

Optimization Methods in Management Science

MIT 15.053

RECITATION 7

TAs: GIACOMO NANNICINI, EBRAHIM NASRABADI

Problem 1

Suppose branch and bound is being applied to a 0–1 integer program in which we are maximizing. By node 11 the first three variables have been fixed. An incumbent has already been found (in another part of the tree) and has value 15. Furthermore, x_4 has been chosen to be the next variable to branch on. This situation is illustrated in the following fragment of the branch and bound tree:

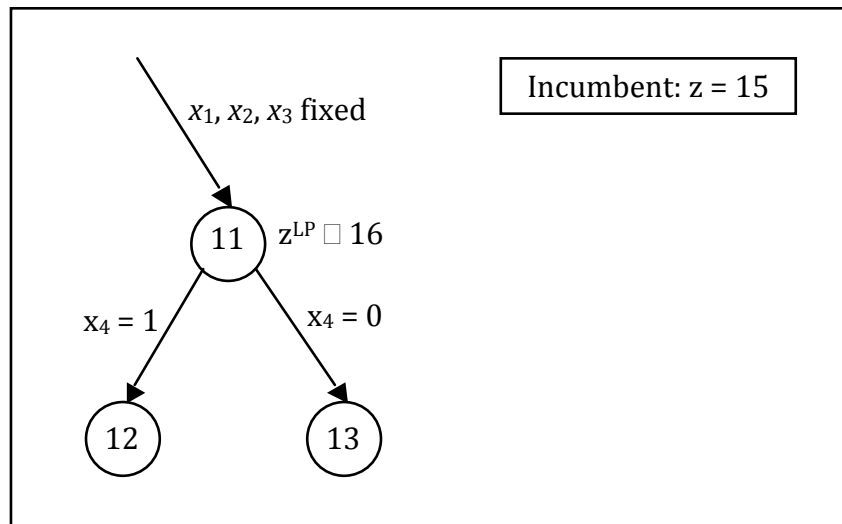


Figure 1: A fragment of the branch and bound tree for Problem 1.

The LP relaxation solved at node 11 (which has x_4 and x_5 as free variables) was:

$$\begin{aligned} \max \quad & 18x_4 + ax_5 + 10 \\ \text{s.t.} \quad & 8x_4 + 10x_5 \leq 5, \\ & 0 \leq x_4 \leq 1, \\ & 0 \leq x_5 \leq 1, \end{aligned}$$

(Hint: For the following questions, the LP relaxation essentially becomes a one variable problem that can be solved by inspection.)

Part A.1

Suppose x_4 is set to 1. Does node 12 get pruned (fathomed)? Justify your answer.

Part B.1

Suppose x_4 is set to 0 and parameter $a = 12$. Does node 13 get pruned (fathomed)? Justify your answer.

Part C.1

Suppose x_4 is set to 0 and parameter $a = 8$. Does node 13 get pruned (fathomed)? Justify your answer.

Problem 2

Consider the following integer program:

$$\begin{aligned} \max \quad & x_1 + 5x_2 \\ \text{s.t.} \quad & -4x_1 + 3x_2 \leq 6, \\ & 3x_1 + 2x_2 \leq 18, \\ & x_1, x_2 \geq 0 \text{ and integer.} \end{aligned}$$

Part A.2

Graph the the set of feasible solutions in the Cartesian plane.

Part B.2

Apply the branch-and-bound algorithm to solve the problem (use the geometric method to solve each linear program encountered) and interpret the branch-and-bound procedure graphically.

Problem 3

Consider the knapsack problem with the following decision variables for $i = 1$ to 4:

$$x_i = \begin{cases} 1 & \text{if item } i \text{ is selected;} \\ 0 & \text{otherwise.} \end{cases}$$

The knapsack problem is formulated as follows:

$$\begin{aligned} \max \quad & 19x_1 + 23x_2 + 30x_3 + 40x_4 \\ \text{s.t.} \quad & 6x_1 + 8x_2 + 10x_3 + 13x_4 \leq 25, \\ & x_i \in \{0, 1\}, \quad \text{for } i = 1, \dots, 4. \end{aligned}$$

Apply the branch-and-bound algorithm to solve the problem (notice that the the LP relaxation of a knapsack problem can be easily solved by selecting the items .

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