MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department
Earth, Atmospheric, and Planetary Sciences Department

Physics 8.282J – EAPS 12.402J May 20, 2005

Final Exam

Name ____ **(please print)**
Last **First**

1. Do any **6** of the first **8** problems - indicate which of these you want graded.
2. Do problems **9** and **10**.
3. Each numbered problem (i.e., #1–#10) is worth 13 points.
4. Closed book exam; you may use two pages of notes.
5. Wherever possible, try to solve the problems using general analytic expressions. Plug in numbers only as a last step.

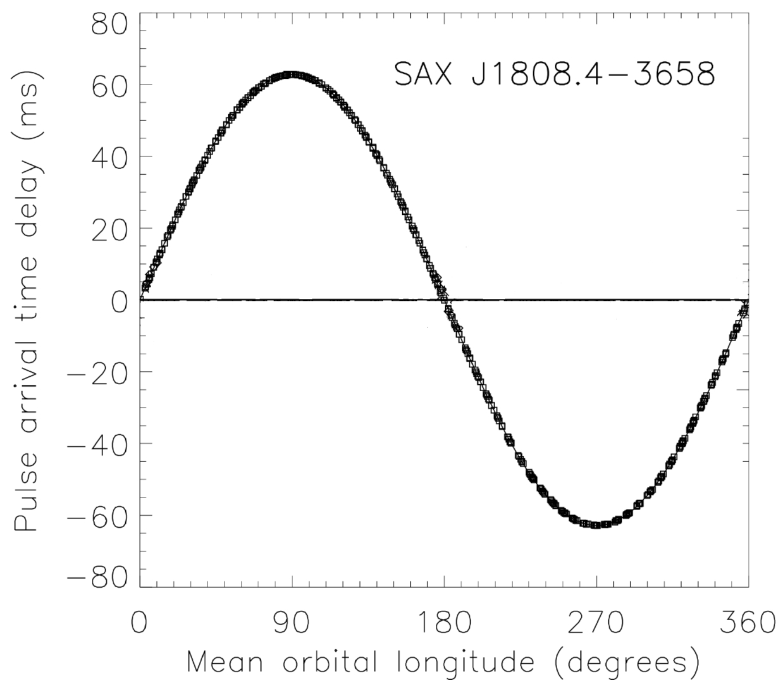
Problem	Grade	Grader
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
Total		

APPROXIMATE VALUES OF USEFUL CONSTANTS

Constant	cgs units		mks units	
c (speed of light)	3×10^{10}	cm/sec	3×10^{8}	m/sec
G (gravitation constant)	7×10^{-8}	dyne-cm^2/g^2	7×10^{-11}	N-m^2/kg^2
k (Boltzmann's constant)	1.4×10^{-16}	erg/K	1.4×10^{-23}	J/K
h (Planck's constant)	6.6×10^{-27}	erg-sec	6.6×10^{-34}	J-sec
$m_{\rm proton}$	1.6×10^{-24}	g	1.6×10^{-27}	kg
eV (electron Volt)	1.6×10^{-12}	erg	1.6×10^{-19}	J
$M_\odot$ (solar mass)	2×10^{33}	g	2×10^{30}	kg
$L_\odot$ (solar luminosity)	4×10^{33}	erg/sec	4×10^{26}	J/sec
$R_\odot$ (solar radius)	7×10^{10}	cm	7×10^{8}	m
σ (Stefan-Boltzmann cons)	6×10^{-5}	erg/cm^2-sec-K^4	6×10^{-8}	J/m^2-sec-K^4
Å(Angstrom)	10^{-8}	cm	10^{-10}	m
km (kilometer)	10^{5}	cm	10^{3}	m
pc (parsec)	3×10^{18}	cm	3×10^{16}	m
kpc (kiloparsec)	3×10^{21}	cm	3×10^{19}	m
Mpc (megaparsec)	3×10^{24}	cm	3×10^{22}	m
year	3×10^{7}	sec	3×10^{7}	sec
day	86400	sec	86400	sec
AU	1.5×10^{13}	cm	1.5×10^{11}	m
$1'$ (arc minute)	2.9×10^{-4}	rad	2.9×10^{-4}	rad
$1''$ (arc second)	4.9×10^{-6}	rad	4.9×10^{-6}	rad

Problem 1 (Binary Orbit)

The orbit of the first accretion-powered millisecond X-ray pulsar was measured here at M.I.T. using data from the Rossi X-Ray Timing Explorer Satellite. The Doppler delay curve for the motion of the neutron star is given in the figure below. The amplitude of the sine curve indicates the projected light travel time across the orbit of the neutron star around the center of mass of the binary.



(a) Use the figure to estimate the value of $a_{\rm ns}\sin(i)$ for this system, where i is the orbital inclination angle. Express your answer in cm or m.

(b) Assume that the neutron star has a mass of $M_{\rm ns} = 1.4\,M_\odot$ and that the orbital inclination is $i = 30°$. The orbital period of the binary is $P_{\rm orb} = 2$ hours. Compute the mass of the companion star, m_c. Hint: in order to solve for m_c you will have to make the approximation that $m_c/M_{\rm ns} \ll 1$ and, equivalently, $M_{\rm ns}/m_c \gg 1$.

Problem 2 (Planetary Atmosphere)

A planet of radius, R, and mass, M, has a very hot atmosphere that extends out to several times the planet's radius. However, the amount of mass contained in the atmosphere is completely negligible compared to the mass of the planet, i.e., gravity is determined entirely by the planet. Note: *gravity is not a constant in this problem!*

(a) Write down the equation of hydrostatic equilibrium in a form relevant to describing the atmosphere in this problem.

(b) Assume that the temperature of the planet's atmosphere is a constant, T_0, independent of height above the surface. Further assume that the atmospheric gas, consisting of atoms of mass, m, acts as an ideal gas. Solve the equation of hydrostatic equilibrium to find the density, $\rho(r)$ as a function of radial distance, r from the center of the planet. Your answer should be in terms of the density, $\rho_0(R)$, at the surface of the planet.

Problem 3 (Nuclear Burning)

A horizontal branch star has radius, $R_{\rm star}$, and photospheric temperature, $T_e = 7000$ K. The stellar luminosity is $40\ L_\odot$. The mass and radius of the He-burning core are $R_{\rm core} = 0.07\ R_\odot$ and $M_{\rm core} = 0.5\ M_\odot$, respectively.

(a) Compute the overall radius of the horizontal branch star, $R_{\rm star}$.

(b) The expression for the nuclear energy generation rate per gram for the triple alpha reaction is:

$$\mathcal{E}_{3\alpha}(\rho, T) \simeq 4 \times 10^{11} \rho^2 e^{-4295/T_6} \quad \text{ergs gm}^{-1}\ \text{sec}^{-1} \quad ,$$

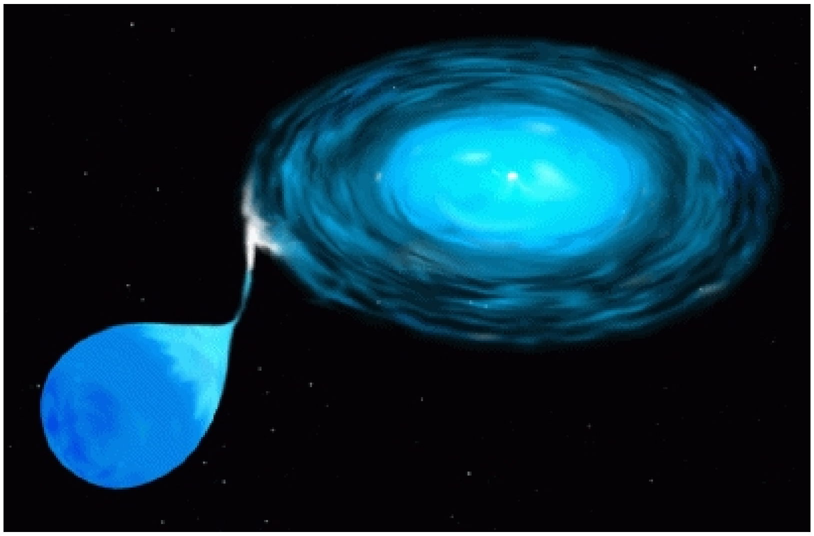
where ρ is the density in the core (in grams cm^{-3}) and T_6 is the core temperature in units of 10^6 K. If you wish to use mks units:

$$\mathcal{E}_{3\alpha}(\rho, T) \simeq 40 \rho^2 e^{-4295/T_6} \quad \text{Watts kg}^{-1} \quad ,$$

where ρ is expressed in kg m^{-3}. Take both the density and temperature in the core to be constant throughout the core. Compute the density, ρ, and temperature, T, *in the core.*

Problem 4 (Accretion Disk)

Matter from a companion star flows through the L1 point of the Roche lobe and spills into the accretion disk of a compact accreting star (e.g., a neutron star). The situation is illustrated in the sketch below (taken from APOD).



(a) A parcel of matter of mass, Δm, makes it through the accretion disk from a very large outer radius (taken to be infinity) to a radius, R, from the compact star. If the mass of the compact star is M, compute the decrease in potential energy of the parcel. Ignore the gravity of the companion star (shown in the lower left corner of the figure).

(b) If mass passes through the disk at radius, R, at a rate $\dot{m} = \Delta m/\Delta t$, compute the luminosity, L that must have been released in the disk for all radii $> R$.

(c) Use the result of part (b) to compute the amount of luminosity, ΔL, released in an annulus of the disk between R and $R + \Delta R$.

(d) Assume that all of the released potential energy is radiated by the accretion disk as black-body radiation. The disk has a surface temperature, $T(R)$, which depends on the radial distance from the central compact accretor. Equate the power radiated by the disk in black-body radiation, in an annulus between R and $R + \Delta R$, to ΔL found above in part (c) due to the potential energy decrease of the matter moving through the disk. Find $T(R)$ in terms of quantities specified in the problem and fundamental constants.

Problem 5 (Short Answers)

(a) Suppose that an interstellar cloud of mass, M, and radius, R, undergoes essentially free-fall collapse in the initial stage of star formation. Assume that the cloud is spherically symmetric in its density structure. Write down the equation of motion for a test mass at the outer boundary of this cloud that essentially describes the free-fall collapse. Solve for the collapse time by using a dimensional analysis, and evaluate this time in units of years if $M = 100\ M_\odot$ and $R = 10$ pc.

(b) Use the Virial Theorem to estimate the mean internal temperature, T, of the Sun. Useful relation: $1/2m\,\langle v^2 \rangle = 3/2\,kT$, where m is the mass of a gas atom, v is its speed, k is Boltzmann's constant, and T is the temperature of the gas. For purposes of this problem, assume that the Sun is made entirely of hydrogen. Give your answer in degrees K.

Problem 6 (21-cm Line of Hydrogen)

The 21-cm line of hydrogen is observed from a distant galaxy. The galaxy is not resolved, i.e., the 21-cm radiation from the entire galaxy is detected in a single receiver. The rest-frame frequency of the 21-cm line is 1420.4 MHz. The radio observations of this particular galaxy show a broadened line centered at 1390 MHz and having a width of ± 1.5 MHz.

(a) What would you infer about the distance to this galaxy?

(b) What approximate rotation speed would you infer?

Problem 7 (Galaxy Rotation Curve)

A spiral galaxy resides in a dark matter halo of density:

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{r_0}\right)\left(1 + \frac{r}{r_0}\right)}$$

where ρ_0 is a constant with the dimensions of density, r is the radial distance from the galaxy center, and r_0 is a scale length of the problem (also a constant). For purposes of this problem assume that the mass in visible stars, i.e., the spiral galaxy, is negligible compared with the dark matter.

Starting from $F = ma$, find the *rotation* curve, i.e., $v(r)$, for stars in this galaxy. Assume simple circular motion for the stars. Express your answer in terms of radial distance from the galaxy center, r, the two parameters of the problem, ρ_0 and r_0, and fundamental constants.

Possibly useful integrals:

$$\int \frac{dx}{(1+x)} = \ln(x) \ ; \int \frac{dx}{x(1+x)} = \ln\left(\frac{x}{1+x}\right) \ ; \int \frac{xdx}{(1+x)} = x - \ln(1+x)$$

Problem 8 (Supernovae)

Compare the energy released, $\mathcal{E}$, in a Type II supernova with that of a Type Ia supernova.

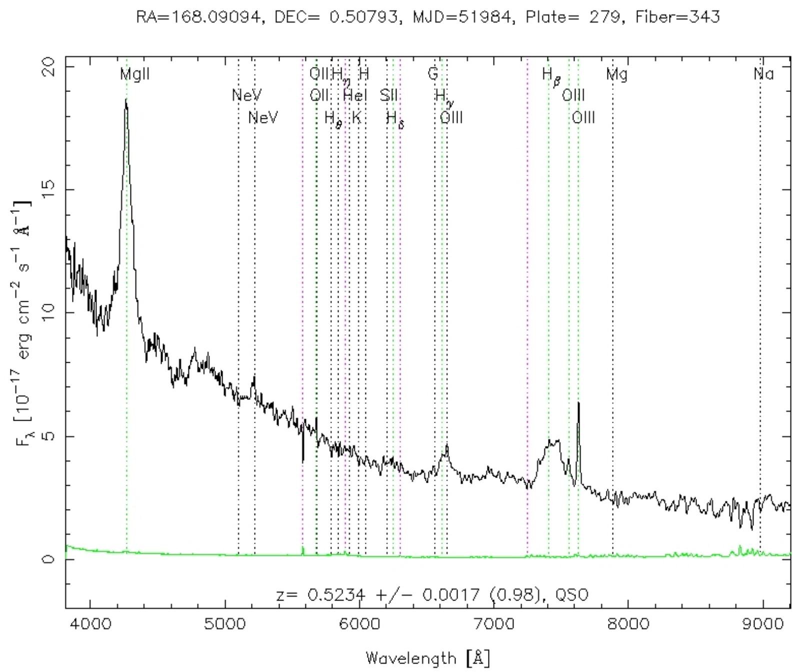
(a) For the case of a Type II supernova consider the gravitational potential energy released when the degenerate core (of Fe) exceeds the Chandrasekhar limit and collapses from white dwarf dimensions to form a neutron star. Adopt some plausible values for the radii of white dwarfs and neutron stars. Compute $\mathcal{E}_{II}$.

(b) For the case of a Type Ia supernova suppose for simplicity that this is triggered when a white dwarf made of carbon exceeds the Chandrasekhar limit, burns to Fe, and explodes. The atomic mass excesses of these two nuclei are 0.0 MeV for ${}_6C^{12}$ and -60 MeV for ${}_{26}Fe^{56}$. Compute $\mathcal{E}_{Ia}$.

(c) In fact, Type II supernovae are actually less luminous than Type Ia's. Can you suggest why this might be true in spite of your results in part (a) and part (b)?

Problem 9 (Quasar Luminosity)

The figure below shows the spectrum of a quasar discovered with the Sloan Digital Sky Survey. Two of the prominent emission lines are due to Hβ and MgII whose rest wavelengths are 4861 Å and 2798 Å, respectively. The bolometric flux from this quasar is $F = 10^{-12}$ ergs cm^{-2} sec^{-1} (or, $F = 10^{-15}$ Watts m^{-2}).



(a) Compute the z of this quasar (or equivalently the Doppler shift). Use this information and your choice of Hubble constant to compute the distance to the object. (You may use the simple linear relation between z and distance even though z for this object is a bit too large for this to be quite right.)

(b) Find the luminosity of this quasar.

(c) If the quasar is powered by a $10^7\ M_\odot$ black hole, estimate the accretion rate required to produce the observed luminosity. If you are unable to do parts (a) or (b) simply assume that the luminosity of the quasar is 10^{13} times the luminosity of the Sun (not necessarily meant to be the correct value).

Problem 10 (Cosmic Evolution)

The equation governing the evolution of the "scale factor" of the universe is given by:

$$\dot{a}^2 = H_0^2 \left[(1 - \Omega_M) + \Omega_M / a \right] \quad ,$$

if we neglect the effects of radiation and "dark energy". Here, Ω_M is the ratio of the current matter density to the critical density.

(a) For the case where $\Omega_M = 1$, solve for the scale factor as a function of time. Leave H_0 in symbolic form. Factors of order unity *do count.*

(b) Again for the case where $\Omega_M = 1$, find the age of the universe when light was emitted that we now observe to have a redshift z, i.e., the $t(z)$ relation.

(c) If the cosmic microwave background (currently with $T = 2.72$ K) was formed when the universe had a temperature of 3000 K, use the result of part (a) or part (b) to find the age of the universe (in years) when the CMB was formed. Take $H_0 = 70$ km sec^{-1} Mpc^{-1}.