Let's look at how sports scheduling can be done with optimization by starting with a small example.

Suppose we're trying to schedule a tournament between four teams-- Atlanta, Boston, Chicago, and Detroit.

We'll call these teams A, B, C, and D.

These teams are divided into two divisions.

Atlanta and Boston are in one division, and Chicago and Detroit are in the second division.

Each team plays one game a week for a total of four weeks.

During the four weeks, each team plays the other team in the same division twice, and each team plays the teams in the other divisions once.

The team with the most wins from each division will play in the championship game.

For this reason, teams prefer to play divisional games later.

If team A plays team C and D in the first two weeks and wins both games while team B also plays team C and D and loses both games, A knows that they only need to win one of the games against B to beat B in terms of wins and to go to the championship.

We can schedule this tournament by using an optimization model.

Our objective is to maximize team preferences, which are that teams would like to play divisional games later.

Our decisions are which teams should play each other each week.

And we have three types of constraints.

Each team needs to play the other team in their division twice, each team needs to play the teams in the other division once, and each team should play exactly one team each week.

Let's start by discussing our decision variables.

We need to decide which teams will play each other each week.

To do this, let's define decision variables which we'll call x_ijk.

If team i plays team j in week k, then x_ijk will be equal to 1.

Otherwise, x_{ijk} equals 0.

As an example, suppose team A plays team C in week 2.

Then x_{AC2} would equal 1.

Since A only plays C once, we should have then that x_{AC1}, or A playing C in week 1, should be equal to 0.

Similarly, x_{AC3} should equal 0, and x_{AC4} should equal 0.

This is called a binary decision variable since it's a decision variable that can only take two values, 0 and 1.

This is a new type of decision variable, and it's what makes integer optimization different from linear optimization.

The decision variables in integer optimization can only take integer values.

This includes binary decision variables, like the ones we have here, that can only be either 0 or 1.

These variables can model decisions like where to build a new warehouse, whether or not to invest in a stock, or assigning nurses to shifts.

Integer optimization problems can also have integer decision variables that take values 1, 2, 3, 4, 5, etc.

These variables can model decisions like the number of new machines to purchase, the number of workers to assign for a shift, and the number of items to stock in a store.

Other than the new types of variables, integer optimization is exactly like linear optimization.

But we'll see this week how integer optimization variables, and especially binary variables, can increase our modeling capabilities.

Now let's go back to our formulation.

As we said before, our decisions are which teams should play each other each week.

We'll model this with the binary decision variables we just discussed-- x_{ijk} which equal 1 if team i plays team j in week k.

Now let's use these decision variables to form our constraints.

The first constraint is that each team should play the other team in their division twice.

So teams A and B should play each other twice in the four weeks.

This can be modeled with the constraint $x_{AB1} + x_{AB2} + x_{AB3} + x_{AB4} = 2$.

This will force two of these decision variables to be equal to 1, and the other two decision variables to be equal to 0.

The ones that are equal to 1 will correspond to the weeks that A and B will play each other.

We'll have a similar constraint for teams C and D.

Our next constraint is that each team should play the teams in the other division once.

So teams A and C should play each other once in the four weeks.

This can be modeled with a constraint $x_{AC1} + x_{AC2} + x_{AC3} + x_{AC4} = 1$.

This is very similar to the previous constraint, except this time only one of the decision variables will have value 1.

We'll have similar constraints for teams A and D, teams B and C, and teams B and D. Our last type of constraint is that each team should play exactly one other team each week.

This means that A should play B, C or D in week 1.

This can be modeled with a constraint $x_{AB1} + x_{AC1} + x_{AD1} = 1$.

Exactly one of these decision variables will be equal to 1, meaning that A will play that team in week 1.

We'll have a similar constraint for every team and week pair.

Now, let's model our objective.

Let's assume that teams have a preference of 1 for playing divisional games in week 1, a preference of 2 for playing divisional games in week 2, a preference of 4 for playing divisional games in week 3, and a preference of 8 for playing divisional games in week 4.

So the preference doubles with each later week.

Then we can model our objective as $x_{AB1} + 2*x_{AB2} + 4*x_{AB3} + 8*x_{AB4} + x_{CD1} + 2*x_{CD2} + 4*x_{CD3} + 8*x_{CD4}$.

Then if team AB plays in week 3, we'll add 4 to our objective.

If they play in week 1, then we'll only add 1 to our objective.

If they don't play in a week, that term will be 0 and will not contribute to the objective.

Now that we've set up our problem, we're ready to solve it.

In the next video, we'll set up and solve our problem in LibreOffice.