suppressed by 1 over beta squared. And it would be good to review that. And what is the consequence?

So if you shoot a laser beam to the moon, the size of the laser beam will be very large. After you learn 8.03, you know that the size of the laser beam is going to be very, very large due to interference between all the point-like sources from the laser beam. And finally, we can put them all together. So the single-slit diffraction and the multi-slit interference, you can put them all together, and basically, what you get is the following. So basically, you have a multi-slit interference pattern, which is showing there. But now the intensity of the multi-slit pattern is modulated by the single-slit diffraction pattern. And of course, the full formula will be given to you. But on the other hand, you are also requested to know how to calculate, just to add the contribution from multi-slit together in case if we change the amplitude of the incident light or we change the phase, like what we did in the homework. And I think that is one important point, and you should review that. And if you are not sure about how to proceed with that, it would be good to review Lecture 22, Lecture 23.

So finally, we talk about the connection to quantum mechanics. Einstein already told us that "I have said so many times, God doesn't play dice with the world." But what we actually find is that there are two very interesting things which we found. The first thing is that if we have a single photon source, and basically, if we don't play dice, we cannot explain the intensity of the-- after this single photon source passes through two polarizers. And what happens is the following. Basically, the result of a single photon source tells you that you really need to play dice so that you can get the resulting polarized light intensity.

And also, the second pseudo-experiment we discussed is that if you have billiard balls, basically, you have them pass through the two-slit experiment, what you are going to get is two piles, Gaussian-like distribution. And if you have a single electron source, what it does is that it interferes with itself. An electron, a single electron, can interfere with itself and produce a pattern which is very similar to what we see in the double-slit interference pattern. So that is really remarkable. And also, we talked about a single-slit-- single electron experiment. That gives you also a diffraction pattern. We have to use the wave function to describe the position-- the probability density of the position of the electron on the screen. And know this issue closely connected to the uncertainty principle, which we discussed earlier, delta, p, delta, x is greater than or equal to h bar over 2. So if you have a very narrow window, that means you have very similar delta x, so you have very, very good confidence about the location of the electron. And then the momentum is in the x-- in the momentum in the x direction, you have large uncertainty, according to this equation. And that can be seen from this single-slit diffraction pattern and it is closely connected to what we have learned before.

So where is this-- how to actually describe what this is really the dispersion relation of the probability density wave is actually coming from Schrodinger's Equation. And this is given here. We briefly talked about that. And the consequence is the following. You can describe the evolution of the wave function as a function of time by using this wave equation. And this wave equation is slightly different from what we have learned before. And we also can use what we have learned from 8.03 to solve a particle in a box problem, which is covered in lecture number 23.

And I just wanted to say that you need to know the general principle, but I'm not teaching 8.04, so I'm not expecting you to solve a quantum mechanics problem. But I would like to say that OK, from this point, it's motivating you to take 8.04, right? Because there can be a lot of fun there as well. And it is closely related to what we have learned from 8.03.

So I just want to say, the last point is that this is really not the end of the vibrations and waves. It's just the beginning. And that there is a path toward the peak. And it may take a long time to reach the peak. All right. And I would like to let you know that I'm really, really very happy to be your lecturer this semester. And I really enjoyed teaching this class and getting your responses when I asked questions. Thank you for the support. And I would like to say good luck with the final exam. And we have 800 contributions on Piazza, many thanks to Yinan, who is actually doing all the hard work, day and night. And thank you very much, and see you around MIT in the future. Thank you.