Game Theory
for
Strategic Advantage

15.025

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Overview of Foundations

Rationality

Equilibrium

Commitment

Backward Induction

Bargaining & Wars of Attrition
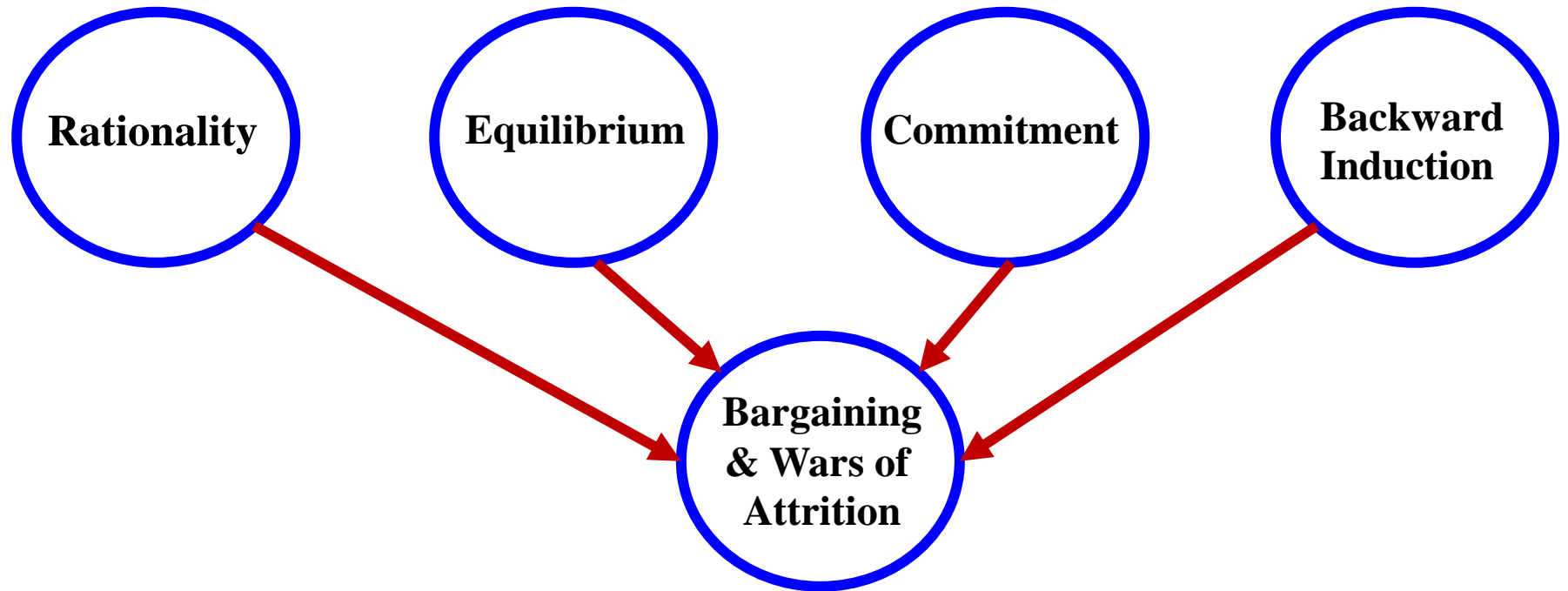
Bargaining

Last Class: Fundamentals

- Players

- Added Values

- Creating (and selling) scarcity

Today's Class: Reinforcement

- Procedures & clauses

- Backward induction #2

- Holding out for a better deal (war of attrition)

- Competitor Analysis (Ryanair)

Alternating Offers

- New bargaining protocol

- Sequential version of the demands game

- First mover: what do you ask for?   **Ultimatum**

Ultimatum Game

- Dividing $10 million

- Player **1** makes a first

  and final offer

- Player **2** can accept or reject

- Game tree?

Pl. 1

$x_1 \geq 0$

Pl. 2

Reject　　Accept

$(\mathbf{0} , \mathbf{0})$　　$(x_1 , \mathbf{10} - x_1)$
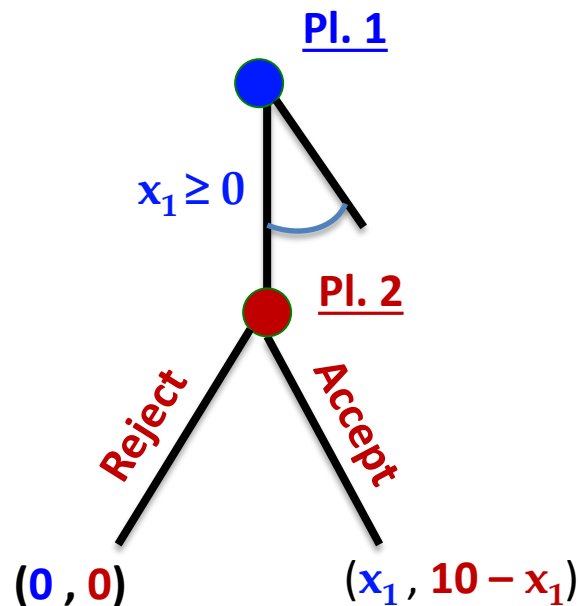
- *B.I.* outcome: *{ demand $x_1 = 10$, accept }*

- Culture & background matter: **what does zero really mean?**

Alternating Offers

- Bargaining protocol matters!

- Sequential version of the demands game

- First mover: what do you ask for?   **Ultimatum**

  – Knowledge of rationality

  – Knowledge of the game

- What if the other player can make a counter-offer?

- How can you change the rules to your advantage?

Right of First Refusal
NBA COLLECTIVE BARGAINING AGREEMENT

EXHIBIT G

OFFER SHEET

Name of Player:

Date:

Address of Player:

Name of New Team:

Name and Address of
Player's Representative
Authorized to Act for Player:

Name of ROFR Team:

Address of ROFR Team:

Attached hereto is an unsigned Player Contract that the New Team has offered to the Player and that the Player desires to accept. The attached Player Contract separately specifies in its exhibits those Principal Terms that will be included in the Player Contract with the ROFR Team if that Team gives the Player a timely First Refusal Exercise Notice.

Player:

New Team:

By_____

By_____

EXHIBIT H

FIRST REFUSAL EXERCISE NOTICE

Name of Player:

Date:

Address of Player:

Name of New Team:

Name and Address of
Player's Representative
Authorized to Act for Player

Name of ROFR Team:
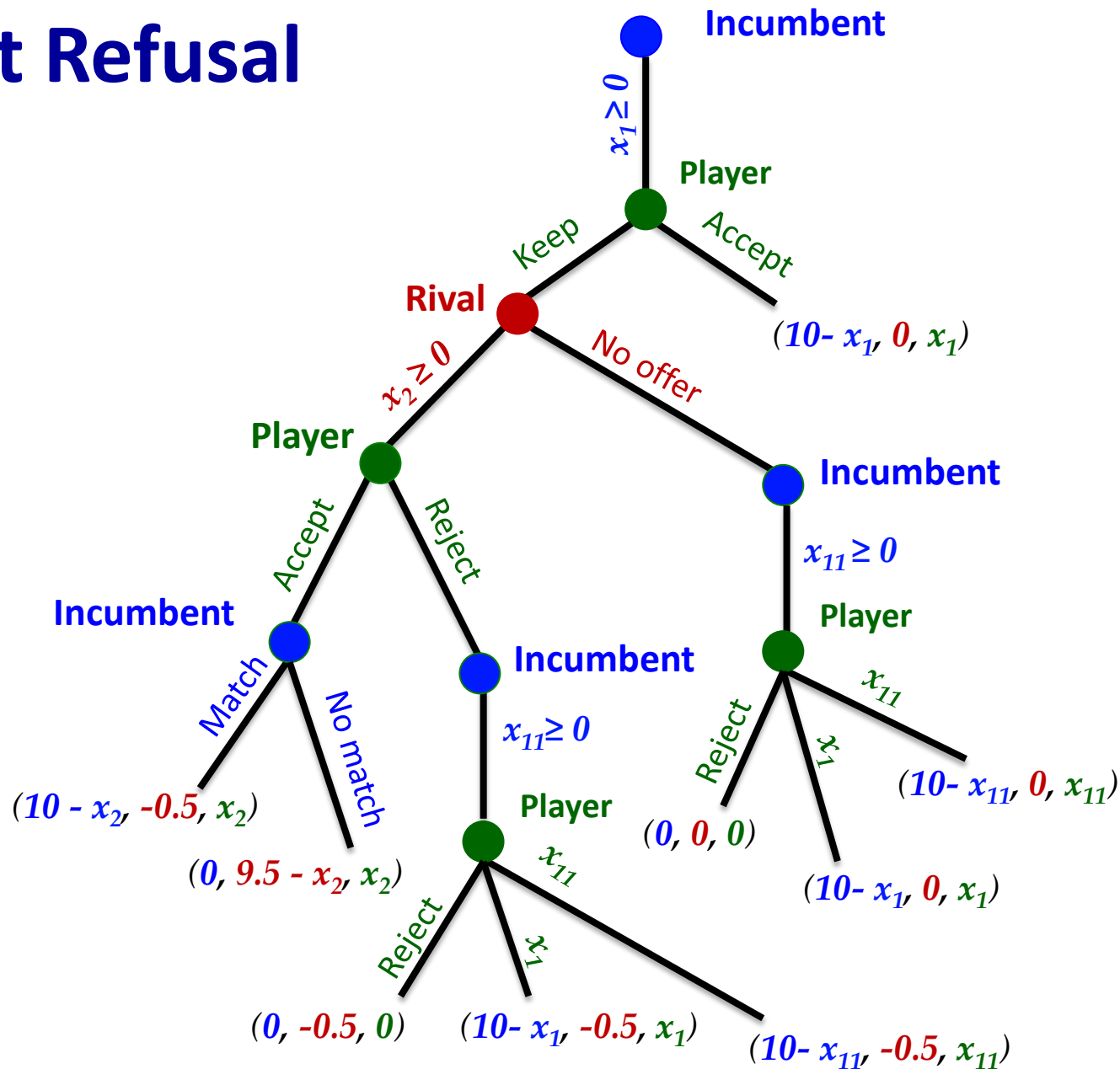
Address of ROFR Team:

The undersigned member of the NBA hereby exercises its Right of First Refusal so as to create a binding agreement with the Player containing the Principal Terms set forth in the Player Contract annexed to the Player's Offer Sheet (a copy of which is attached hereto).

ROFR Team:

By _____

Bargaining clauses as "commitment devices"

Right of First Refusal

- **Incumbent** makes **offer** x_1
- Player accepts or keeps
- **Rival** can make (costly!) offer x_2
- **Player** may sign or reject
- If sign: Incumbent can match
- If reject: Incumbent can make new offer
- Player chooses one of incumbent's offers (if any)

Incumbent

$x_1 \geq 0$

Player

Keep — Accept

$(10 - x_1, 0, x_1)$

Rival

$x_2 \geq 0$ — No offer

Player

Accept — Reject

Incumbent

Match — No match

$(10 - x_2, -0.5, x_2)$

$(0, 9.5 - x_2, x_2)$

Incumbent

$x_{11} \geq 0$

Player

Reject — x_{11}

$(0, -0.5, 0)$

$(10 - x_1, -0.5, x_1)$

$(10 - x_{11}, -0.5, x_{11})$

Incumbent

$x_{11} \geq 0$

Player

Reject — x_1 — x_{11}

$(0, 0, 0)$

$(10 - x_1, 0, x_1)$

$(10 - x_{11}, 0, x_{11})$

Right of First Refusal

- If player doesn't sign offer sheet, incumbent won't upgrade offer

- Player will accept original offer

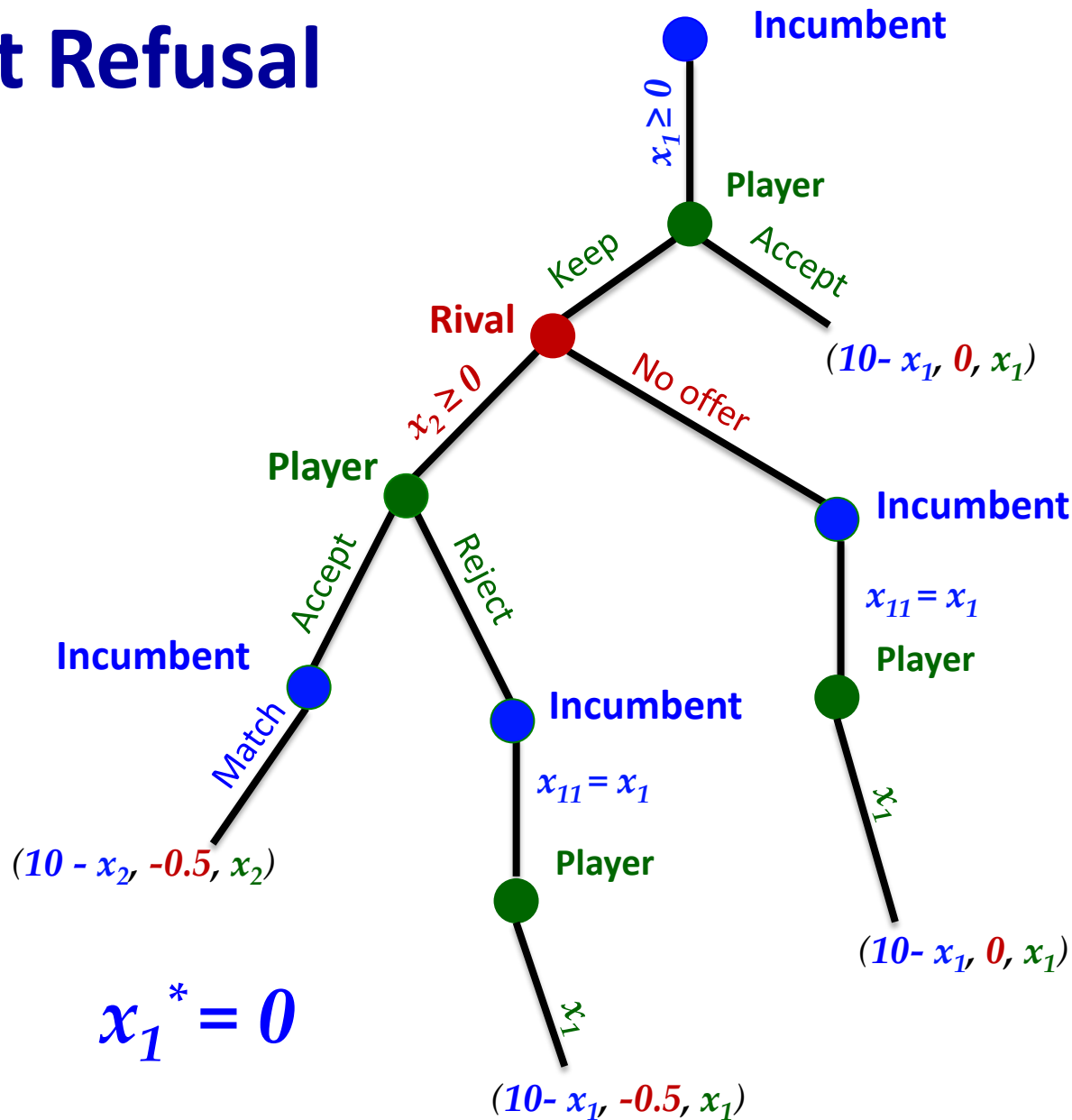- Incumbent would match any offer of $10m or less

Incumbent

$x_1 \geq 0$

Player

Keep / Accept

Rival

$x_2 \geq 0$ / No offer

$(10 - x_1, 0, x_1)$

Player

Accept / Reject

Incumbent

$x_{11} = x_1$

Incumbent

Match / No match

Incumbent

$x_{11} = x_1$

Player

Reject / x_1 / x_{11}

$(10 - x_2, -0.5, x_2)$

$(0, 9.5 - x_2, x_2)$

Player

Reject / x_1 / x_{11}

$(0, 0, 0)$

$(10 - x_{11}, 0, x_{11})$

$(10 - x_1, 0, x_1)$

$(0, -0.5, 0)$

$(10 - x_1, -0.5, x_1)$

$(10 - x_{11}, -0.5, x_{11})$
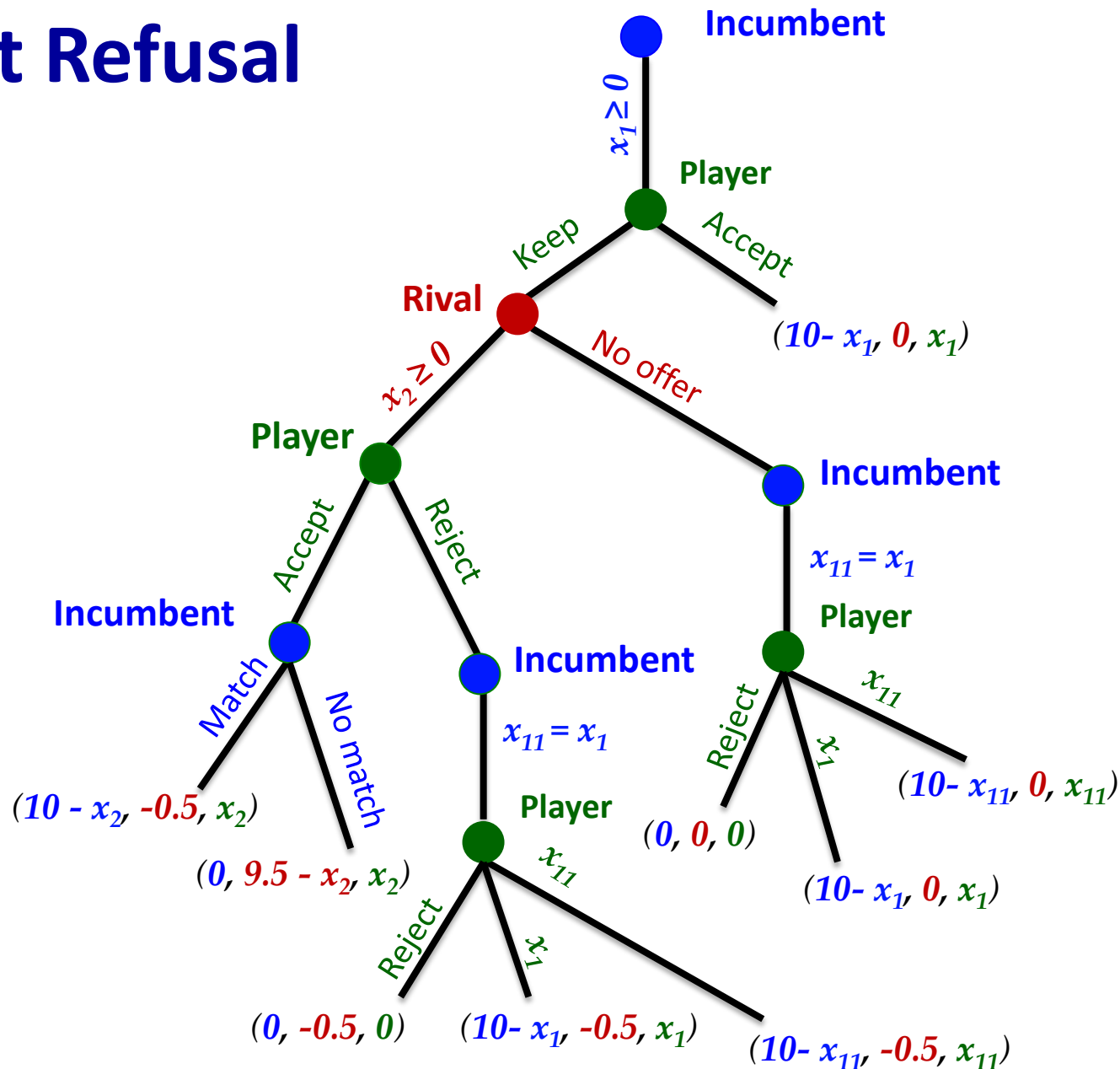
Right of First Refusal

- Whatever the player's action, the Rival loses by making an offer

- Two backwards-induction outcomes

- Incumbent wins

Incumbent

$x_1 \geq 0$

Player

Keep Accept

Rival

$x_2 \geq 0$ No offer

$(10 - x_1, 0, x_1)$

Player

Accept Reject

Incumbent

Match

$(10 - x_2, -0.5, x_2)$

Incumbent

$x_{11} = x_1$

Player

x_1

$(10 - x_1, 0, x_1)$

Incumbent

$x_{11} = x_1$

Player

x_1

$(10 - x_1, -0.5, x_1)$

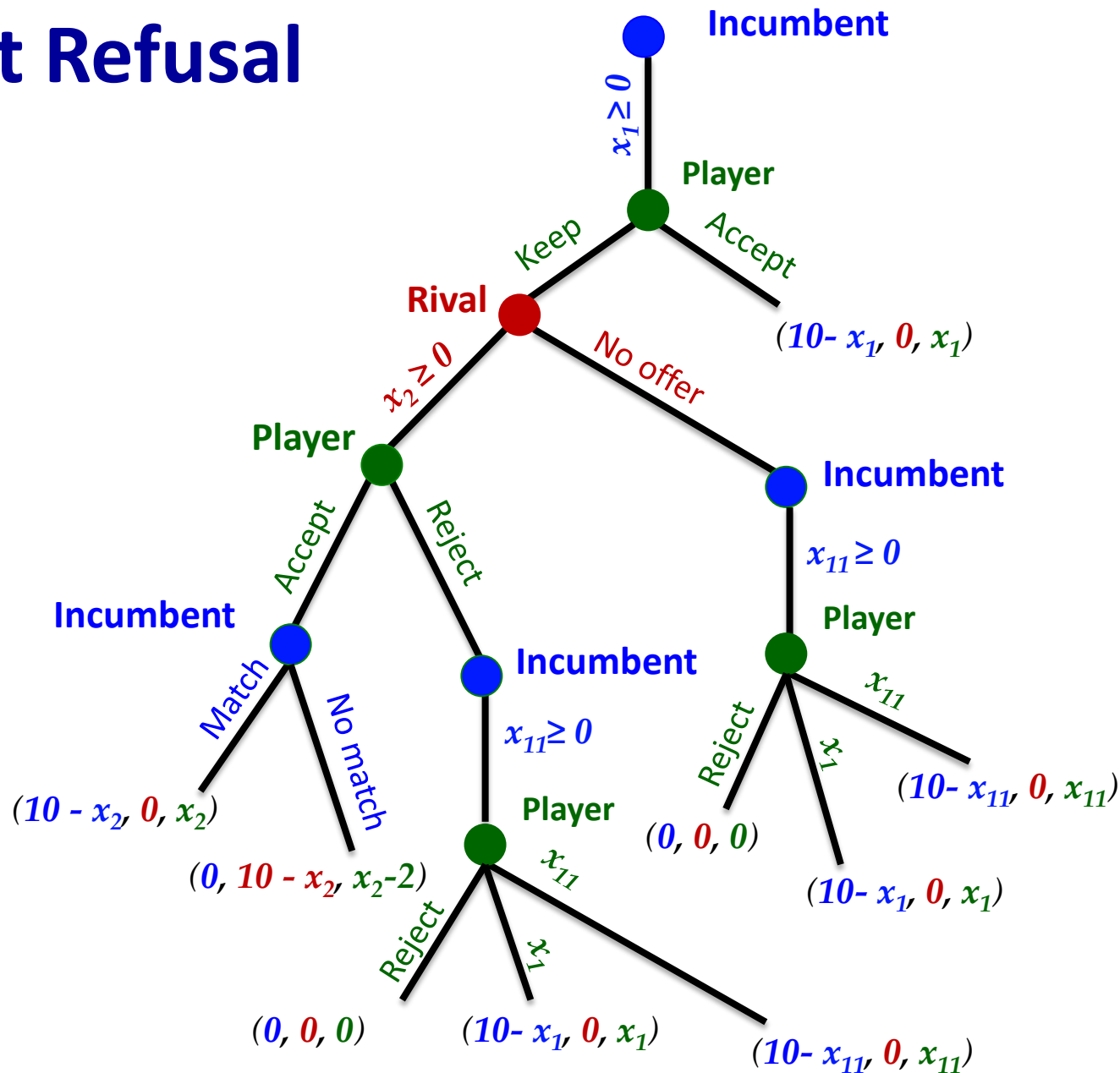$$x_1^* = 0$$

RoFR: Winners and Losers

- **Incumbent** wins with an offer of (close to) zero!

- Why does the **player** lose out?   **OFFERS ARE COSTLY**

- Would you make an offer (as the **Rival**)?

  – What are the actual payoffs?

  – Symmetric game?

  – Salary cap?

  – Repeated interaction?

Player's Switching Cost

- Player worth $10m to both teams

- Offers are free

- However, the player would take a $2m pay cut to play for the incumbent

- What happens without the RoFR?

Incumbent wins for $8 million
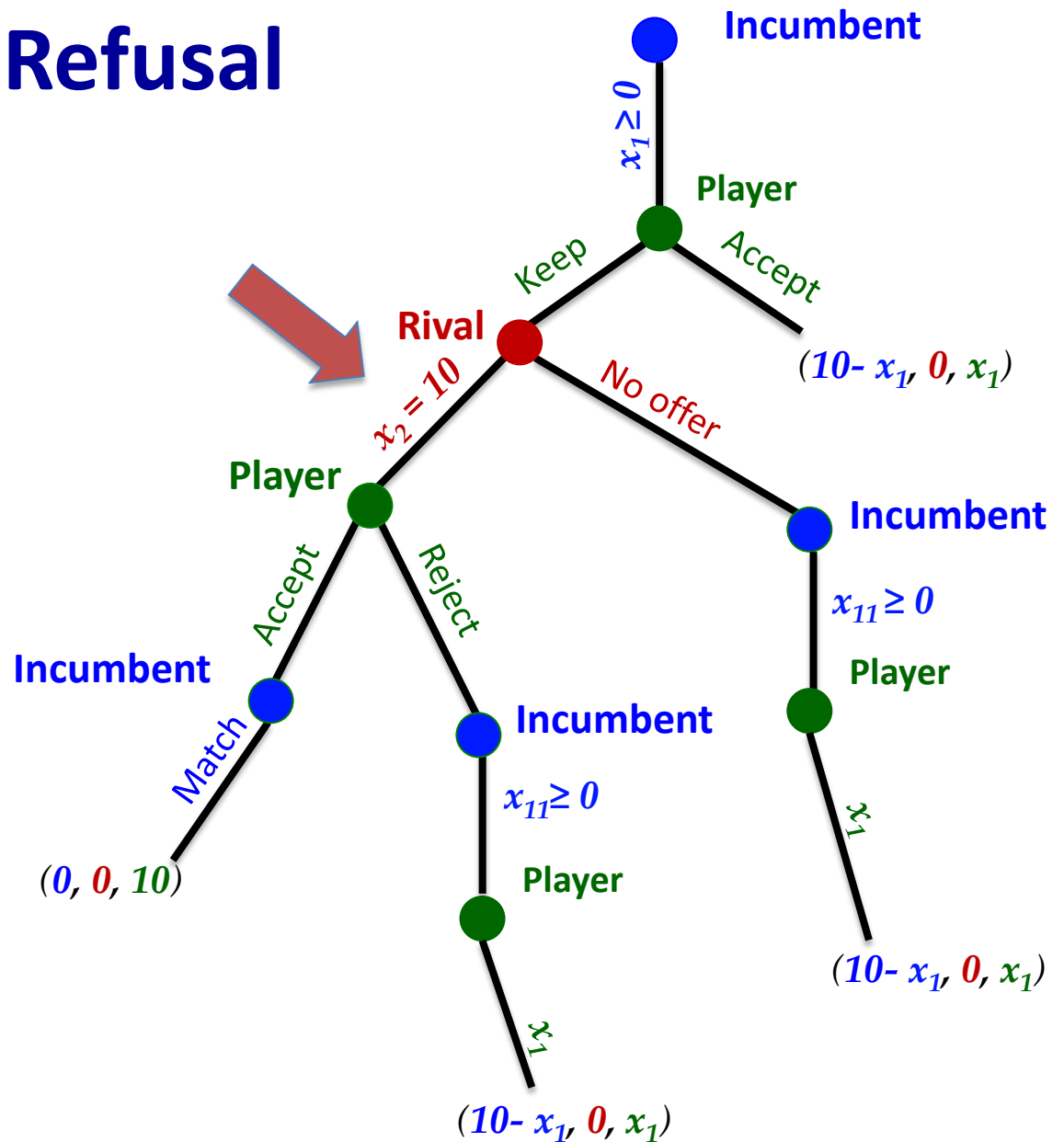
- What happens with the RoFR?

Right of First Refusal

- Incumbent makes offer

- Player accepts or keeps

- Rival can make an offer

- Player may sign or reject

- <u>If sign:</u> Incumbent can match

- <u>If reject:</u> Incumbent can make new offer

- Player chooses one of incumbent's offers (if any)

Incumbent

$x_1 \geq 0$

Player

Keep — Accept

$(10 - x_1, 0, x_1)$

Rival

$x_2 \geq 0$ — No offer

Player

Accept — Reject

Incumbent — **Incumbent**

Match — No match

$x_{11} \geq 0$

$(10 - x_2, 0, x_2)$

$(0, 10 - x_2, x_2 - 2)$

Player

Reject — x_1 — x_{11}

$(0, 0, 0)$

$(10 - x_1, 0, x_1)$

$(10 - x_{11}, 0, x_{11})$

Incumbent

$x_{11} \geq 0$

Player

Reject — x_1 — x_{11}

$(0, 0, 0)$

$(10 - x_1, 0, x_1)$

$(10 - x_{11}, 0, x_{11})$

Right of First Refusal

- Rival can now make any offer (risk-free!)

- Rival can offer 10!

- Player should accept it

- Incumbent will match!

Incumbent

$x_1 \geq 0$

Player

Keep   Accept

Rival

$x_2 = 10$ No offer

$(10 - x_1,\ 0,\ x_1)$

Player

Accept   Reject

Incumbent

Match

$(0,\ 0,\ 10)$

Incumbent

$x_{11} \geq 0$

Player

x_1

$(10 - x_1,\ 0,\ x_1)$

Incumbent

$x_{11} \geq 0$

Player

x_1

$(10 - x_1,\ 0,\ x_1)$

Incumbent

$x_{11} \geq 0$

Player

x_1

$(10 - x_1,\ 0,\ x_1)$

Player's Switching Cost

- Without the RoFR: the incumbent exploits the switching-cost advantage (worth $2)

- With the RoFR: the player can be offered the whole $10 million <u>by the incumbent</u> – how?

- Why does RoFR help?

- The player <u>commits to rejecting</u> a lower offer!

Takeaways

1) Relative scarcity ➜ value added ➜ bargaining power

2) Rules can play in your favor

3) Costly offers are **barriers to entry**

4) Clauses as commitments

Wars of Attrition – How Long to Hold Out?

- WW1 / Military escalation

- BSB-Sky Television

- Price and console wars

- Lobbying / campaign contributions

- Labor negotiations / strikes

- Litigation (broadly defined)

High-Stakes Games!!

- Two teams with great (similar!) ideas.

- One "long" presentation slot (next week)

- Simultaneous choices {Fight, Quit}

- 1 team quits ➔ other team presents

- Both quit ➔ neither presents

- Both fight ➔ pay $5, play again
  (Natallia enforces, proceeds go towards breakfast)

- Suppose that NPV(slot) = $10... How long do you fight?

Key Strategic Elements

- Why might a war last so long?

- If player believes that the **concession probability** by the rival is high enough ➔ it pays to keep fighting

- How do you judge this probability?

  – Financial capabilities

  – Reputation / past actions

  – Estimates of valuation of "prize" to rival

- Competitor analysis

Two-Period Game

- 2 players, choose Fight or Quit

- Game ends in stage 1 if someone Q's

- If the other player quits first, you win v

- Each period in which both Fight ➜ pay cost $-c$

- If both quit at the same time ➜ 0

- Easier if we assume: $v > c$, and $r = 0$

The Complete Game

B

	Q1	F1
Q1	0 , 0	0 , v
F1	v , 0	?

A

B

	Q2	F2
Q2	-c , -c	-c , v-c
F2	v-c , -c	-2c , -2c

A

Second Stage

- Use Backwards-Induction!

$$-c + \quad A$$

B

	F2	Q2
F2	-c , -c	v , 0
Q2	0 , v	0 , 0

SUNK COST

- Two pure-strategy NE in this stage-game

$$(F2 , Q2) , (Q2 , F2)$$

- Payoffs $\quad (v , 0) \quad , \quad (0 , v)$

First Stage Revisited

B

		F1	Q1
A	F1	-c + Stage2 NE payoff , -c + Stage2 NE payoff	v , 0
	Q1	0 , v	0 , 0

- More general procedure: consider first stage
- Plug-in continuation payoffs

if Stage2 NE → (F2, Q2) …

B

	F1	Q1
F1	-c + v , -c	v , 0
Q1	0 , v	0 , 0

A

- General result: "if we both know I'm going to win tomorrow, then I win today."

- 2 Backwards-Induction, pure-strategy equilibria: *{(F1, F2), (Q1, Q2)}* and *{(Q1, Q2), (F1, F2)}*

"Mixed-Strategy" Equilibrium in Stage 2

B

	F2	Q2
F2	-c , -c	v , 0
Q2	0 , v	0 , 0

$- c +$ A

SUNK COST

- If **B** fights with probability **p**
- **A's exp.** payoff of Fighting = **-c p + v (1-p)**
- **A's exp.** payoff of Quitting = **0**
- "Stable point" requires indifference **(recall the cities game)**

Mixed-Strategy Equilibrium

- Exploit indifference condition

$$- c + A$$

SUNK COST

	F2	Q2
F2	-c , -c	v , 0
Q2	0 , v	0 , 0

B
F2   Q2

- Need: *-c p + v (1-p) = 0*
- *Equilibrium **Prob[F2]** = p_2^* = v/(v+c)*
- Expected payoffs in the "mixed" equilibrium **= 0**
- "Full value dissipation"

Back to the first stage

B

		F1	Q1
A	F1	-c + Stage2 NE payoff, -c + Stage2 NE payoff	v , 0
	Q1	0 , v	0 , 0

if mixed equilibrium in stage 2 …

B

		F1	Q1
	F1	-c+0 , -c+0	v , 0
A	Q1	0 , v	0 , 0

- Mixed equilibrium in stage 1 too!
- Then: $p_1^* = p_2^* = p^* = v/(v+c)$
- Mixed B-I equilibrium: $\{(p^*, p^*), (p^*, p^*)\}$

Summary Statistics

- Pr [game goes to stage 2] = $(v/(v+c))^2$
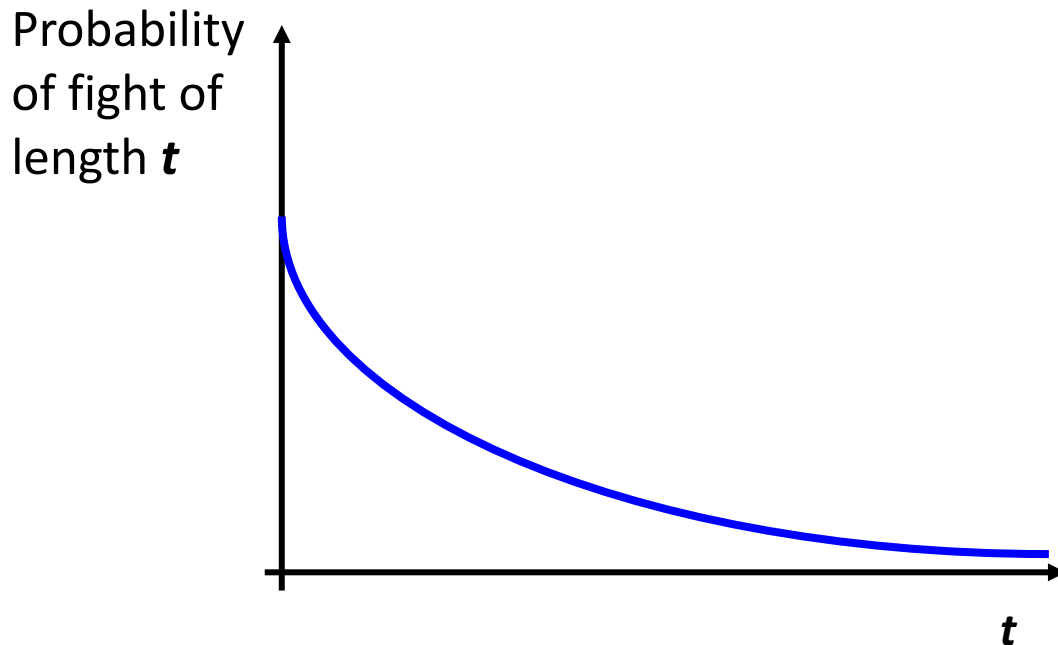 decreasing in c

- Pr [game ends without winner]
  = $(v/(v+c))^4 + (c/(v+c))^2(v/(v+c))^2$
  decreasing in c

  What about longer games?

- Expected costs paid
  = $c(v/(v+c))^2 + 2c(v/(v+c))^4$
  hump-shaped in c

Expected Outcome

- Not pride, not craziness
- Each period probability of a fight = p^{*2} = $(v/(v+c))^2$
- Increasing in v, decreasing in c

Probability
of fight of
length t

t

Empirical Predictions

- Higher stakes ➔ longer wars of attrition

- Length of the war up until time *t* has no effect on the likelihood of war ending

For example:

- Probability of settling a patent lawsuit is independent of length of litigation.

W-of-A: Takeaways

1. Overconfidence Bias: game theory helps you *calibrate* the probability of opponent conceding

2. Sunk-cost fallacy: the "break even" period plays no role in the appropriate strategy

3. Escalation of commitment: costlier fights are shorter, but not overall cheaper

Course Recap through W-of-A

- Putting yourself in your opponent's shoes

- Who am I playing?

- Backwards Induction

- Focal points

- Changing the game through strategic moves

- Playing for the Long Run

Repeated games after the break

15.025 Game Theory for Strategic Advantage
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