

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.282

March 8, 2000

Quiz 1

Name Solutions
Last First (please print)

1. Closed book exam; you may use one page of notes.
2. Be sure to attempt all problems.
3. **Important:** Wherever possible, try to solve the problems using general analytic expressions. Plug in numbers only as a last step.

Problem Grade Grader

1	25	
2	25	
3	25	
4	25	
Total	100	

Problem 1 (25 points)

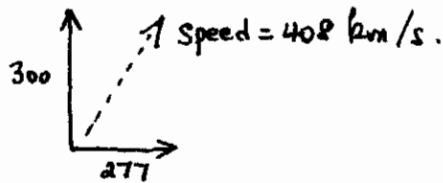
A spectrum is taken of a single star (i.e., one *not* in a binary). Among the observed spectral lines is one from oxygen whose *rest* wavelength is 5007 Å. The Doppler shifted oxygen line from this star is observed to be at a wavelength of 5012 Å. The star is also observed to have a proper motion, μ , of 1 arc second per year (which corresponds to $\sim 1.5 \times 10^{-13}$ radians per second of time). It is located at a distance of 60 pc from the Earth.

(a - 19 pts) What is the 3 dimensional velocity of the star?

(b - 6 pts) What parallax would be measured for this star?

a)
$$V_r = \frac{\Delta\lambda}{\lambda} c = \frac{5}{5000} c = 300 \text{ km/s}$$

$$V_T = 1.5 \times 10^{-13} \times D = \dot{\theta} D = \mu D = 277 \text{ km/s}$$

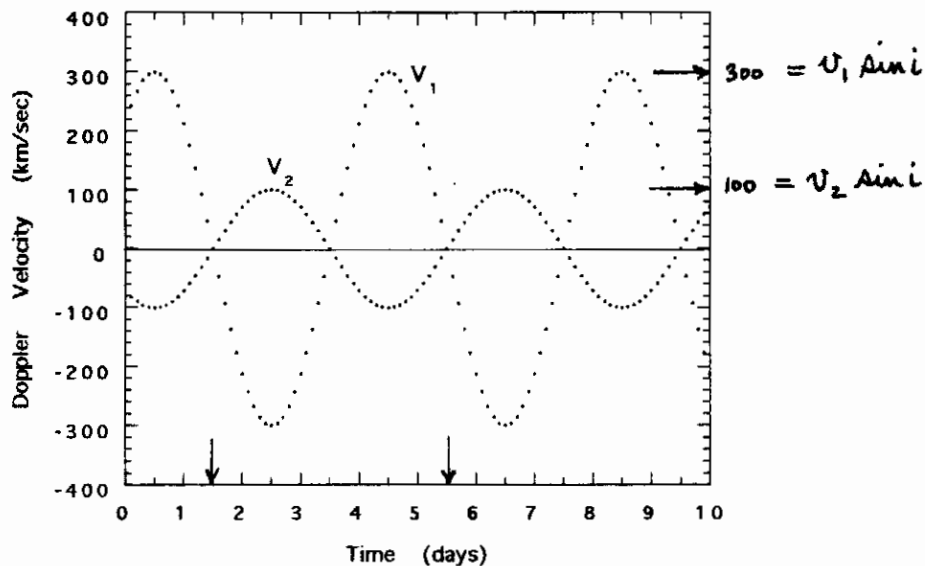


b)
$$\text{parallax} = \frac{1''}{60}$$

Problem 2 (25 points)

The velocity curve for a double-line spectroscopic binary is shown in the sketch. The system is viewed edge on, i.e., with an inclination angle of $i = 90^\circ$, so that the maximum possible Doppler shifts for this system are observed.

- (a - 3 pts) Find the orbital period.
 (b - 3 pts) Find the orbital speed of star 1 and star 2.
 (c - 3 pts) What is the mass ratio of the two stars?
 (d - 16 pts) Find the masses of the two stars, preferably in solar units.



- a) $P = 5.5 - 1.5 = 4 \text{ days}$
 b) $v_1 = 300 \text{ km/s}$ $v_2 = 100 \text{ km/s}$ (since $i = 90^\circ$, $\sin i = 1$)
 c) $m_1 v_1 = m_2 v_2$ $\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{100}{300} = \frac{1}{3}$ so, $m_1 < m_2$

d) $\frac{G m_1 m_2}{(r_1 + r_2)^2} = \frac{m_1 v_1^2}{r_1}$ $\frac{G m_1 m_2}{r_1 (1 + \frac{m_1}{m_2})^2} = m_1 v_1^2$
 From $\vec{F} = m\vec{a}$
 $m_2 = \frac{v_1^2 r_1 (1 + \frac{m_1}{m_2})^2}{G}$ But $\frac{2\pi r_1}{P} = v_1$
 $m_2 = \frac{v_1^2 v_1 P}{2\pi G} (1 + \frac{m_1}{m_2})^2 = \frac{v_1^3 P (1 + \frac{1}{3})^2}{2\pi G}$

$$m_2 = 3.96 \times 10^{34} \text{ g} = 19.8 M_\odot$$

$$m_1 = 1.39 \times 10^{34} \text{ g} = 6.6 M_\odot$$

Problem 3 (25 points)

(a - 9 pts) A candle has a power in the visual band of ~3 Watts. When this candle is placed at a distance of 3 km it has the same apparent brightness as a certain star. Assume that this star has the same luminosity as the Sun in the visual band (~ 10^{26} Watts). How far away is the star?

$$\text{Same flux} \Rightarrow \frac{3}{(3 \text{ km})^2} = \frac{10^{26}}{d^2}$$

$$d = 10^{13} \times \frac{3}{\sqrt{3}} = 1.7 \times 10^{13} \text{ km}$$

$$\underline{d = 0.56 \text{ pc}}$$

(b - 10 pts) Two stars have the same surface temperature. Star 1 has a radius that is 2.5 times larger than the radius of star 2. Star 1 is ten times farther away than star 2. What is the difference in apparent magnitude between the two stars?

$$m_1 = -2.5 \log \left[\frac{T_1^4 R_1^2}{d_1^2} \right] + \text{const}$$

$$m_2 = -2.5 \log \left[\frac{T_2^4 R_2^2}{d_2^2} \right] + \text{const}$$

$$m_1 - m_2 = -2.5 \log \left[\frac{R_1^2}{R_2^2} \right] - 2.5 \log \left[\frac{d_2^2}{d_1^2} \right] \quad (\text{Since } T_1 = T_2)$$

$$= -5 \log \left[\frac{R_1}{R_2} \right] + 5 \log \left[\frac{d_1}{d_2} \right]$$

$$= -5 \log 2.5 + 5 \log 10$$

$$= -2 + 5 = 3$$

$$\underline{m_1 - m_2 = 3}$$

(c - 6 pts) A certain red giant has a radius that is 500 times that of the Sun, and a temperature that is 1/2 that of the Sun's temperature. Find its bolometric (total) luminosity in units of the bolometric luminosity of the Sun.

$$L_{\text{bol}*} = 4\pi\sigma T_*^4 R_*^2$$

$$L_{\odot} = 4\pi\sigma T_{\odot}^4 R_{\odot}^2$$

$$L_{\text{bol}*} = \left(\frac{T_*}{T_{\odot}} \right)^4 \left(\frac{R_*}{R_{\odot}} \right)^2 L_{\odot} = \left(\frac{1}{2} \right)^4 (500)^2 = \underline{1.56 \times 10^4 L_{\odot}}$$

Problem 4 (25 points)

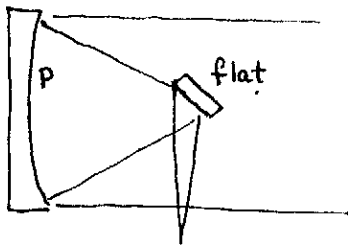
Short answer questions:

(a - 9 pts) A radio interferometer, operating at a wavelength of 1 cm, consists of 100 small dishes, each 1 m in diameter, distributed randomly within a 1 km diameter circle. What is the angular resolution of a single dish? What is the angular resolution of the interferometer array for a source directly overhead?

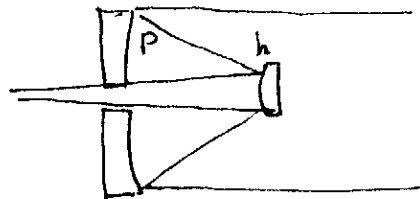
$$\theta_{\text{single}} \approx \frac{\lambda}{D_1} = \frac{1}{100} = 10^{-2} \text{ radians} \approx 0.6^\circ$$

$$\theta_{\text{array}} = \frac{\lambda}{D_{\text{array}}} = \frac{1}{10^5} = 10^{-5} \text{ radians} \approx 2''$$

(b - 8 pts) Sketch the mirrors for one type of optical telescope and label what shapes their surfaces must have. Sketch a few light rays incident along the optical axis until the point where they reach the focal plane.



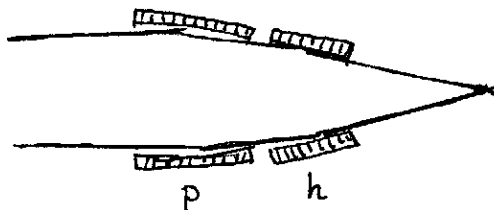
Newtonian



Cassegrain

(c - 8 pts) Briefly describe and sketch how an X-ray telescope works?

Double reflection from a combination paraboloid-hyperboloid, at grazing angles smaller than the critical angle for "total external reflection".



P = paraboloid
h = hyperboloid