PROFESSOR: Then let me just-- I gave you two things about the last class, one is the question about the electron and factors of 2 and what precesses at what frequency. Think about it, I mean, write down for yourself what is a magnetic moment of an electron? What is a magnetic moment of a classical particles which is one unit of angular momentum and try to sort of re derive it.

The way how I did it is deceptively simple. But whenever you think about it, you will get confused about some factors. But this is, of course, where I showed you here is the quantum mechanical mystery about the g-factor of 2 that you have an electron which has one more magneton of magnetic moment, and you have a classical particle which has one more magneton. The energy levels of up and down what are the same. The energy is 1.4 megahertz per Gauss.

But the precession of frequency of the electron is 2 times faster than the precession frequency of the classical particle, and this is what the g-factor of 2, which comes out of the Dirac equation means. And at least one intuitive explanation I can give you is a frequency which is observed as a precession frequency in the laboratory is the beat frequency between two neighboring levels. And here you have a level in between. So the beat frequency between two neighboring levels is 1.4 megahertz, and here the beat frequency is 2.8 megahertz.

And you will actually see that if you take a classic system which has much, much larger angular momentum, maybe L equals 10. It will have this level structure, but it will go from minus 10 to plus 10. But the precession frequency is the beat frequency between two levels. Or if you don't like the word beat frequency, it precesses and you can increase the precession.

So if you're not increasing the impression, you can increase the energy of the precessing system by driving it at the resonance frequency which is a precession frequency. And the resonance is always the energy difference between two levels. So no matter how many levels you have in a classical system with a g-factor of 1, you're always driving it step by step, and each step is the resonance and this is the precession frequency of the system.

Questions about it? I know people get confused all the time, but just think about it. I think I gave you all the different angles you can use to look at it. Questions about that? OK.

Next thing I just want to point out is rotation. Yes, we are spending quite some time in class just to figure out that something is rotating. So what we figured out here is that if you go to a rotating frame, if you have angular momentum, and we go to a rotating frame, that that simply means that the effect of a rotating frame is to simply add a fictitious magnetic fields to the real magnetic field.

Well, that's very handy. For instance, we know then immediately if you have a real magnetic field and we pick the fictitious magnetic field in such a way that the real magnetic field that the total field cancels it. It cancels the total magnetic field, well, then we have a system viewed from the rotating frame where the effective field is 0. Well, the spin is zero field does nothing and then we know when you go back into the lab frame, all this spin is doing it rotates. It's an exact solution.

I want to use it today again but in a different way, but I want to just point out today we will not picking the fictitious field to make the magnetic field zero. Today we are talking about what happens to a spin in a rotating field. Complicated. A time-dependent problem, time-dependent Hamiltonian. But if you go in a rotating frame, if you rotate with the field, then in the rotating frame it becomes a time-independent problem which we can immediately solve.

So today we use the same transformation again, but we pick our frequency not to cancel some static field. We pick our frequency to co-rotate with an external rotating field that in our rotating frame now everything is stationary. So that's what you're going to do today.

And finally just to give you an outlook, just try to sort of make you aware that often what we're doing is the same thing in a different angle. So this is one way to deal with rotation. I'll give you an exact solution by going to rotating frame. I will later show you today that quantum mechanically the solution to the Heisenberg equation of motion for angular momentum or magnetic moment in a magnetic field is exactly a rotation.

So we'll again show you that a quantum mechanical solution, the solution of a time-dependent problem is exactly rotation. And later on when we use the spin-1/2 Hamiltonian and we write down the wave function, we solve sometimes the wave function by transforming the wave function. And this will be, again, the rotating frame transformation. It's not always called like this, but it's always the same. You go to some form of rotating frame, and we'll do that in three different ways.

This is the first way just as a general classical physics transformation to rotating frame. We will do it again for the expectation value as a solution of Heisenberg's equation of motion, and then we do it again when we transform the wave function with a unitary transformation. A lot of time for simple rotations, but its good thing. It really provides a lot of insight.

Anyway, this is more sort of an outlook over today's lecture and part of Tuesday's lecture. Any question about the summary and the outlook? Because before I really come back to the rotating frame, I quickly want to do something I couldn't. I ran out of time on Monday. We talked so much about harmonic oscillators, the precision at which we can determine the frequency of the harmonic oscillator, and I really want to give you examples. I want give you two outstanding examples for atomic clocks.

And the two extreme examples are, well, the two best atomic clocks in the world. One is the cesium atom with a fountain clock. Well, we'll talk about it later, but some of you know that the cesium atom is hyper fine structure. In one state, the electron and nucleus spin are parallel. In the other state, they are antiparallel, and the transition frequency is 10 gigahertz, well, 9.something, but for the matter of this discussion, it's 10 gigahertz.

And the definition of time, the definition for 1 second is in terms of so-and-so many cycles of this transition. So this transition has this frequency, and for decades, this frequency was determined in an atomic beam. You have an atomic cesium beam, and you interrogate it with microwave fields. But now with cold atoms, we can achieve much, much longer interrogation times by almost completely eliminating the atomic motion.

In the current experiment, is that you have a cloud [? lace ?] are cooled to micro Kelvin temperature. You launch it into a fountain. The cloud goes up and goes down, and you interrogate it twice and the interrogation time is 1 second. And this interrogation time is no longer as in the convention atomic clocks limited by the thermal velocity of the atom, it's limited by gravity. If you want to increase a time to 10 seconds, you need a 100-meter tower and nobody wants to build that. I mean, it would be a really big atomic clock.

So therefore you usually deal with an interrogation time on the order of 1 second. So we know that based on Fourier's theorem, delta is a factor of 2 pi, but the line widths, if you would recall now the spectrum, the line widths, delta omega, would be that's a factor of 2, but it would be 1 over 1 second. And therefore the fractional accuracy is 10 to the 11, is on the order of 10 to the minus 11. However, the accuracy of the base cesium fountains is now 10 to the minus 16.

So people are able to split the line to 1 part in 100,000, which requires exquisite knowledge, exquisite knowledge of the line shape and also systematic effects, but well, an atomic clock is a piece of art. And I just want to emphasize that I sometimes feel when I talk about the uncertainty at which you can measure a classical oscillator or such is almost trivial, and you should know all about it. But I can just say without looking at anybody that I just had a lunch discussion with some of my graduate student and we talked about laser stabilization. And one graduate student asked, but if the natural line widths of a transition is 10 megahertz, can we stabilized a laser to better than a megahertz?

Of course, we can. I mean, here we have a natural line width of 10 to the minus 11, but we can get an a accuracy of a signal, which is 10 to the minus 16. So a microwave oscillator, which is the microwave accrual end of a laser can now be locked to the cesium transition with a precision of 10 to the minus 16, 100,000 times better than the line width.

Let me give you a second example. Just noticing the red color. This is red. I wanted to highlight it. There's probably nothing we can do. It seems the projector is not showing the color.

Well, in the lecture post, the lecture notes, this will be bright red, but maybe I should use more blue and green and yellow for highlighting it today. That's really odd. Yellow-- OK. The red is completely missing. The other example I want to give you is the strontium-- yeah, it's blue-- the strontium optical clock.

And there was a really nice paper in *Nature* just a few weeks ago, and well, here is a level diagram of atomic strontium what happens is there is a very fast transition s to p transition for laser cooling and trapping and all that, but then there is a very, very slow transition [? forbidden ?] to a triplet state, which is meta stable. And this transition, those states have a very long lifetime, and therefore this transition is extremely level.

So what is state of the art for the strontium clock? Well, in the experiment, they use an interrogation time. So they observe the atom for delta t, which is on the order of 160 milliseconds and putting in the 2 pis in the right place that would mean the frequency resolution is 1 Hertz.

And you see that on the left-hand side when we record the resonance, the blue one is about 1 Hertz and there is something broader. This is when they are not actively feeding back the magnetic field. So for this fantastic position, you have to control everything, but the blue line is sort of what we record as a clock transition. It's about 1 Hertz.

Now, compared to a cesium clock, the big advantage is that the strontium clock operates in the optical domain, and this is a frequency at 5 times 10 to the 14 Hertz. So we're seeing if the frequency is much, much faster even if you have a shorter interrogation time, your relative accuracy is better. And here the Q value the mu over delta mu is on the order of 10 times 10 to the minus 15. Fantastic. 15 orders of magnitude, and they are splitting the line, not as extreme as the cesium atomic clock by a factor of 100,000, by a factor which is on the order of 300. And with that, they have an accuracy, which is now really the record for the base performance of any atomic clock, which is 6 times 10 to the minus 18.

Now, this is now in the optical domain. You may wonder about the laser, how stable is the laser. The laser is stabilized to an optical activity, but the laser because of thermal fluctuations, not terminal fluctuations, thermal fluctuations in the mirror because of thermal noise is limited in the short term to 10 to the minus 16.

So the short term stability between 1 and 1,000 second is 10 to the minus 16. So they use a laser which has a stability of 10 to the minus 16. Every 1.3 seconds, they take a data point. Each data point or the spectral widths is 2 times 10 to the-- the laser is 10 to the 16. The line we record is 10 to the 15, but seen since the thermal noise is completely random and by averaging it, they can determine the line center to better than 10 to the 17.

So I think this just illustrates the precision at the which you can observe the harmonic oscillator, and what I like about this example, it shows you that both the transition you are recording and the laser itself-- 10 to the 15, 10 to the 16-- are worse than the final precision of the measurement, which is better than 10 to the 17. Make sense, but yes, that's what it is. Questions about that? Yes, Collin.

AUDIENCE: What was the accuracy of the [INAUDIBLE] result? Was it better than 6 times 10 to the minus 18?

PROFESSOR: No. This 6 times 10 to the minus 18 is really the [INAUDIBLE] record.

AUDIENCE: Is that the [? atomic ?] clock?

PROFESSOR: There was an aluminum ion clock, which had, I think, 10 the minus 17 precision. But

I think it's 6 times 10 to the minus 18 is close to that. So they are close or they are better now than the aluminum clock, but the aluminum clock is a single ion. You have to ever reach for much, much longer time to get this precision.

So that's a big advantage for the strontium clock, which is many, many atoms in an optical lattice, and they say that they have improved on the best previous lattice clock by a factor of 20. But I think the improvement effect of 20, you always have to distinguish between sensitivity and absolute precision. In the absolute precision, you also have to control all systematic effects.

And the big step here which was really boosting the absolute precision was they completely controlled the black body environment. So you really have to know with high precision what is the effective temperature of the black body radiation because at the 10 to the minus 18 level, it causes a shift of the atomic resonance.

We talk later about it, but it's the ac Stark effect of the black body radiation, which becomes an important systematic at that level of precision. I don't know exactly the number of the previous atomic clocks, but this is sort of now the gold standard of frequency methodology.

OK. Let's go. Yes, what you have here is you have sort of classical harmonic oscillator, but the classical harmonic oscillator is based on doing measurements on seamless strontium atoms and single cesium atoms. So you see that the accuracy, and we had a discussion on it last class, the accuracy at which you can observe a quantum mechanical oscillator. In a classical oscillator, it's pretty much the same. It's the same kind of-- if you have a good signal to noise, you can improve your precision for the line center substantially.

From those results, and exciting atomic clocks, let's go back to classical physics. So we want to go back to our classical magnetic moment and understand the motion of it. But in addition to what we discussed so far, a stationary magnetic field, we now want to add to it a rotating magnetic field.

So the situation is that-- just use another color-- we have a magnetic moment. We assume it's classical, and we have a magnetic field, which we assume points along the z-axis. And then, of course, we know that from our previous discussion that the spin undergoes precession. It's sort of precesses around the magnetic field at the Larmor frequency.

And now, let's assume we have-- the yellow shows up. We have a rotating field. We add a rotating field eight B1, and just to keep things simple, we want to assume that the rotating field rotates at the same frequency as a magnetic moment, so we are on a resonance.

Because what happens now is we can simply do a transformation to the rotating frame. The rotating frame is now at the Larmor frequency and we have just learned that in the rotating frame, of course, in the rotating frame the rotating field stands still, so it becomes a static field which points in the x direction.

And we have the static field in the z direction, but now that's what I just reviewed. We have a fictitious magnetic field, which comes from the transformation to the rotating frame and on resonance, at the Larmor frequency, this is exactly the negative of B naught.

So in other words, we started out to expose a magnetic moment to a time-dependent field. So we had a rotating field cosine omega Lt sine omega Lt. But in the rotating frame, this becomes now the x prime x, and it's stationary in the rotating frame. So this was the field in the lab frame.

In the rotating frame, we have an effective field, which is the field in the lab frame minus the fictitious field, and the fictitious field was given by that. It just cancels the B naught component. And in the rotating frame, this vector and this vector cancels and we are just left with a field of strengths B1, which points in the x-axis, or actually the x-axis in the rotating frame is what I call x prime.

So therefore, we have a very simple problem that in the rotating frame. We have a static field of value B1. And the good thing is, we know already what a magnetic moment does with a static field. The magnetic moment is just precessing around the static magnetic field.

So therefore, our solution is now we have transformed the time-dependent field and now we know that mu precesses around this field, and the precession frequency is the Rabi frequency, which we discussed previously. The Rabi frequency is the gyromagnetic ratio times B1. So therefore, if we would start out with a magnetic moment aligned with the z-axis. If we would wait half a Rabi's cycle, the magnetic moment would now be inverted.

So the situation is we have a magnetic moment which points in the z-axis with a field in the z-axis. But we expose it to a rotating field which rotates in the xy plane. So in the rotating frame, we have a stationary field which points in x. In this rotating frame, the spin is simply precessing around what is now a steady field, and after half a Ravi cycle, the spin points down.

So therefore we know-- going back to lab frame-- that this rotating field has caused in quantum mechanics I would say a spin flip, a full reversal of the magnetic moment, and this is what we call the pi pulse-- it has already rotated the spin by pi-- or we call it a spin flip, but it's a completely classical system. Any questions?

Well, then I would question for you. I've discussed with you the case that the rotating field rotates at the Larmor frequency. But now I want to discuss the case that the rotating field is not at the Larmor frequency, it's at the frequency omega 1, which is faster than the Larmor frequency.

And the question is now what will happen to the spin or the magnetic moment. I explained to you that on a resonance, the magnetic moment was just flipping over, rotating precessing at the Rabi frequency. And I want you to think about it for a moment and then decide if we go off resonant, if we drive this system away from the resonance with a rotating field which is faster than the Larmor frequency, what is now the oscillation frequency of the magnetic moment.

And so the choices are, is it larger, smaller, or the same? And this was, of course, compared to the Rabi frequency. So I've explained to you that the spin flip, the Rabi flopping, or the pi pulse-- and this picture was at the magnetic moment-- does Rabi flopping rotates plus z minus, z plus, z minus, z at the Rabi frequency.

But now we drive the system faster than the Larmor frequency and the question is, is whatever this magnetic moment does, is it faster, slower, or does it always happen at the Rabi frequency? Larger. Good.

Well, let me then immediately add another twist. What would happen-- let's ask the same question, but now we are driving it at a lower frequency. The yellow. This projector has a problem.

So now same question as before, but instead of driving the system with a faster rotating field, faster than the Larmor frequency, we are driving it with a smaller frequency. Is now the response of the magnetic moment, the effective precession frequency, larger, smaller, or the same as the Rabi frequency as the resonant case?

OK, good. So that means I can go very quickly about the explanation. It's correct. Whenever you are off resonant, this system precesses faster, so let me summarize what probably is obvious to all of you. What happens when we have an off-resonant rotating field. When we have an off-resonant rotating field, we go to the rotating frame, but of course, the rotating frame we go to is now not rotating at the Larmor frequency because the purpose of going to rotating frame is get rid of the time dependence of the rotating field. So we go to rotating frame which rotates at the frequency of the rotating field.

So if the rotating field rotates at a frequency omega, we have a fictitious field, which is omega over gamma. Gamma is the gyromagnetic ratio. For the resonant case, we were just completely canceling the static field in the z direction, but for the off-resonant case, when omega is larger or smaller in both cases, this one here is no longer 0.

And our total effective field is now the quadrature sum of what we have in the z direction, and what we have in the x direction or x prime direction, this is B1. So in the z direction, we have the static field minus the fictitious field and then the two are added up

On resonance, the angle theta is 90 degrees, but for the off-resonant case, the angle is different given by the simple geometric result. And the effective field is the quadrature sum of B1 squared plus B naught minus-- and the fact is this adds something to the effective field in the rotating frame whether we drive it above or below resonance.

So therefore the magnetic moment precesses at what is called the generalized Rabi frequency, which I now call-- this would be red-- let's use green instead-- the generalized Rabi frequency, which is gamma times B effective, and this is the quadrature sum of the detuning plus the capital letter omega, omega R, is the Rabi frequency at resonance, and this is nothing else than a measure for the drive field for the strengths of the drive field B1 in frequencies. So therefore, the generalized Rabi frequency is the resonant Rabi frequency added in quadrature with the detuning squared. Any questions?

So because it's an exact result and I like the result Rabi flopping at the generalized Rabi frequency, I want to derive it for you. So I want to figure out what is the dynamic of a spin which is originally aligned, and now it undergoes-- it is driven by the rotating field.

Remember, the resonant case was very simple. The spin was just doing Rabi flopping. It was fully inverted, came back, and just did this at that Rabi frequency. Now we know that in the off-resonant case, there will be an effective magnetic field and it will precess at a faster frequency, which is a generalized Rabi frequency, but since the effective magnetic field is not transverse, it has a z component, the spin will never fully invert.

So geometrically, it's very easy. We start out with a magnetic moment at zero time, and I can immediately draw to you the complete solution. The complete solution is that in the rotating frame, this sort of precesses around the effective magnetic field.

This is the solution. But I just wanted to do is because it takes me three or four minutes, I want to read from this graph, from this drawing, one of two trigonometric identities and derive for you the explicit expression, what is the value of the magnetic moment as a function of time. But it's clear from that it will have a maximum value. It precesses around the tilt direction, and when it's over there, it has a minimum value, but it will never completely invert. Well, quantum mechanically, if you drive a system not on resonance, you cannot completely invert the population, but we'll come to the later.

So what do I need? Well, the spin is moving here on a circle. I need a few angles. So let's say the spin was here at one time. At another time, it is there, and that would mean that on the circle, it has moved an angle phi. The tilt angle between the spin and the magnetic field is what I call theta.

And the angle between the initial magnetic moment and the magnetic moment at time, t, is what I call alpha. Yeah, these other the three relevant angles. The tip of the magnetic moment goes in a circle, and this circle has a radius, which is mu times sine theta. Sine theta is nothing else than the rotating magnetic field over the effective magnetic field, which is nothing else than the resonant Rabi frequency divided by the generalized Rabi frequency.

I said what I want to determinant is the magnetic moment in the z direction as a function of time, and for that, I defined the angle cosine alpha. So the way how I derive it the easiest way, its geometry, its three dimension, and triangles, and all that. But the best way how I can describe it for you, let me introduce this auxiliary line, which connects the tip of the magnetic moment at time t equals 0 and at time t.

And I call the length of this line A. And I want to derive-- I want to have now two triangles where one side is A, determine A in two different ways, combine the equation, we are done. So the first way is that we have the magnetic moment at time 0, the magnetic moment at time, t. We said the angle is alpha, and the two tips are connected by A, and you know in every triangle you have this would Pythagoras, but in the general case, this is valid for general triangle.

So applying that to this triangle, we have a square. b square is mu square. c square is mu square, so we get 2 mu square. And then this term 2 ab is 2 mu times mu times cosine alpha, so therefore, this is 1 minus cosine alpha. So we've taken care of the first triangle.

I hope the drawing is not completely confusing at this point, but why don't we just look at the drawing looking down the effective field. We look down at the effective field and then we see the circle where the magnetic moment precesses, and the radius of the circle I've already given to you.

So now we want to look down the effective magnetic moment. We see the circle. We want to lock down the effective magnetic field. We see the circular where the magnetic moment precesses. It's has precessed form here to there.

We connect this line. This was our A, and the angle at which the magnetic moment has precesses is phi. And the radius as we derived before was mu times sine theta.

So now just using the same equation for this triangle, we find that A square is 2 mu square sine square theta times 1 minus cosine phi. And for cosine phi, I want to use the trig identity and express it by half the angle.

All right, now we are done. We're pretty much looking at this-- the drawing is clear precession around a tilt angle, and we're just doing geometry here. And we have now two expressions for A square. We can set the two expression equal and solve for the unknown, which is cosine over [? phi. ?] And with that, we find the cosine alpha is 1 minus 2 sine square theta sine square phi over 2.

And the purpose of this exercise was that cosine alpha tells us the tilt angle of the magnetic moment away form the vertical axis, so therefore, we have done what we wanted to do. We know the z component of the magnetic moment as a function of time is this times 1 minus generalized Rabi frequency squared times sine square. And now we know the precession phi, the precession at which the keep of the magnetic moment moves in a circle, we discussed it already before, and you gave the correct answer with the clicker, is this generalized Rabi frequency omega Rabi.

So that's a nice formula, but before we lean back and look at it, let me just do one tiny step. Based on our quantum mechanical background, we can now define that the probability that the spin has been flipped is the relative difference in the z component appropriately normalized. This is just the normalized change in the z component, and if I call that the probability that, that which is on the left-hand side, then I find that the probability for this classically expression for which expresses how much the z component of the magnetic moment has changed, and this is exactly the celebrated formula for spin flips in a spin-1/2 system.

So we have derived exactly the solution for the motion for the precession of a magnetic moment in a magnetic field plus the rotating magnetic field, and what we found is, we found that the magnetic moment precesses at the generalized Rabi frequency. And as I will show you next week, this result is exactly the same as for quantum mechanical expectation values. Question, yeah?

AUDIENCE: So what if we have magnetic field oscillating just in one direction instead of rotation magnetic fields?

PROFESSOR: Oh, that's a much more complicated problem. What happens when we have the magnetic field which is linearly polarized, which is only oscillating in one direction? Well, light or a vector which is oscillating linearly in one direction can be regarded as a superposition of a left and right rotating field. In other words, if you superimpose left-handed and right-handed circular polarized light, the sum of the two is linearly polarized light.

So now we have actually the situation that linearly polarized magnetic field-- that's what we usually do when the lab. I mean, we have coils. We connect them to synthesizer and the field is not going in a circle. It's going back and forth. It's linearly polarized.

This corresponds to a magnetic field which corresponds to two magnetic field, one rotates left and one rotates right. But the problem is if you now do a transformation in the rotating frame, do we want to rotate omega to the left or omega to the right?

So what we can do is, we can pick our rotating frame and we are now in the rotating frame. One of the rotating fields has become time independent, the other rotates now at 2 omega. And at that point, we need the celebrated rotating wave approximation that we keep the one term [? via ?] rectified and the other one at two omega rotates so rapidly that we say these rapid oscillations do nothing and we discard it.

We'll discussed it later in this course. But the gist is, if you have linearly-polarized light, linearly-polarized magnetic fields, we usually have to do an additional approximation, the rotating wave approximation. And since the rotating wave approximation is done always in almost any treatment, any paper you can find, we think it's always necessary. But what I've shown to you is, when we have a rotating field, we don't need any approximation. The transformation in the rotating frame is exact, but that's the beauty of it the when we assume rotating frames, we can hold on to exact solutions for longer and only later then discuss what happens when we introduce linearly-polarized magnetic fields.

But that's something we definitely do later in its full beauty. 15 minutes. OK. There's one thing I want to do about classical spins and then we do with the full quantum treatment. And this aspect of classical spins is called rapid adiabatic passage.

So we want to add one more piece to our discussion. So far we have assumed a static field and a rotating field, which rotates at one frequency. But now we want to change the frequency of the rotating field. So we have our magnetic moment in a static magnetic field, which is our quantization axis. And now we have a rotating field, but we increase the frequency of the rotating field from slow to fast and ask what happens.

And the result is that by increasing the frequency, sweeping the frequency through the resonance, we can do something very useful. We can invert the spin. We can turn over the magnetic moment in a very robust way, and this is the concept of rapid adiabatic passage.

A lot of you may be familiar with the concept a Landau-Zener transition, what I'm telling you what is exactly the classical counterpart of the Landau-Zener transition. actually in many cases when it comes to spin physics, the classical physics and the quantum physics is the same. So that's why I want to discuss rapid adiabatic passage and Landau-Zener physics first in the classical environment.

So rapid adiabatic passage is a technique for inverting turning around spins or magnetic moments by sweeping the frequency of your drive field across the resonance. So adiabatic means that this frequency sweep has to be slow. Slow is slow compared to the Larmor frequency. We get any given moment the magnetic moment precesses at the Larmor frequency, which is the gyromagnetic ratio times g-factor magnetic field, so it's sort of quasi stationary. The spin precesses around the effective magnetic field and we want to change the frequency of the drive slow compared to that motion.

Well, the word rapid adiabatic passage has the word adiabatic, which means slow but also rapid. The word rapid means we have to be rapid compared to relaxation processes, which we do not discuss here in an idealized environment. For instance, if you do rapid adiabatic passage in the environment where the atoms can collide, rapid means you have to do it fast enough before you have decoherence in two collisions, so slow compared to the Larmor frequency and rapid compared to all the things I'm not mentioning here-- rapid compared to decoherence and relaxation processes.

I will not set up differential equation and solve them. I want to give you the intuitive picture of what goes on, but then also derive what is sort of the criterion for adiabadicity, which we have to fulfill.

So what are our ingredients? We have a magnetic moment, mu. We have a static field B naught. We have a drive field B1, which rotates at a frequency omega. And we assume that the rotating field is smaller than the static field.

It's not absolutely necessary, but you apply a big static field and then you have a repetitive drive. That's a standard situation. Or we always want to have quantization axis, which is given with the z-axis as defined by the static magnetic field, but it's only defined by the static magnetic field if the transverse field is not much larger than the static field, otherwise, we are talking about a somewhat different problem.

And let me just assume to be specific we later discuss what happens if it's not the case. We assume that we start with a frequency omega, which is much, much smaller than the Larmor frequency, much, much smaller than the resonance.

So what does that mean for our effective magnetic field? Let me just sketch it for you. Remember our effective magnetic field, we have a field B naught. We have a drive field B1, which we assume is smaller. But then when we go to the rotating frame, we have a fictitious field, but if the frequency omega is below the Larmor frequency, this fictitious field is very small. So therefore, we start out in a situation where the effective field is pretty much pointing along the z direction.

So just to remind you, so we have a situation where the effective field is just at a tiny angle, and if we started out with our magnetic moment aligned in z, and we assume B1 is really perturbative, pretty much the magnetic moment is very tightly coupled. It has a very small precession angle, or if you want, if you take the magnetic field B1 to be perturbative, you can say the magnetic moment is aligned with the effective field. That's the limit that the cone angle of precession is very small.

So let we write that down. This is what's omega much, much smaller than the Larmor frequency. So now we want to turn up the knob on the frequency of the drive. We want to rotate the drive field B1 faster and faster. And the picture you should have is-- I just wiped it away, but what that means is, the effective field, the fictitious field is no longer and longer component and on resonance, the fictitious magnetic field will cancel B naught.

So in other words, at this point, when B naught is canceled, the effective field has only the B1 component in the x direction. So therefore, when be go to delta equal 0, the effective field is only B1 and points in the x direction. So what we have done is by changing the frequency, by ramping up the frequency to resonance, we have tilted the effective magnetic field from the z direction into the x direction.

And at any given moment, I mean, we know what the exact solution of the magnetic moment is. At any given moment, the magnetic moment precesses around the effective magnetic field. And if the precession is very fast and the effective magnetic field is slowly rotating, the magnetic moment is just following.

So therefore, at this point when we are at resonance, we have tilted the magnetic moment by 90 degrees. Just one second. And if we go with a frequency much higher than the Larmor frequency, then our fictitious magnetic field is much larger than B naught and therefore, the effective magnetic field points now in the minus z direction.

So the idea is that as long as this rotation of the effective field from being plus z in the x direction and into minus z is slow enough, the rapid precession is locking, is keeping the magnetic moment aligned with the effective magnetic field and we have just a handle we invert. We move around the magnetic moment.

So in the adiabatic limit, the spin precesses tightly, and by tightly I mean the angle theta is small around B ef, B effective, and follows the direction of the effective magnetic field. So we are rotating an effective field. We are rotating the magnetic moment, but we're not rotating anything in the laboratory. The only thing we are doing is, we are changing the frequency of the rotating magnetic field.

Questions? Jenny.

AUDIENCE: Oh, yeah. I was thinking, can you also do this by keeping the frequency the same and just ramping up the strength? Like say, put it at the-- make the B1 equal to the frequency of that Larmor frequency and just start from 0 and ramp up?

PROFESSOR: Yes. I mean, the essence here is that the effective magnetic field is [INAUDIBLE] resonance, and what you are suggesting is if the frequency is constant, that would mean the fictitious magnetic field is constant. But if you were given a fictitious magnetic field and we a huge field B naught, the effective magnetic field points out, points up. But if we now make this static field B naught smaller and smaller, both through resonance and make it even smaller, we have also done an inversion of the effective magnetic field. The result is the same.

Actually, sometimes in the laboratory, if you have a synthesizer which is not easily computer controlled, we do an analog sweep of the magnetic field, so we actually change the Larmor frequency of the atom instead of changing the drive frequency. What really matters in the whole business is the relative frequency between the two.

AUDIENCE: Is it necessarily true that the spin resistance is tightly around the effective fields? Isn't it at resonance the radius of the circle in precession is equal to mu? Is it my [INAUDIBLE] before we found the radius which was mu sine theta. Then sine theta cosine Omega R--

PROFESSOR: Yes.

AUDIENCE: --divided into--

PROFESSOR: But the way is we start-- the way we've started it up here. Now if we are far away from resonance, the angle theta is infinitesimally small. Something confuses you.

AUDIENCE: Yeah. I mean, it's just [INAUDIBLE].

PROFESSOR: See, what we have solved before, we have done something else before. What we have discussed before is, that we have a spin which is aligned and then we looked at the-- you can say the transient solution, we switched suddenly on the rotating field. So that may sort of described that. We have a magnetic field, which is in the z direction. Obviously we don't. Our spin is aligned in the z direction.

And if you then suddenly switch on a rotating field, that could mean you have suddenly created an effective field, which is tilted. So then by the sudden switch on of your drive, you have an angle theta and the magnetic moment precesses with the precession angle theta. But what we are here doing is, we are ever so slightly changing the angle theta and then the spin stays aligned. That's the difference.

So before I give you another example and discuss the limit of adiabadicity, let's just have a quicker question to make sure that everybody follows. So OK, what I think we have at least understood in the adiabatic limit is, that when we start with a magnetic moment, which is aligned, and now we do a sweep, we start with a small rotating frequency and we make it large.

We sweep from low to high frequency through resonance, then we can invert the magnetic moment. My question for you now is, what happens when we start with a spin but we switch on a drive, which is a buff resonance and we sweep it with small frequencies? What will now happen when we do the opposite sweep? Will the spin just stay, will it be flipped, or will it sort of get diffused or get disoriented?

So this is answer A, this is answer B, and this is answer C. So what happens? I have assumed in already discussion before that we start with a very low frequency where the fictitious magnetic field is negligible. And then we change the frequency, we sweep the frequency. It cause the Larmor frequency to very high frequency.

My question now is, what happens when we reverse the frequency of the sweep? OK. There is some distribution. The correct answer is indeed B, we flip the spin. What happens is the following. The spin is in the z direction, but at high frequency, the fictitious magnetic field is very large.

So therefore, the effective magnetic field is now a large effective magnetic field pointing in the minus z direction. So now we have a situation where the spin tightly precesses around the magnetic effective magnetic field at an angle of 180 degrees, not 0 degrees, but 180 degrees. And now, when we change the detuning, the effective magnetic field tilts, but the spin keeps on precessing at 180 degrees and eventually when we sweep through the resonance, the effective magnetic field at low frequency of the rotating field becomes the real magnetic field is now pointing in the z direction and the spin has followed the 180 degree rotation.

So in other words, it does not matter whether you go from low to high or from high to low frequency. Whenever you go through the resonance, you flip the spin. Let's close that Let's see. It is that.

So the more generalized answer is that the rapid adiabatic passage always swaps the spin state no matter which way you sweep. When you start in spin up, you wind up in spin down. When you start in spin down, you wind up in spin up because what you're doing is you're inverting the direction of the effective magnetic field and the spin just follows.

So therefore it goes both ways. It applies to-- you can say to the ground state with the lowest energy state, it applies to the highest energy state, and as such, it is actually a swap operation like the pi over 2 pulse. When you have a pi over 2 pulse, you have a spin and you just pulse on a rotating magnetic field, which in the rotating flame means you have an x field. When the spin was up at the Rabi frequency, rotates down. When the spin was down at the Rabi frequency, it rotates up.

And quantum mechanically it just means you take the population in two states and your unitary time evolution is a swap operation. And the classical counterpart is what we just discussed, the rapid adiabatic passage. However, there is one big experimental advantage of doing rapid adiabatic passage over pi over 2 pulse. And I think many of you use rapid adiabatic passage in the lab.

Remember, if you give yourself the drop, you want to transfer the magnetic moment form spin up to spin down. If you want to do it with the pi pulse-- if you want to do it with the-- I said pi over 2 pulse, I meant pi pulse. The only way how with a pulse you can rotate the magnetic moment by 180 degrees, if you are exactly on resonance, where the fictitious field cancels the static field and what you have is in the rotating frame is only a field in x. And then you can rotate around the x-axis.

But if for some reason you're not exactly on resonance, then the fictitious field does not cancel the static field, and your effective magnetic field is at an angle. And if you rotate an effective magnetic field, which is not in the x-axis but at an angle, you cannot do a full inversion of the magnetic moment. So in other words, if you want to use a pi pulse to flip over a spin, you have to pulse on your drive exactly on resonance.

And if you have MBN magnetic fields which drift by a few milligals and you're not sure where the resonance is, you cannot do a perfect pi pulse. But with the sweep, you just have to sweep from point A to point B, and if you know you cross the resonance, you have a perfect inversion of the magnetic moment, which is robust against frequency drifts.

Of course, you have to do a longer sweep. The pi pulse is the shortest possible wave. If you heat the cloud on resonance, you first sweep, nothing happens, nothing happens, and you go through resonance and just view further and further, you waste some of your time. So you pay a price for it, but often we want precision and we have the time to do it and then the job is done by rapid adiabatic passage.

Let me just mention one more thing and then I stop. I have discussed the physics of keeping-- with rapid adiabatic passage, I've discussed with you the physics that rapid precession keeps a magnetic moment aligned with an effective magnetic field. Let me now discuss the same phenomenon but in a very different environment.

And this is a similar process happen in a magnetic trap. In a magnetic trap, we don't have any drive or we have a time-dependent field. But what happens is, we have a magnetic field, but the atoms move through the atom trap. So the atom sees a changing magnetic field.

And in many of our experiments here at MIT, we use a quadrupolar field. We've discussed some of these aspects in 8.422, so a quadrupolar field, the field has to be homogeneous in order to provide trapping forces, so we often use quadrupolar fields, a lot of advantages. That's how we build the tightest magnetic traps.

But what happens now is, if an atom moves along this projector, it moves up in the laboratory, here t let's say the atom is in spin up. Here it's now a aligned. The magnetic field is opposite to the spin and now it moves up, and now the magnetic field up here is pointing up. The physics I explain to you rapid adiabatic passage means that the rapid precession off this spin means that as the atom moves, the spin stays aligned with the magnetic field.

So you find the same physics here in a different environment, but the mathematical description is the same. Of course, and that's my last comment for today, if you have a spheric quadrupole trap and you go right through the origin, you're out of luck because here, the atom sees the magnetic field is down. It gets smaller, the magnetic field gets smaller, gets smaller. The magnet field gets zero.

Oopsy, the magnetic field points in the other direction. And there was no warning. The magnetic field has jumped from 0 degree to 180 degree. There was never, ever any transverse field around which the atom could precess and change its orientation.

So therefore, when an atom is aligned with a magnetic field moves through the origin, oopsy, it's anti aligned. It has lost its orientation with respect to the magnetic field, and this is the breakdown of rapid adiabatic passage because there's no adiabadicity. It is not an adiabatic change of the direction of the magnetic field, it's a sudden field.

And the consequences are bad. You lose your atoms from the magnetic trap. It's called Majorana losses. A lot of people know what I'm talking about, but I'm not explaining it in detail. Time is over. Any question about what I've discussed? Yes.

AUDIENCE: [INAUDIBLE].

PROFESSOR: You said the weights of the--

AUDIENCE: Yeah, so [INAUDIBLE].

PROFESSOR: I would say-- the question is about the frequency weights and if you switch on the frequency drive, it's not a delta function. If we are far away from resonance, it doesn't really matter. That is, of course, a criterion that the effective weights of the frequency should not be comparable to the detuning. So then you switch it on, but you're so far away from resonance that it doesn't matter if you have a small weights, the effective detuning is still large. You scan for resonance.

But these are sort of the boundary condition. The exact solution in the Landau-Zener problem, for instance, assumes you go from minus infinity to plus infinity in the detuning. Nobody does that. So we're discussing sort of sweep finite duration effects, but usually we are pretty close to the idealized assumption.

OK, enjoy the Monday holiday. We meet on Tuesday, and Tuesday is in building 37 in our standard classroom.