Assignment #4 **PHYSICS 8.284**

Due 11:04 am Friday 2006 March 10

- Reading: Clayton (reserve) §2-4 (Polytropes); Hansen and Kawaler §§7.1-7.2.2.

(1-4) This is a long, time consuming problem, requiring that you use a computer to solve a differential equation. It counts as *four* problems! But this is the way stellar structure is done, and there's no better way to get the flavor of it than doing it. You may, if you like, use the 4th order Runge-Kutta routine supplied (courtesy of Prof. E. Bertschinger) accessible through the 8.284 home page. You may also want to look at *Numerical Recipies* by W. H. Press *et al.* §16.0-16.1. The explanatory text (which gives a tutorial on numerical integration of differential equations) can be browsed at `http://www.nr.com/`. The code isn't available at that URL but it is available on MIT server.

a) Solve the Lane-Emden equation for a polytrope of index 3. Include a copy of your code with your assignment.

b) Make 3 plots: the enclosed (dimensionless) mass as a function of dimensionless radius r/a, the dimensionless density as a function of r/a, and the decimal logarithm of the dimensionless density, again as a a function of r/a.

Constants of the Lane-Emden Functions

n	ξ_1	$-\xi_1^2\left(\frac{d\phi}{d\xi}\right)_{\xi_1}$	$\frac{\rho_c}{\bar{\rho}}$
1.5	3.65375	2.71406	5.99071
2.0	4.35287	2.41105	11.40254
2.5	5.35528	2.18720	23.40646
3.0	6.89685	2.01824	54.18250

(5) The present average baryon density in the universe is roughly $\rho = 10^{-31}$ gm/cm^3. The temperature of the microwave background radiation is $T = 2.728$ K. Suppose that this baryon-photon fluid is 100% hydrogen, that it is in thermodynamic equilibrium and it is compressed adiabatically (PV^γ = constant). As the temperature increases,

$$H + 13.6\text{ev} \leftrightarrow H^+ + e^-$$

the ionized fraction $H^+/(H + H^+)$ increases.

a) Use the Saha equation to determine by what factor f must the volume decrease to produce an ionized fraction of 50%?

b) Assuming all 3 dimensions change by the same amount, what is the corresponding change in linear dimension?

advice: You will not be able to solve this problem in closed form. Reduce the problem to a dimensionless equation that you'd like to solve, and then solve it numerically. For this particular problem iteration works well –make a guess of f, substitute on one side of the equation and then solve for a new f on the other. Two or three iterations should suffice.

6. Use the equation of hydrostatic equilibrium and the equation of state for an ideal gas to derive a differential equation for the density of the Earth's atmosphere as a function of distance z above the Earth's surface. Assume that the atmosphere is isothermal, i.e., T is constant, and that $z << R$, the Earth's radius. Solve the equation. Does the equation suggest a natural "scale height" for the problem? What is that scale height, in kilometers?