

Problem set #11

1) Equations for supersymmetric gauge theories.

a) Look up in books and in papers the form of the $N=2$ supersymmetric action in 4 dimensions for a gauge group $U(k)$ with one adjoint hypermultiplet and n fundamental hypermultiplets.

b) Write this action in an $N=1$ supersymmetric notation by explicitly writing down the D term and the superpotential.

c) Write the bosonic part of this action in components form, expressing the kinetic terms and potentials for gauge fields and scalars.

d) Find the vacuum equations.

e) Find the possible solutions and identify the Coulomb and Higgs branches. It is enough to do this in $N=1$ language but it can also be done in component form, depending on taste.

f) Remember to use the brane intuition in every step of the computation. Show that the equations coincide with the brane picture which was presented in the lectures.

g) Show that mixed branches of Coulomb and Higgs branch are solutions to the vacuum equations.

2) With one hypermultiplet in the vector (5 dimensional) representation of $SO(5)$ how many hypermultiplets in the spinor representation are allowed by anomaly cancellation in 6 dimensions?

3) Five dimensional states

a) Consider a D4 brane next to an $O8^-$ plane. Write down the quiver gauge theory that lives on the D4 brane. How many parameters does this theory have? How many moduli?

b) Find the mass of the W boson for this theory. What brane represents this state? Draw it.

c) What is the instanton of this theory? What is its mass?

d) What is the 't Hooft Polyakov monopole of this theory? What is its tension?

e) Write the masses and tensions in terms of gauge theory quantities only - no string coupling and no string scale.

f) Write down a simple relation between the tension of the monopole and the masses of the instanton and the W boson.

g) Consider the strong coupling limit of the gauge theory. What is the global symmetry group in this case? In what representation of this global symmetry group does the instanton and W boson transform?