# Simulation: Conley Fisheries



Summer 2003

# *Outline*

- Motivating the need for simulation
- The Conley Fisheries problem
- Random number generators
- Sampling from discrete and continuous distributions
- Simulating the Conley Fisheries Problem in Excel (without Crystal Ball)
- Analyzing the Simulation Results…

# *The Need for Simulation*

- Consider the following simple mathematical model:

  Profits = total revenue – total cost

- If we know the data with *certainty* (as we supposedly do when preparing financial statements based on historical data), calculating profits is trivial…

- However, calculating *future* profits requires that we consider the *uncertainties* associated with revenues and costs (i.e. revenues and cost are random variables)

- These uncertainties result in a large number of profit scenarios…

- Just looking at a *single* profit figure based on expected values totally ignores the fact that actual results may deviate significantly from the expected value of profits.

- In general, we see that profits, P, is a function of two random variables: total revenue, R, and total cost, C:

$$P=f(R,C)$$

- Since P is *a function of random variables* R and C, then P *is also a random variable*.

- Deriving the probability distribution for functions of random variables is impossible in most practical cases.

- Using *simulation*, we can calculate the distribution of the random variable of interest empirically by generating a large number of scenarios for the underlying r.v.s.

- In this case, we can generate scenarios for P by generating random values of R and C, and then construct the empirical distribution of P (frequency table/Histogram)

- The probability distribution of P gives us the detailed information we need to answer many questions….

# *Conley Fisheries Inc.: "one boat problem"*

- For the Conley Fisheries example, please see Chapter 5 in the course textbook.

# *Equivalent Distribution for Q*

| Quantity (Q in lbs) | Probability | CDF |
|---|---|---|
| 0 | 0.02 | 0.02 |
| 1,000 | 0.03 | 0.05 |
| 2,000 | 0.05 | 0.10 |
| 3,000 | 0.08 | 0.18 |
| 3,500 | 0.82 | 1.00 |

E(Q)= 3240 lbs

Since P and Q are independent we have that:

E(R)= (3.65 $/lb) (3240 lbs) - 10000 = $1826

*Notice that the units of E(R) are $ = ($/lb) (lbs)

In practice, calculating the expected value of a simulated random variable is not straightforward…

# *General procedure to generate random values for a r.v.*

i. Generate a uniform random value, v , in the interval [0,1]

ii. Use v to calculate a random value, x, for the probability distributions of interest using the inverse of the CDF:

$$\underline{x = F^{-1}(v)}$$

**Cumulative Distribution Function F(X) of the Random Variable X**
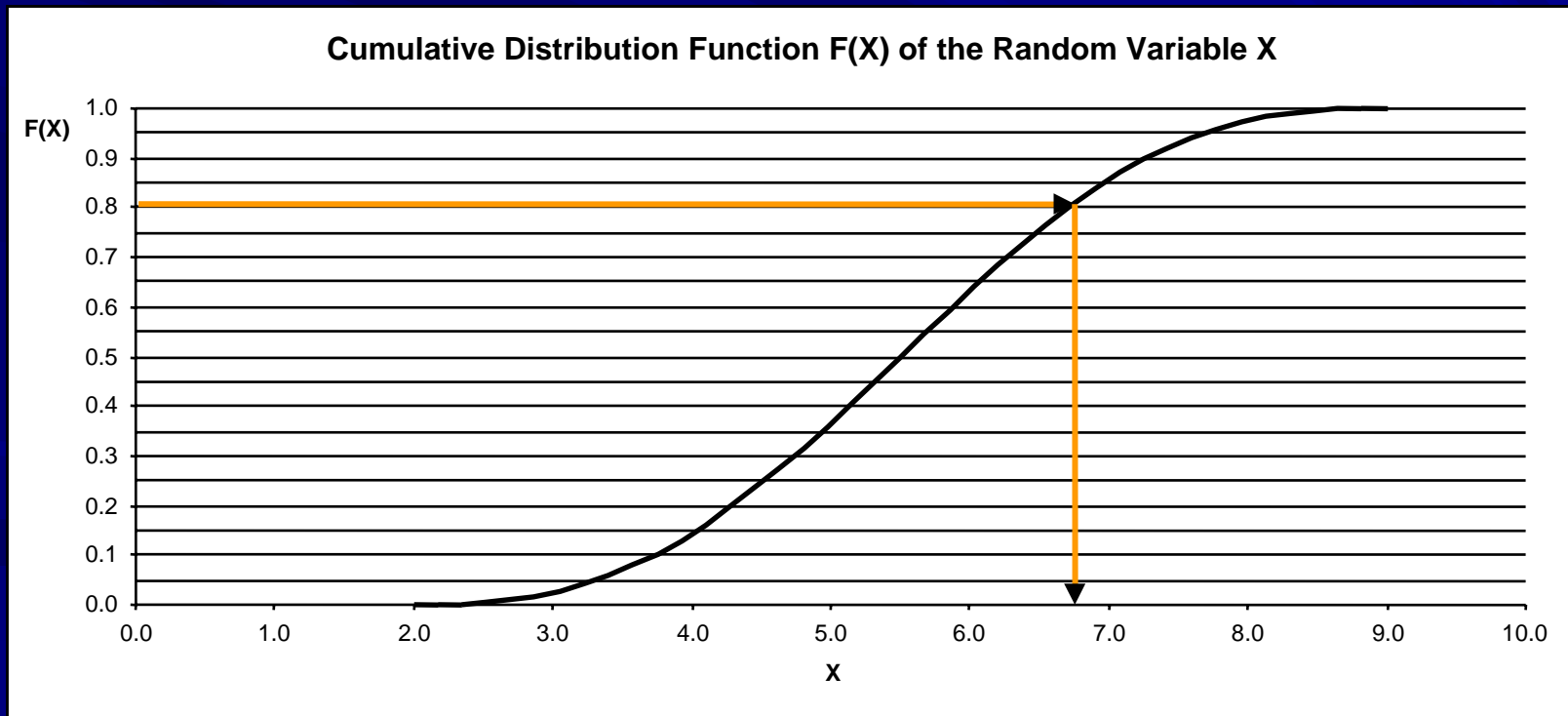
# *Calculating a Profit Scenario*

1. Generate a random value, q, for quantity (r.v. Q)

2. Generate a random value, p, for price (r.v. P)

3. Calculate Rockport profits: qp – 10,000 ($)

   To perform 1 and 2 we use a random number generator (a uniform [0,1] r.v.) to:

   i. Generate a random value, v, in the interval [0,1]

   ii. Use v to calculate a random value, x, for the probability distributions of interest using the inverse of the CDF:

$$x = F^{-1}(v)$$

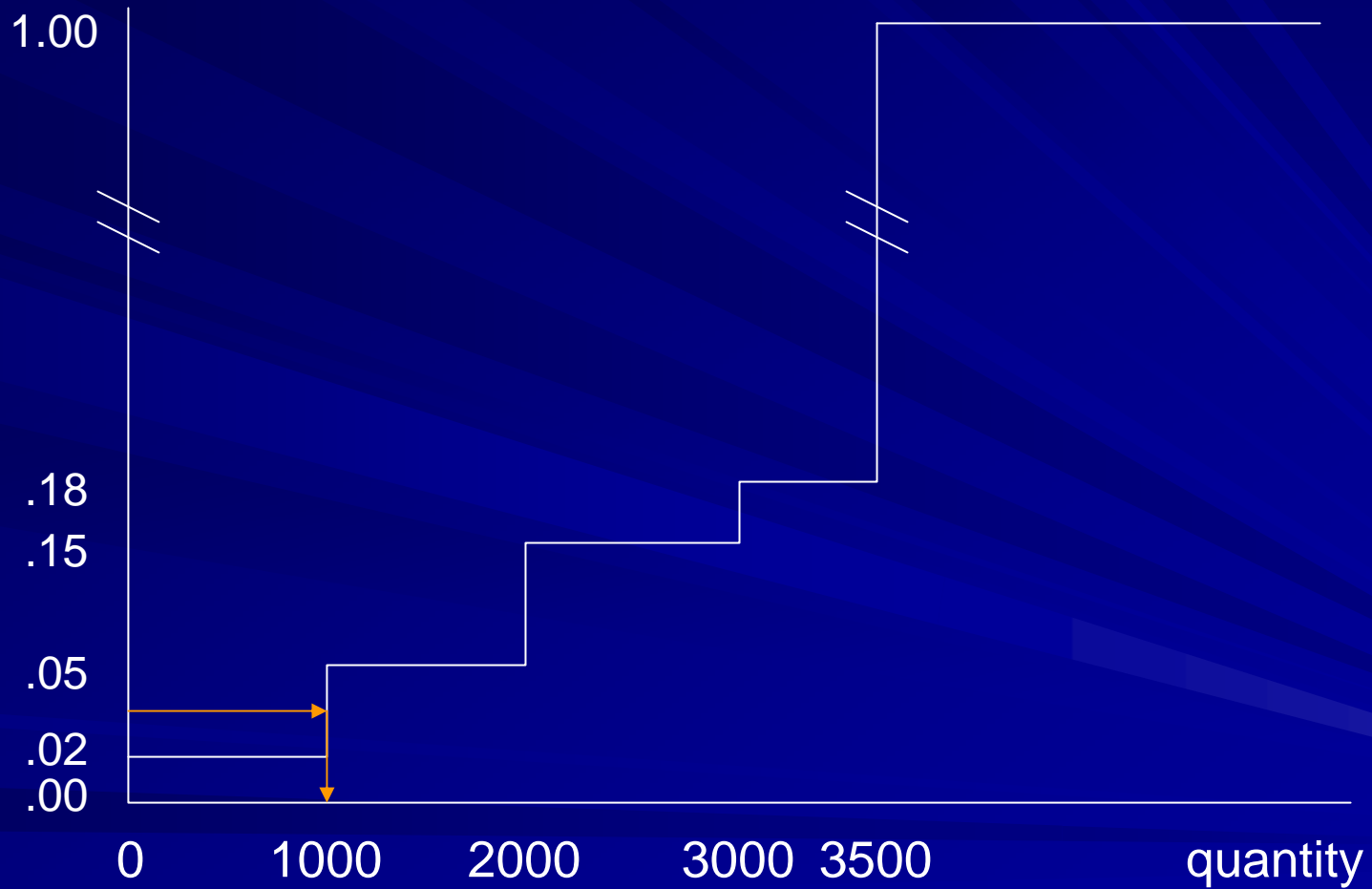# *Generating a random value for Q*

1. Generate a random value, v, in the interval [0,1] (e.g., using Excel RAND() we get v=0.0345)

2. Generate a random value, d, by reading the CDF table for d backwards (implementing this in Excel is easy!):

   In this case, d= 1000 since $F(0)=0.02 < v \leq 0.05 = F(1000)$

3. Calculate q= min (3500,d) = 1000 lbs.

   (Important!:  Notice that we arbitrarily assign the upper end of the interval if v=F(). In theory, this is a probability zero event.  However, this <u>can happen in practice</u> as computers have finite architectures!  So, if v=.0500, then d=1000, not the next higher value)

# Reading the CDF for Q

# *Generating a random value for P*

1.  Generate a random value, v, in the interval [0,1] (e.g. using Excel RAND() we get v=0.4321)

2.  Read the CDF table for Z N(0,1) backwards or use Excel NORMINV(). In this case z= -0.17103

3.  Use the z formula to calculate a value for X N(3.65 $/lb, 0.20 $/lb):    $x = \mu + z\sigma = 3.65 + (-0.17103)(0.20) = 3.62$*

4.  Calculate P = max (0,x)= 3.62 $/lb

    (* Notice that x can be calculated using NORMINV() if we give 3.65 and .20 as the mean and SD arguments instead of 0 and 1, i.e. N[3.65,.20].)

# A Random Value for P



Cumulative Distribution Function of the Random Variable P (Price)
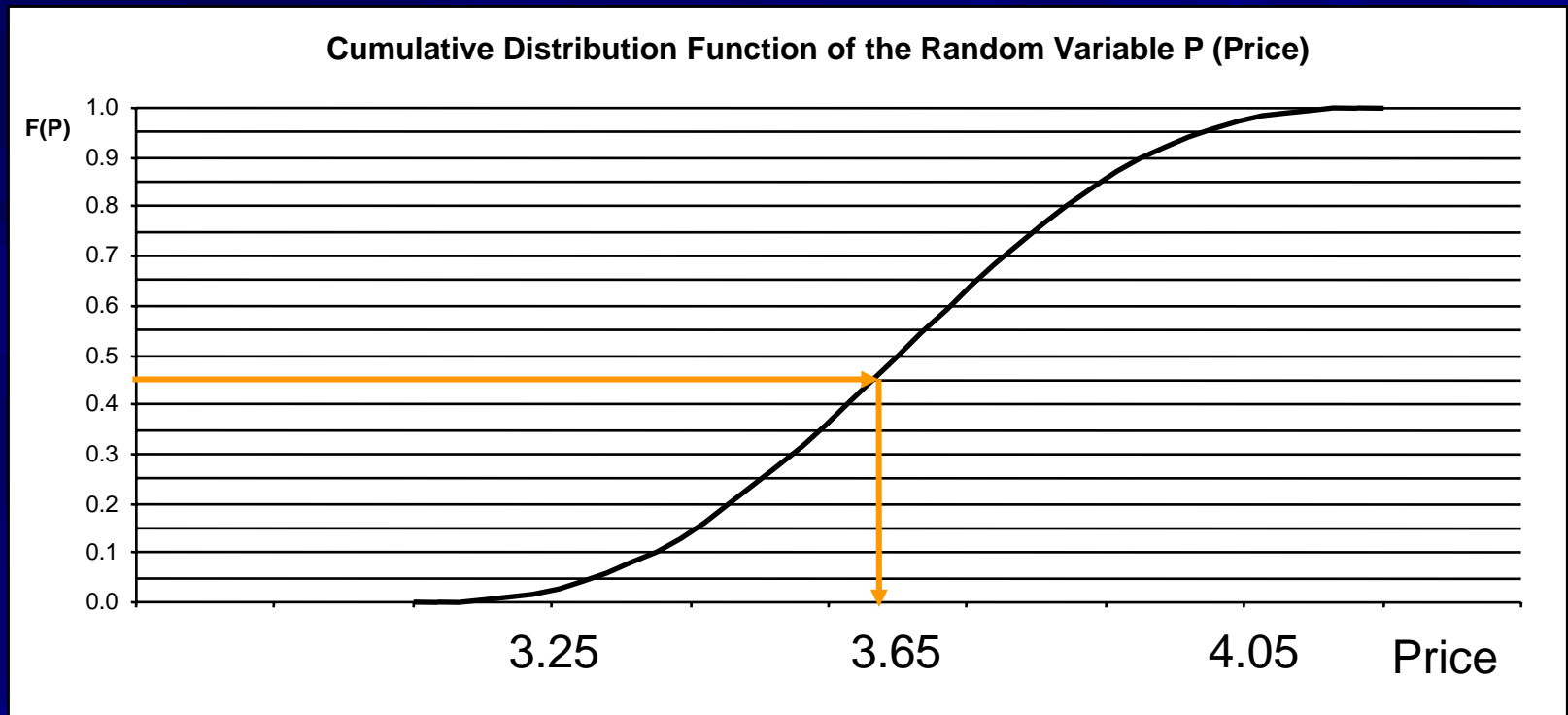
# Out of Sight, Out of Mind

- Remember that simulations are driven by the decision maker's understanding
- If you don't consider key sources of uncertainty, you can't identify their contributions
- If you don't include branches in your decision tree or outcomes on the branches, they won't be considered in your decision

# Fault Tree Example

# Fault Tree Example

Car won't start

Low battery

Fuel system fault

Other engine fault

All others
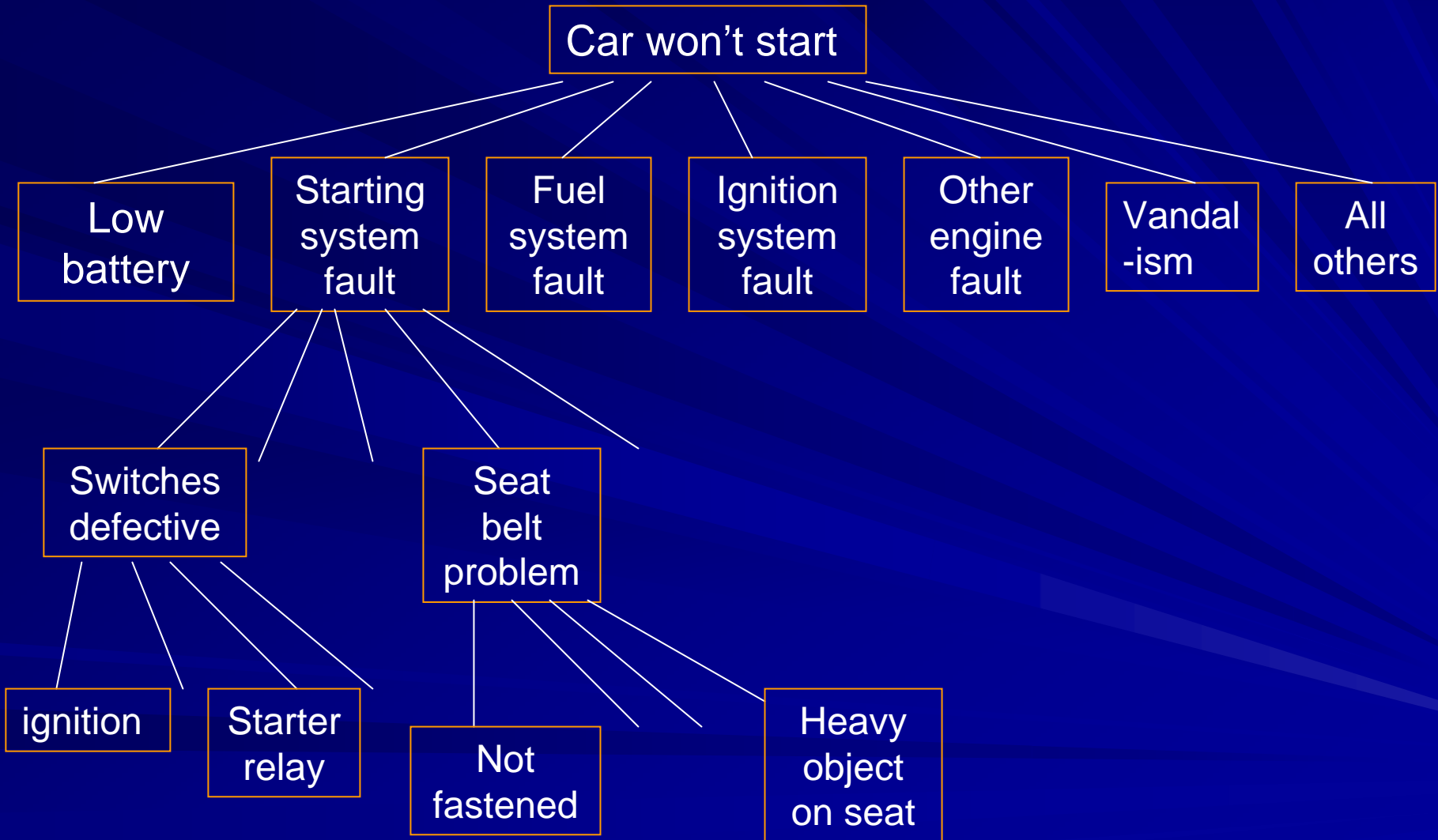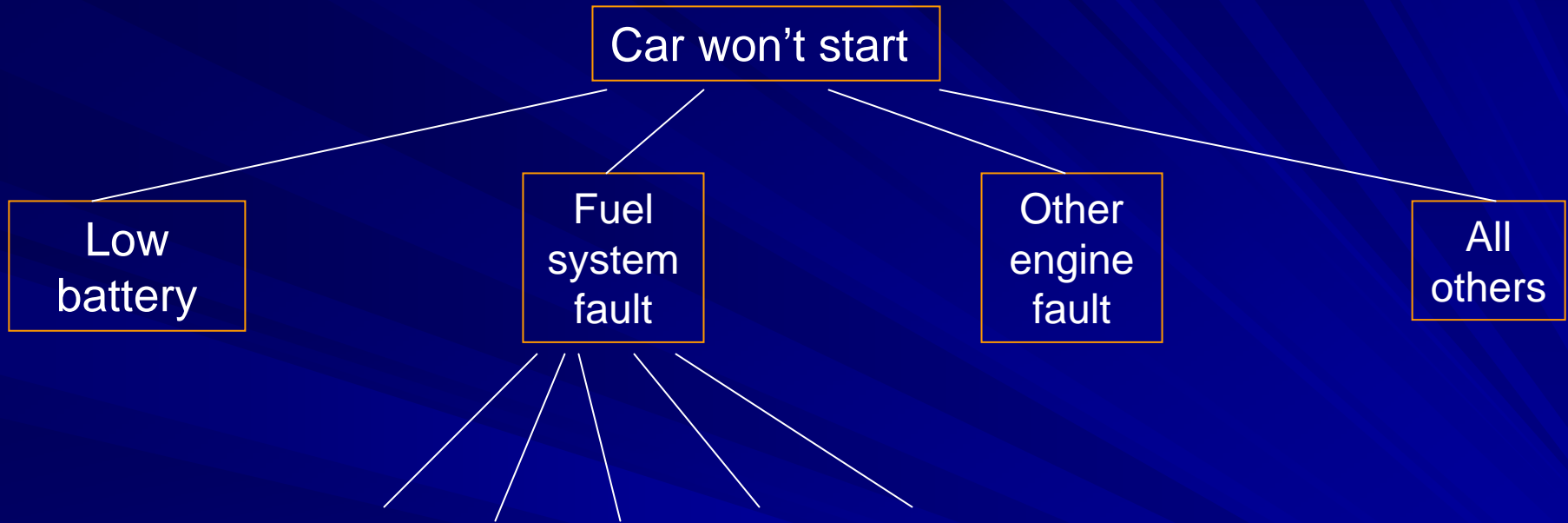
# Results

| Students | Battery | Starting system | Fuel system | Ignition system | Engine | Mis-chief | Other |
|---|---|---|---|---|---|---|---|
| Full | 26% | 20% | 19% | 14% | 7% | 5% | 8% |
| Pruned | 43% | | 31% | | 12% | | 14% (47%) |
| Car Mech-anics | | | | | | | |
| Full | 41% | 11% | 10% | 25% | 5% | 3% | 6% |
| Pruned | 48% | | 23% | | 7% | | 22% (45%) |

# Summary and Look Ahead

- Simulation is useful to understand the impact of uncertainty on the distribution of expected outcomes

- Fortunately, computers can be a big help in generating large numbers of scenarios

- Computers can't think by themselves; they need you to tell them what to think about (out of sight, out of mind)

- Prepare the Gentle Lentil case for tomorrow but nothing to hand in.  Try to use Crystal Ball, but you are not required to hand anything in.