8.851 Homework 3

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Problem 1) Mixing and 4-quark operators

a) Compute the divergent part of the one-loop graphs that renormalize the operators in Eq.(1.125) of the text and derive the matrix Z_{ij} discussed in class.

b) Verify that in the case $\mu \to e\nu_\mu\bar{\nu}_e$ the four fermion operator is not renormalized at order e^2. Explain why.

Problem 2) Electric Dipole Moments in the Standard Model

Electric dipole moments violates CP. Lets explore contributions in the SM. (No explicit loop computations are necessary for this problem.)

a) Write down an operator that couples two up-quarks and a photon which can contribute to the neutron electric dipole moment, d_n. Check that your operator violates CP.

b) CP-violation in the SM requires weak interactions. Draw the SM one-loop graphs that could give contributions of the form of your operator in a) and demonstrate that they do not introduce CP-violation.

c) Interestingly, graphs with two electroweak loops and a single quark line also do not contribute (this occurs in the sum of graphs and there is no need for you to prove it). In general, graphs involving two quark lines in the neutron, or an extra gluon loop can contribute. Would you expect these graphs to still contribute in the limit where all quarks are massless? Why or why not?

Problem 3) Operators for $b \to s\gamma$

After integrating out the top, W, Z, and Higgs, local operators are induced which mediate the flavor changing charge neutral process $b \to s\gamma$. The basis of operators that are induced include the 4-quark operators discussed in class, and two quark bilinear operators

$$\begin{aligned} Q_{7\gamma} &= e m_b\, \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}\,, \\ Q_{8G} &= g m_b\, \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}\,. \end{aligned} \tag{1}$$

(Here we neglect m_s relative to m_b.) Other operators can be reduced to these by the equations of motion.

a) Write down the QCD equations of motion for the gluons and for the quark fields b_L, b_R, and s_L.

b) Use the equations of motion to reduce the operators

$$\begin{aligned} O_1 &= \bar{s}_L \not{D} D_\mu D^\mu b_L \,, \\ O_2 &= g G^a_{\mu\nu} \bar{s}_L T^a \gamma^\mu D^\nu b_L \,, \end{aligned} \tag{2}$$

to our basis taking $m_s = 0$ but $m_b \neq 0$. Here $D_\mu = \partial_\mu + igT^a A^a_\mu + ieQA_\mu$ and Q is the charge operator. (You do not need to write the 4-quark operators out explicitly, but do write "+ 4-quark" to indicate when your manipulations have induced them. The identity $2D^\mu = \{\gamma^\mu, \not{D}\}$ might be useful.)

c) Determine the Feynman rule for $Q_{7\gamma}$.