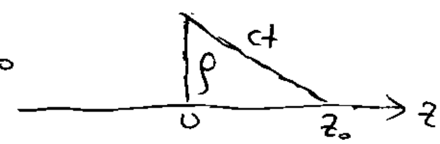
8.311 Problem Set # 11 Solutions

1. $\phi(t)=0$ since $\rho=0$; $\quad \vec{A}(t)=\frac{1}{c}\int\frac{\vec{j}(\vec{r}',t')}{|\vec{r}-\vec{r}'|}d^3r' \qquad t'=t-\frac{|\vec{r}-\vec{r}'|}{c}$

$$\vec{j}(\vec{r},t)=\begin{cases} I_0\,\delta(x)\delta(y)\,\hat{z} & (t>0)\\ 0 & (t<0)\end{cases} \quad\Rightarrow \text{ only } |\vec{r}-\vec{r}'|<ct \text{ contributes}$$

$\vec{A}$ is only a function of ρ and t, not a function of z $\Rightarrow$ let $z=0$

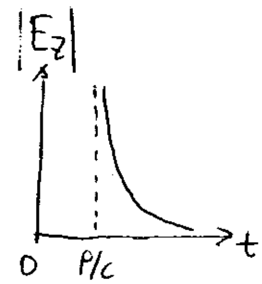


$$\Rightarrow \vec{A}(t)=\frac{\hat{z}I_0}{c}\int_{-z_0}^{z_0}\frac{dz'}{\sqrt{\rho^2+z'^2}} \qquad z_0=\sqrt{c^2t^2-\rho^2}$$

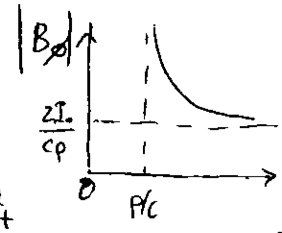
$$=\frac{2I_0}{c}\hat{z}\ln\left[\frac{ct}{\rho}+\sqrt{\left(\frac{ct}{\rho}\right)^2-1}\right] \quad \text{when } \rho<ct$$

$\vec{A}(t)=0$ when $\rho>ct$

$$\vec{E}=-\frac{\partial\vec{A}}{c\partial t}=\begin{cases}-\frac{2I_0}{c}\hat{z}\frac{1}{\sqrt{c^2t^2-\rho^2}} & (\rho<ct)\\ 0 & (\rho>ct)\end{cases}$$

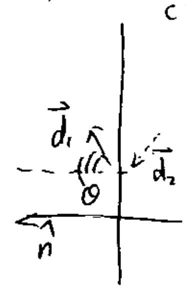


$$\vec{B}=\vec{\nabla}\times\vec{A}=\begin{cases}\frac{2I_0}{c\rho}\frac{ct}{\sqrt{c^2t^2-\rho^2}}\hat{e}_\phi & (\rho<ct)\\ 0 & (\rho>ct)\end{cases}$$



$$\vec{S}=\vec{E}\times\vec{B}\frac{c}{4\pi}=\begin{cases}\frac{I_0^2 t}{\pi\rho\left(\sqrt{c^2t^2-\rho^2}\right)^2} & (\rho<ct)=\frac{I_0^2t}{\pi\rho(c^2t^2-\rho^2)}\\ 0 & (\rho>ct)\end{cases} \qquad \text{Flux}=\frac{2I_0^2Lt}{c^2t^2-\rho^2}$$

2. Equivalent dipole on surface $\vec{d}=\vec{d}_1+\vec{d}_2=2d\cos\theta\,\hat{n}$



$\Rightarrow$ total power of emission $=\frac{1}{2}\times\frac{\omega^4}{3c^3}(2d\cos\theta)^2=\frac{\omega^4d^2}{3c^3}2\cos^2\theta$

Enhanced radiation if $|\theta|<\pi/4$, $\pi/2>$ Otherwise attenuated radiation if $|\theta|>\pi/4$

3. $$\frac{dP}{d\Omega}=\frac{I^2}{2\pi c}\begin{cases}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} & \text{half-wave}\\ 4\frac{\cos^4\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} & \text{full-wave}\end{cases}$$