15.063: Communicating with Data
Summer 2003

Recitation 5

Simulation

Today’s Content

- Simulation
- Crystal Ball
- Problems

Simulation

- Why?
- What?
- Pros?
- Cons?

Crystal Ball

- Simulation package
- Inputs are RV: *assumptions*
- Functions of assumptions are *forecasts*
- Output:

  statistics of forecasts after randomly generating values for the assumptions

Crystal Ball

- Problem
  - define random variables
  - define function f of random variables
  - obtain theoretical results for f if possible.
- Crystal Ball
  - define assumption
  - construct forecast f
  - obtain statistical data for f after simulation.

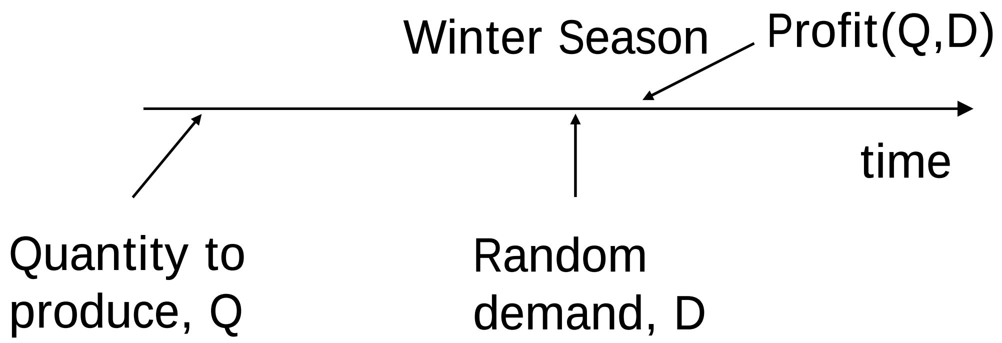
Exercise 3.11

- See example 3.11 in the course textbook:

  Data, Models, and Decisions: The Fundamentals of Management Science by Dimitris Bertsimas and Robert M. Freund, Southwestern College Publishing, 2000.

Ski Jacket Production

The problem is to decide how many ski jackets to produce given an uncertain level of demand in order to maximize profit



Ski Jacket

- Want to: select Q that maximizes profit
- Since demand D is *unknown* (RV), pick Q that maximizes the *expected* profit
- The problem is: $\max_Q E[\text{Profit}(Q,D)]$

Ski Jacket Production

- Costs:
  - Variable prod. cost per unit (C): $100
  - Selling price per unit (S): $125
  - Salvage value per unit (V): $27.50
  - Fixed production cost (F): $100,000
- Let Q denote the quantity of ski jackets to produce (decision variable).

Ski Jacket

- Managers have estimated the demand for ski jackets to be normally distributed with:
  - mean $\mu = 12000$
  - standard deviation $\sigma = 2750$

Ski Jacket

- What is the general formula for the profit given production Q and demand D?
  Profit(Q,D)=?
  - 2 cases:
    - What happens if $D \geq Q$?
    - What happens if $D < Q$?

Ski Jacket

General formula for the profit given production Q and demand D

- Profit(Q,D)=

$$=\begin{cases} 125*Q-100*Q-100{,}000 & D \geq Q \\ 125*D-100*Q+27.5*(Q-D)-100{,}000 & D < Q \end{cases}$$

Ski Jacket

- How do we solve $\max_Q E[\text{Profit}(Q,D)]$?
- By simulation:
  - Choose Q
  - Simulate n random demands: $D_1,\ldots,D_n$
  - Compute profits Profit(Q,D_1),...,Profit(Q,D_n)
  - Estimate expected profit E[Profit(Q,D)] = {Profit(Q,D_1)+...+Profit(Q,D_n)} / n
  - Repeat process with another value of Q

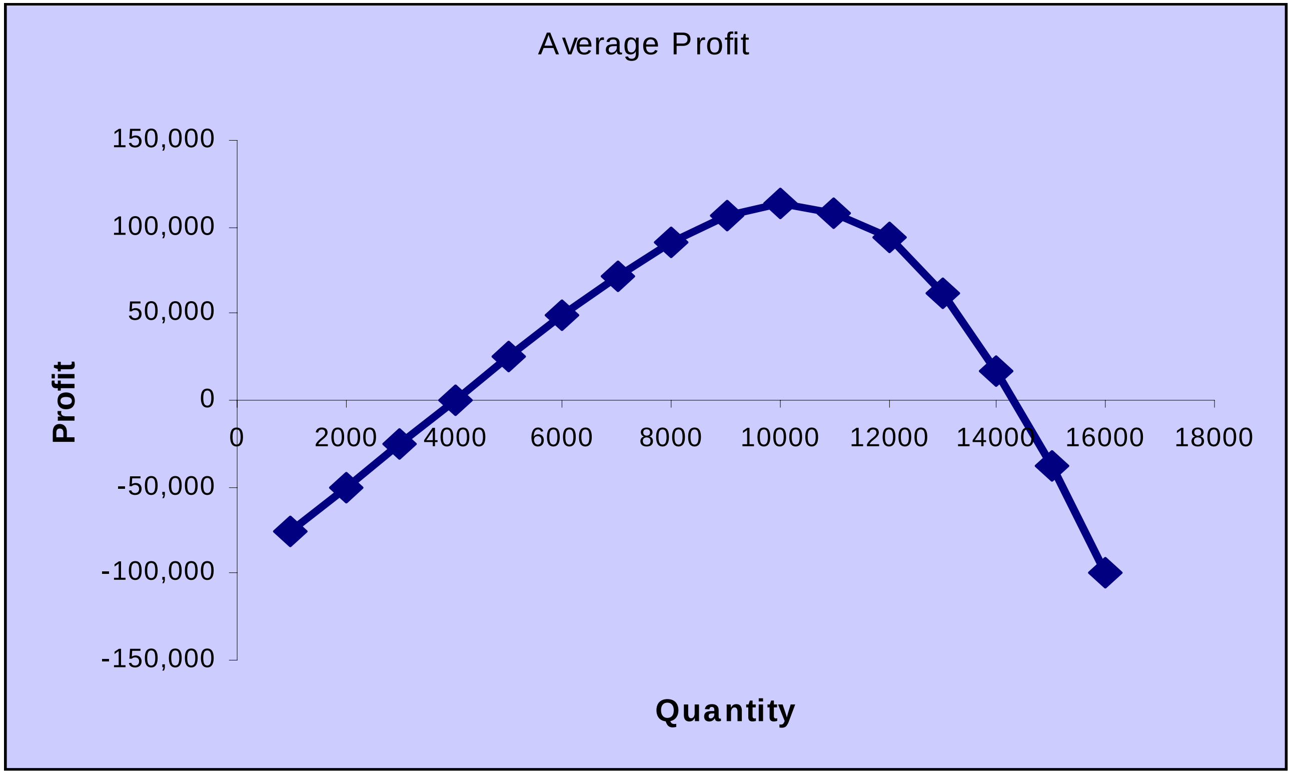
Ski Jacket

- Look at worksheet page 'ski'
- We have done simulations for 16 different values of Q. 1000, 2000, ...,16000.
- From these values we analyze the means, and the standard deviation.

Ski Jacket

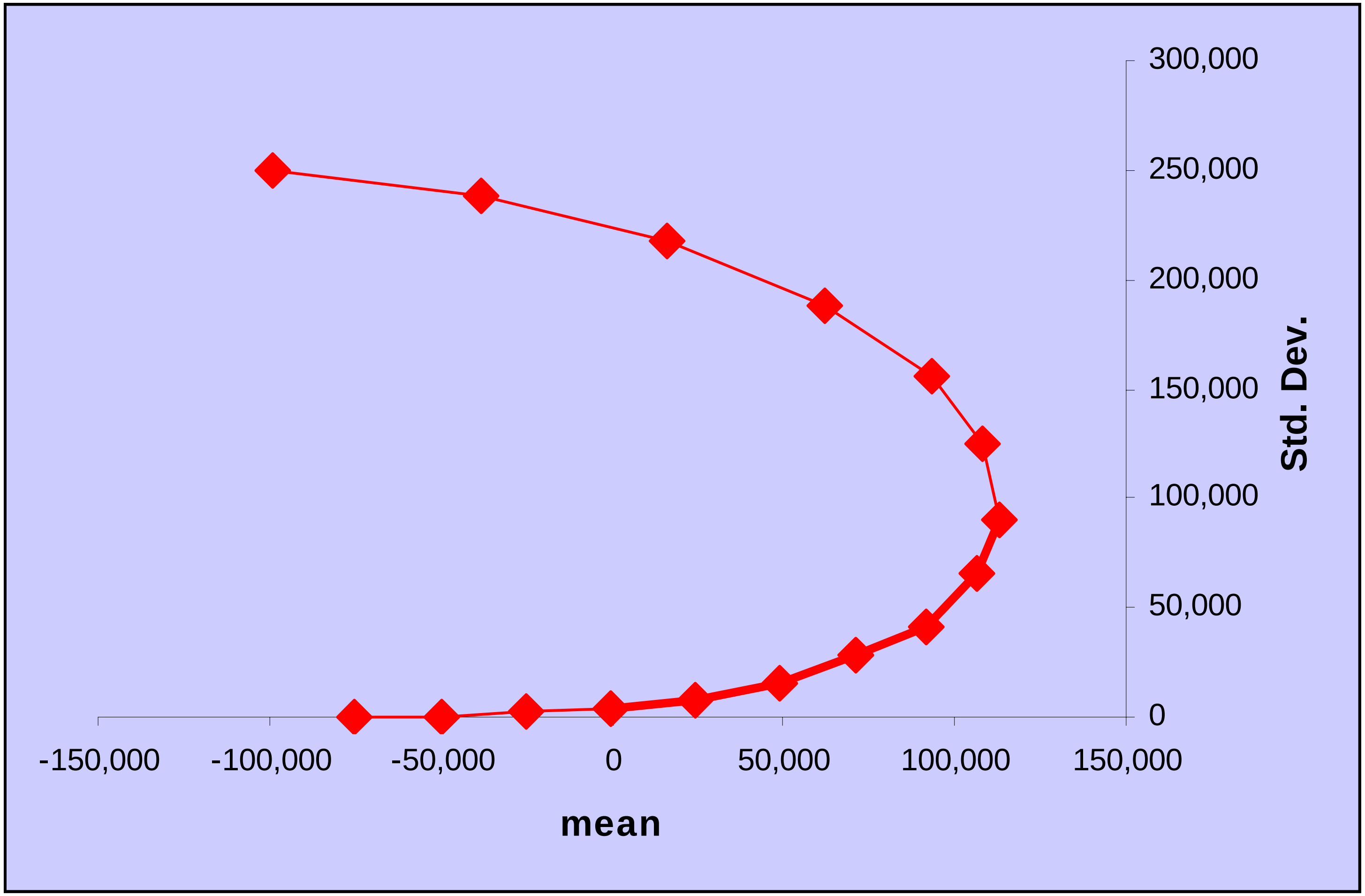
Estimates for Profit(Q) from Simulations		
Each simulation 10000 trials		
Cuantity (Q)	**Mean**	**Std. Dev.**
1000	-75,000	0
2000	-50,000	0
3000	-25,053	2,450
4000	-123	3,450
5000	24,636	7,257
6000	48,589	15,428
7000	71,013	27,836
8000	91,324	41,675
9000	106,164	65,709
10000	113,184	90,175
11000	108,484	125,305
12000	93,303	156,233
13000	61,994	188,339
14000	16,131	217,111
15000	-38,050	237,658
16000	-99,317	249,653

Ski Jacket



- From this data we conclude that the optimal Q is around 10000

Ski Jacket


300,000
250,000
200,000
150,000
100,000
50,000
0
Std. Dev.
-150,000
-100,000
-50,000
0
50,000
100,000
150,000
mean

Ski Jacket

- To further analyze our solution it is wise to study the distribution of our suggested answer.

- We observe for Q=10000 that with probability greater than 80% we have a profit higher than $115,451.

Percentile	Profit
0%	($773,879)
10%	($1,247)
20%	$115,451
30%	$150,000
40%	$150,000
50%	$150,000
60%	$150,000
70%	$150,000
80%	$150,000
90%	$150,000
100%	$150,000

The End.