

8.513 Quantum many body theory

Fall 2004. ⁽¹⁾

Quantum mechanics of many particle systems

How many?

$\approx 10^{23}$ or so.

What particles/systems?

Electrons in solids, liquid & solid forms of He-4 and He-3,
protons/neutrons in nuclei, trapped ultracold atomic

Bose & Fermi gases, quark matter,

Many of these may be described by a Hamiltonian with the
general structure

$$H = \sum_{i=1}^N \frac{\vec{p}_i^2}{2m} + \sum_i V(\vec{r}_i) + \sum_{ij} U(\vec{r}_i - \vec{r}_j) + \dots$$

+ specification of statistics of the particles.

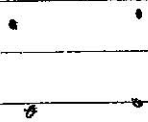
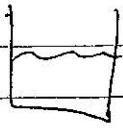
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Some very general ideas/concepts (to be elaborated on in the course)

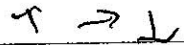
① In equilibrium, behavior governed by quantum statistical mechanics.

② Macroscopic matter organizes itself into distinct forms known as phases

Eg: (i) Liquid versus solid



(ii) Ferromagnet versus para magnet



③ Free energy/other thermodynamic quantities not smooth functions on crossing a phase boundary.

④ Notion of broken symmetry
(or equilibrium if at $T > 0$)

Ground state of many body system may not have

the full symmetry of underlying Hamiltonian.

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Example: Ferromagnetism

Ferromagnet

Spins Aligned

↑ ↑ ↑

↑ ↑ ↑

Paramagnet

Spins disordered

↑ ↑ → ↓

↑ ↓ ↗ ←

Similarly with ^{or} antiferromagnets, superfluids, solids, etc.

5) Notion of order parameter

Example: Spontaneous magnetization in a ferromagnet

Extra thermodynamic variable needed to specify microscopic state of ordered state.

Notions of broken symmetry & order parameters

- powerful unifying framework for thinking generally about variety of ordered phases

Determine many universal properties of phases

Eg: Rigidity of crystals, superflow in superfluids, vortices in superfluids

(4)

(6) Phase transitions

(Partial) classification - "1st order" if order parameter jumps to zero

Second order if order parameter changes continuously.

Landau: Singularities at 2nd order phase transition are due to long distance, long time fluctuations of order parameter degrees of freedom.

Basis for successful theory of critical phenomena in variety of contexts.

(7) Metals - Landau Fermi Liquid Theory.

Electrons in a metal: Quantum fluid of fermions.

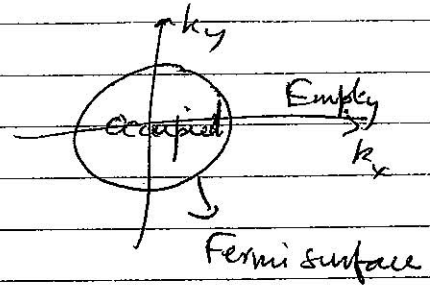
Inter electron spacing $\sim 1 \text{ \AA}$

\Rightarrow Very strong Coulomb repulsion $\sim 1-10 \text{ eV}$.

Nevertheless it is qualitatively correct to pretend that electrons are free & model ^{electrons in} metals as a free electron gas.

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Free electron gas: Electrons fill up Fermi surface in momentum space



Interactions effects - dramatically weakened due to Pauli exclusion.

Important 'quasiparticle' states near the Fermi surface scatter only weakly off each other.

Formalize: Landau Fermi Liquid Theory.

Describes conventional metals extremely well.

Modern problems in condensed matter

- challenges to some of these ideas

Example: (1) $d=1$ many-electron or boson systems.

- ~~many~~ (Carbon nanotubes, quantum wires, ...)

- many unusual phenomena

Breakdown of Fermi liquid theory, fractional quantum #s.

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(2) $d=2$ electron gas in strong magnetic fields
(& possibly $d=2$ bosons in fast rotating traps)

• Integer & fractional QHE.

- "Order" without broken symmetry, fractional charge.

(3) Host of open problems

(a) HiTc superconductivity

(b) ^{Onset of} Magnetic order in some rare earth alloys ("heavy fermions")

- possible breakdown of Landau Fermi Liquid Theory
and/or Landau paradigm for phase transitions.

(c) Phase transitions with no order parameter

Eg: Anderson/Mott insulator - metal transition.

Recent theoretical progress - ~~data~~ concrete demonstrations of failure of Landau paradigms in various contexts.

~~But~~ Conceptually important 0th order answer to questions posed by expt - but much work is ahead.

This course - mostly focus on the basic ideas/pictures established over last several decades .

Will also provide brief introduction to some of the modern ~~theoretical~~ theoretical developments in the features of these basic ideas .