8.311: Electromagnetic Theory   Problem Set # 11   Due: **4/28/04**

Retarded fields, dipole radiation, antennas.

Reading: Schwinger, Chaps. 31, 32, 33, 34.

1. Field buildup around a straight wire.

Current in an infinite straight wire is turned on abruptly at $t = 0$:

$$I(t) = \begin{cases} 0, & t < 0 \\ I_0, & t > 0 \end{cases}$$

The wire remains neutral at all times, $\rho = 0$.

a) Using the retarded potentials formula, find $\mathbf{A}(r, t)$ and $\Phi(r, t)$ around the wire.

b) From the potentials, determine the electric and magnetic field $\mathbf{E}(r, t)$, $\mathbf{B}(r, t)$. Plot the fields schematically as a function of time.

c) Find the energy flux at a distance r from the wire, given by the Poynting vector $\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{B}$ integrated over the surface of a cylinder of radius r and length L.

2. Dipole radiation near a surface.

Consider a radiating dipole placed near a surface of perfect conductor. The effect of the conductor can be described by an image dipole, with the total radiation field being a sum of the contributions due to the primary dipole and its image. Assume that the image dipole oscillates in phase with the primary dipole, which is a reasonable approximation when the distance from the dipole to the surface is smaller than the radiation wavelength $\lambda = 2\pi/k = 2\pi c/\omega$ with ω the dipole oscillation frequency.

For a dipole oscillating at an angle θ with respect to the normal, $\mathbf{d}(t) = \mathbf{d}_0 \cos \omega t$, find the radiated power. At which values of θ the radiated power is enhanced, and for which θ it is suppressed due to the proximity to the conductor? (This problem provides a simple explanation of the so-called Peierls effect, describing the change in luminescence of atoms and molecules adsorbed on a surface of a metal.)

3. Antenna radiation. *(Schwinger, Problem 6, Chap 34)*

Obtain formulas for the angular distribution of radiated power for a half-wave center-fed antenna, $kl = \pi$, and for a full wave antenna. Plot the latter, and compare with Figure 34.4.