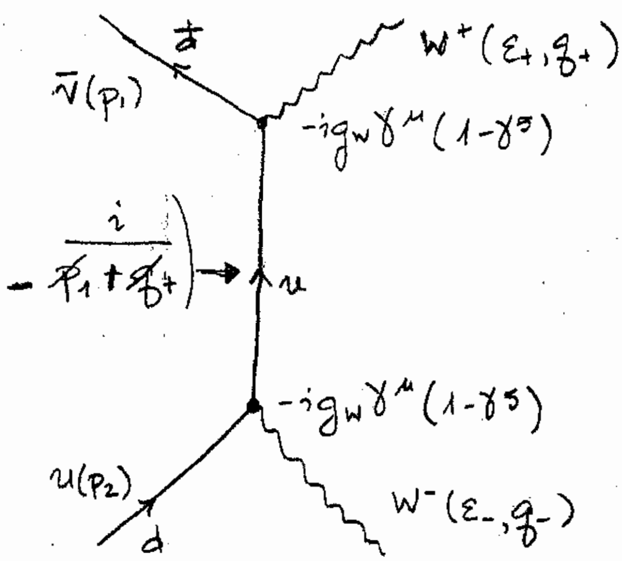
HW $\underline{4}$ 8811

1. In the $d\bar{d} \to W^+ W^-$ reaction, write out the interaction matrix element w/ the u quark exchange diagram & the interaction matrix element w/ the γ intermediate diagram explicitly & show that no cancellation is possible to avoid the divergence in the cross section.



The matrix elem:

$$M_1 = -ig_W^2\, \bar{v}(p_1)\, \gamma^\mu (1-\gamma^5)\, \varepsilon_+ \left(\frac{1}{-\not{p}_1 + \not{q}_+} \right) \varepsilon_-\, \gamma^\mu (1-\gamma^5)\, u(p_2)$$

[using: $\gamma^\mu \gamma^5 = -\gamma^5 \gamma^\mu$]

$$= ig_W^2\, \bar{v}(p_1)\, (1+\gamma^5)\, \not{\varepsilon}_+ \left(\frac{\not{p}_1 - \not{q}_+}{(p_1 - q_+)^2} \right) \not{\varepsilon}_- \,(1-\gamma^5)\, u(p_2)$$

Now, we want to go to the high-E limit, so we replace $\varepsilon_+ \longrightarrow q_+/m_W$

$$M_1 \longrightarrow i\frac{g_W^2}{m_W}\, \bar{v}(p_1)\, (1+\gamma^5)\, \underbrace{\not{q}_+ \left(\frac{\not{p}_1 - \not{q}_+}{p_1^2 + q_+^2 - 2p_1 \cdot q_+} \right)}_{(*)} \not{\varepsilon}_- \,(1-\gamma^5)\, u(p_2)$$

1, cont

going into detail on $(*)$:

$$\not{q}_+ \left(\frac{\not{p}_1 - \not{q}_+}{\underbrace{p_1^2}_{0} + \underbrace{q_+^2}_{0} - 2p_1 \cdot q_+} \right) = \frac{\not{q}_+ \not{p}_1}{-2p_1 \cdot q_+} + \frac{\overbrace{\not{q}_+^2}^{\to 0}}{-2p_1 \cdot q_+}$$

← b/c we're letting $s \to \infty$, thus even m_W is effectively $= 0$

using:

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$$

$$\not{q}_+ \not{p}_1 + \not{p}_1 \not{q}_+ = 2p_1 \cdot q_+$$

our term $(*)$ becomes:

$$(*) = \frac{-\not{q}_+ \not{p}_1}{-2p_1 \cdot q_+} + \frac{2p_1 \cdot q_+}{-2p_1 \cdot q_+}$$

putting this back into $\mathcal{M}_1$:

$$\mathcal{M}_1 \longrightarrow i \frac{g_W^2}{m_W} \bar{v}(p_1)(1+\gamma^5)\left[\frac{\not{q}_+ \not{p}_1}{2p_1 \cdot q_+} - 1\right] \not{\epsilon}_- (1-\gamma^5) u(p_2)$$

but we know from the Dirac eq, for some spinor $u(p)$:

$$(\not{p} - m) u = 0$$

$$\bar{u}(\not{p} - m) = 0$$

Thus:

$$\mathcal{M}_1 \longrightarrow -i \frac{g_W^2}{m_W} \bar{v}(p_1)(1+\gamma^5) \not{\epsilon}_- (1-\gamma^5) u(p_2)$$

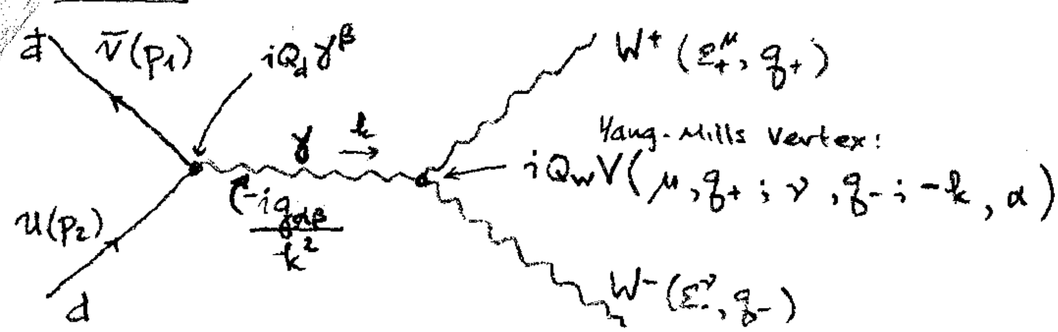
$$= -i \frac{g_W^2}{m_W} \bar{v}(p_1)(1+\gamma^5)^2 \not{\epsilon}_- u(p_2)$$

where:

$$(1+\gamma^5)^2 = 1 + 2\gamma^5 + (\gamma^5)^2 = 2(1+\gamma^5)$$

$$= -2i \frac{g_W^2}{m_W} \bar{v}(p_1)(1+\gamma^5) \not{\epsilon}_- u(p_2)$$

This is clearly a V-A form.



where: $k = q_+ + q_-$

$$M_2 = +iQ_dQ_W\,\bar{v}(p_1)\gamma^\beta u(p_2)\left(\frac{g_{\alpha\beta}}{k^2}\right)V(q_+,\mu;q_-,\nu;-k,\alpha)\,\varepsilon_\alpha\,\varepsilon_{+\mu}\,\varepsilon_{-\nu}$$

where the Yang-Mills Vertex is given by:

$$iQ_W\left[(q_+-q_-)^\alpha g^{\mu\nu}+(q_--k)^\mu g^{\nu\alpha}+(k-q_+)^\nu g^{\alpha\mu}\right]$$

Work on this vertex & the polarizations in M_2: (note $k\to -k$ here)

$$V\,\varepsilon_\alpha\,\varepsilon_{+\mu}\,\varepsilon_{-\nu}=\left[(q_+-q_-)^\alpha g^{\mu\nu}+(q_-+k)^\mu g^{\nu\alpha}+(-k-q_+)^\nu g^{\alpha\mu}\right]\varepsilon_{+\mu}\,\varepsilon_{-\nu}$$

$$=\Big[(2q_+)^\alpha g^{\mu\nu}\varepsilon_{+\mu}\varepsilon_{-\nu}+(q_-+\overbrace{q_++q_-}^{k})^\mu g^{\nu\alpha}\varepsilon_{+\mu}\varepsilon_{-\nu}+(-2q_+-q_-)^\nu g^{\alpha\mu}\varepsilon_{+\mu}\varepsilon_{-\nu}\Big]$$

no ε_α b/c we have a virtual γ

$$=2q_+^\alpha(\varepsilon_+\cdot\varepsilon_-)+\left[(2q_-+q_+)\cdot\varepsilon_+\right]\varepsilon_-^\alpha+\left[(-2q_+-q_-)\cdot\varepsilon_-\right]\varepsilon_+^\alpha$$

let $\varepsilon_+\longrightarrow q_+/m_W$

$$\to\Big[\cancel{2q_+^\alpha(q_+\cdot\varepsilon_-)}+(2q_-\cdot q_+)\varepsilon_-^\alpha+\underbrace{(q_+\cdot q_+)}_{=m_W^2}\varepsilon_-^\alpha-\cancel{(2q_+\cdot\varepsilon_-)q_+^\alpha}-\underbrace{(q_-\cdot\varepsilon_-)}_{\uparrow}q_+^\alpha\Big]\left(\frac{1}{m_W}\right)$$

(which we neglect in the $s\to\infty$ limit)

by Lorentz condit. $q_-\cdot\varepsilon_-=0$

$$\to 2(q_-\cdot q_+)\varepsilon_-^\alpha$$

1, cont.

continuing on the Y-M vertex term:

$$2(q_- \cdot q_+)\,\varepsilon_{-\alpha} = 2(E^2 + |\vec{q}|^2)$$

b/c $q_+ = |q| = -q_-$

$$= 2(2E^2 - m_W^2)$$

$$S = 4E^2 = E_{CM}^2 = (E+E)^2$$

in class, this became (somehow...)

$$= s - m_W^2$$

(I must have misunderstood some steps b/c here I'm confused)

Thus:

$$M_2 \longrightarrow +iQ_d Q_W \bar{\nu}(p_1)\gamma^\alpha u(p_2)\left(\frac{1}{s}\right)\cancel{2}(\cancel{2}s - m_W^2)\,\varepsilon_{\alpha-}$$

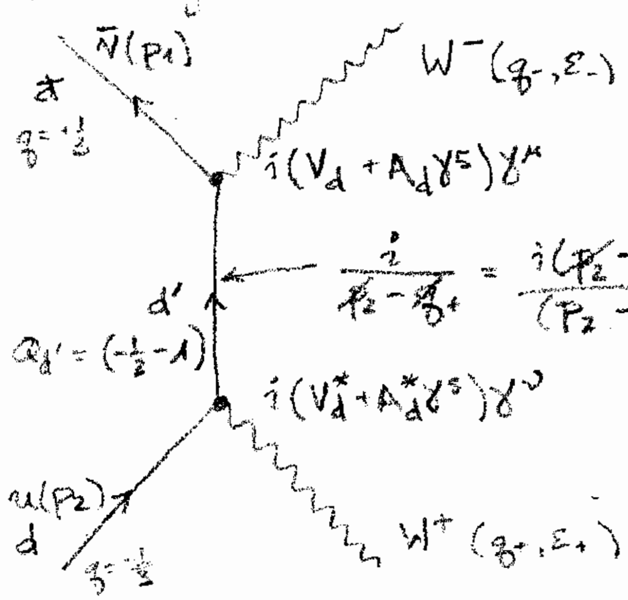
$$\longrightarrow +i\,Q_d Q_W \bar{\nu}(p_1)\,\not{\varepsilon}_-\,u(p_2)\ \frac{s - \cancel{m_W^2}}{s\,m_\omega}$$ neglect

While M_1 was a V–A form, M_2 is simply V.

There is no way that these 2 matrix elements can cancel each other out to keep $\sigma <$ the unitary limit. Thus some other diagrams must contribute.

2. In the same $d\bar{d} \to W^+W^-$ reaction, instead of "inventing" the Z^0 we could propose the existence of a $q_{d'} = -\frac{4}{3}$ d' quark & thus an exchanged diagram w/ this $q_{d'} = -4/3$ quark. Show that cancellation is indeed possible to avoid the divergence in σ. Find the 2 eq's which would determine the V & A coupling consts. of the vertex: $W - d' - \bar{d}$. What are so unusual about these V & A couplings? (p. 74)

The diagram:



$$M_{d'} = -i\left[\bar{v}(p_1)(V_d + A_d\gamma^5)\gamma^\mu \varepsilon_-\right]\left(\frac{i(\not{p}_2 - \not{q}_+)}{p_2^2 + q_+^2 - 2p_2\cdot q_+}\right)\left[\varepsilon_+ (V_d^* + A_d^*\gamma^5)\gamma^\nu u(p_2)\right]$$

$$= -i\,\bar{v}(p_1)(V_d + A_d\gamma^5)\not{\varepsilon}_-\left(\frac{(\not{p}_2 - \not{q}_+)}{\cancel{p_2^2} + \cancel{q_+^2} - 2p_2\cdot q_+}\right)\not{\varepsilon}_+ (V_d^* - A_d^*\gamma^5)\, u(p_2)$$

neglect masses.

let $\varepsilon_+ \to q^+/m_W$

$$M_{d'} \longrightarrow \frac{-i}{m_W}\bar{v}(p_1)(V_d + A_d\gamma^5)\not{\varepsilon}_-\left(\frac{\not{p}_2 - \not{q}_+}{-2p_2\cdot q_+}\right)\not{q}_+ (V_d^* - A_d^*\gamma^5)\, u(p_2)$$

- $\not{q}_+\not{q}_+ = m_W \to 0$ (neglect)
- change the order of $\not{p}_2\not{q}_+$ to get $\not{p}_2 u(p_2)$

$$\not{p}_2\not{q}_+ = 2p_2\cdot q_+ - \not{q}_+\not{p}_2$$

2, cont

$$\mathcal{M}_{d} \to \frac{-i}{m_W}\bar{v}(p_1)(V_d + A_d\gamma^5)\not{\epsilon}_-\left(\frac{2p_2\cdot q_+ - \not{q}_+ \not{p}_2}{-2p_2\cdot q_+}\right)(V_d^* - A_d^*\gamma^5)u(p_2)$$

by Dirac: $\not{p}_2 u(p_2) = m_d u(p_2) \to 0$ (neglect m)

$$\to \frac{+i}{m_W}\bar{v}(p_1)(V_d + A_d\gamma^5)\not{\epsilon}_-(V_d^* - A_d^*\gamma^5)u(p_2) \qquad (?)$$

$(V_d + A_d\gamma^5)(V_d^* + A_d^*\gamma^5) = |V_d|^2 + |A_d|^2 + (V_d A_d^* + A_d V_d^*)\gamma^5$

$$= \frac{i}{m_W}\bar{v}(p_1)\cancel{(V_d + A_d\gamma^5)^2}\not{\epsilon}_- u(p_2)$$

$\cancel{\to = V_d^2 + A_d^2(\gamma^5)^2 + V_d A_d\gamma^5 = (V_d^2 + A_d^2) + V_d A_d}$

To cancel the divergence from $\mathcal{M}_1 + \mathcal{M}_2$ ...

$\because s = 4E^2$

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2$$

$$\to -i\frac{g_W^2}{m_W}\bar{v}(p_1)(1+\gamma^5)\not{\epsilon}_- u(p_2) + i\cancel{4}Q_d Q_W\bar{v}(p_1)\not{\epsilon}_- u(p_2)$$

Thus the V & A terms...

$$V: \; -i2\frac{g_W^2}{\cancel{m_W}} + i\cancel{4}\frac{Q_d Q_W}{\cancel{m_W}} = \frac{i}{\cancel{m_W}}(|V_d|^2 + |A_d|^2)$$

$$A: \; -i2\frac{g_W^2}{\cancel{m_W}} = \frac{i}{\cancel{m_W}}(V_d A_d^* + V_d^* A_d)$$

$$\to V_d = \frac{-g_W^2}{A_d} \qquad \text{use this in V:}$$

$$-\frac{g_W^2}{m_W} - 4Q_d Q_W = \frac{1}{m_W}\left(\frac{g_W^4}{|A_d|^2} + |A_d|^2\right) \quad A_d^2$$

$$A_d^4 + A_d^2(g_W^2 + 4m_W Q_d Q_W) + g_W^4 = 0$$

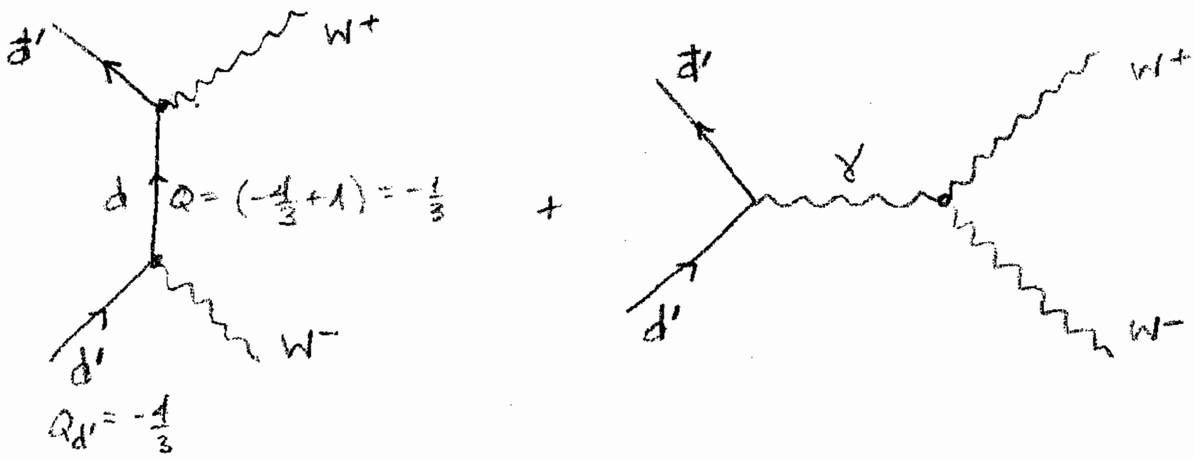
$$A_d^2 = -\frac{1}{2}(g_W^2 + 4m_W Q_d Q_W) \pm \frac{1}{2}\sqrt{(g_W^2 + 4m_W Q_d Q_W)^2 - 4g_W^4}$$

$$\pm\frac{1}{2}\sqrt{g_W^4 + 8g_W^2 m_W Q_d Q_W + 16 m_W Q_d Q_W - 4g_W^4}$$

(3.1)

Explain why the proposed existence of a $Q_{d'} = -\frac{4}{3}$ quark would cause more divergence & thus would be futile.

If we have the d' quark, we must consider diagrams such as:



To cancel the divergence here, we need to introduce another quark, d'' with $Q_{d''} = -\frac{7}{3}$.

Then, we'd be able to draw the above diagrams with the d'' quark, & need to introduce the d''' quark...

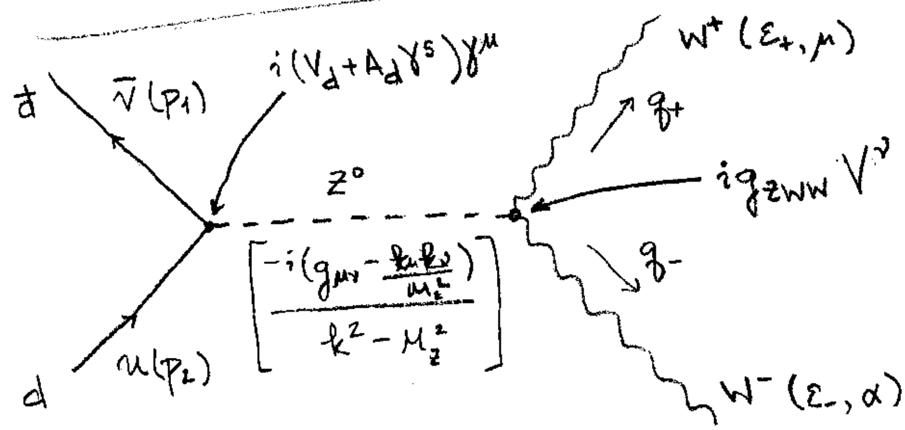
And so on.

$\Rightarrow$ bad idea.

4.1

4. In the $d\bar{d} \to W^+W^-$ reaction, compute the matrix element w/ the Z as the intermediate state (using the proper triple vector boson vertex) at the limit of the polarization vector of the W^+ being replaced by its 4-mom. / its mass in the high-E limit

p.78



The matrix element:

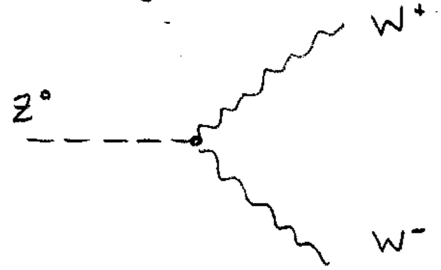
this becomes $kk = M_Z^2 \to 0$

Yang-Mills (Z-W-W) vertex.

$$M_Z = i\,\bar{v}(p_1)\left[V_d + A_d\gamma^5\right]\gamma^\mu u(p_2)\left[\frac{g_{\mu\nu} - \cancel{\frac{k_\mu k_\nu}{M_Z^2}}}{s' \cancel{-M_Z^2}}\right] g_{ZWW}V^\nu$$

neglect

The Yang-Mills vertex:



simplifies as in #1, giving us (in the limit of $\varepsilon_+ \to q^+/m_W$)

$$V\varepsilon_{Z,\nu}\varepsilon_{+\mu}\varepsilon_{-\alpha} \to 2(q_- \cdot q_+)\varepsilon_-^\nu$$
$$\to 2\,(2E^2 - m_W^2)\,\varepsilon_-^\nu$$

Reducing the matrix element to:

$$M_Z \to \frac{ig_{ZWW}}{m_W}\bar{v}(p_1)\left[V_d + A_d\gamma^5\right]\gamma^\mu u(p_2)\frac{g_{\mu\nu}}{s'}(\cdot s \cancel{-m_W^2})\varepsilon_-^\nu$$

neglect

$$\to ig_{ZWW}\,\bar{v}(p_1)\left[V_d + A_d\gamma^5\right]u(p_2)\,\not{\varepsilon}_-\left(\frac{s}{s'}\right)\left(\frac{1}{m_W}\right)$$

$$\boxed{\to \frac{ig_{ZWW}}{m_W}\bar{v}(p_1)\left[V_d + A_d\gamma^5\right]u(p_2)\not{\varepsilon}_-}$$

5. From the above, derive the 2 eq's which determine the vector & axial vector coupling consts. of the vertex Z-d-$\bar{d}$. (5.1)

We want to equate the vector bits & axial bits of $M_1 + M_2$ (from #1) with M_Z from #4.

$$M_1 \to -i\frac{g_W^2}{m_W}\bar{v}(p_1)(1+\gamma^5)\not{\epsilon}_- u(p_2)$$

$$M_2 \to +i\,Q_d Q_W\,\bar{v}(p_1)\not{\epsilon}_- u(p_2)$$

$$M_Z \to +i\frac{g_{ZWW}}{m_W}\bar{v}(p_1)\left[V_d + A_d\gamma^5\right]\not{\epsilon}_- u(p_2)$$

Vector:

$$-i2\frac{g_W^2}{m_W} + i\frac{Q_d Q_W}{m_W} + i\frac{g_{ZWW}}{m_W}V_d = 0 \qquad (V)$$

Axial Vector:

$$-i2\frac{g_W^2}{m_W} + i\frac{g_{ZWW}}{m_W}A_d = 0 \qquad (A)$$

From (V) we have:

$$\frac{m_W}{g_{ZWW}}\left[2\frac{g_W^2}{m_W} - \frac{Q_d Q_W}{m_W}\right] = V_d$$

From (A) we find:

$$\frac{m_W}{g_{ZWW}}\left(\frac{g_W^2}{m_W}\right) = A_d$$

$$\Rightarrow \boxed{A_d = \frac{2g_W^2}{g_{ZWW}} \qquad V_d = \frac{2g_W^2}{g_{ZWW}} - \cancel{m_W}\,Q_d Q_W}$$

5, cont

If we write out

$$g_W = \frac{e}{\sqrt{8}\, s_W}$$

$$g_{ZWW} = Q_W \frac{c_W}{s_W}$$

Then:

$$A_d = \left(\frac{2e^2}{8 s_W^2}\right)\left(\frac{s_W}{Q_W c_W}\right) = \frac{-2e}{8 \sin\theta_W \cos\theta_W} = \frac{-e}{4 \sin\theta_W \cos\theta_W}$$

$$V_d = \frac{-2e}{8 s_W c_W} + \frac{e Q_d s_W}{Q_W c_W}$$

$$= \frac{e}{c_W s_W}\left[\frac{-1}{4} + \frac{|Q_d|}{e} c_W s_W\right] \quad ; \qquad |Q_d| = \frac{2}{3}$$

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