15.401 Finance Theory

MIT Sloan MBA Program

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Lectures 8–9: Forward and Futures Contracts

Critical Concepts

- Motivation
- Forward Contracts
- Futures Contract
- Valuation of Forwards and Futures
- Applications
- Extensions and Qualifications

Readings:

- Brealey, Myers, and Allen Chapters 27

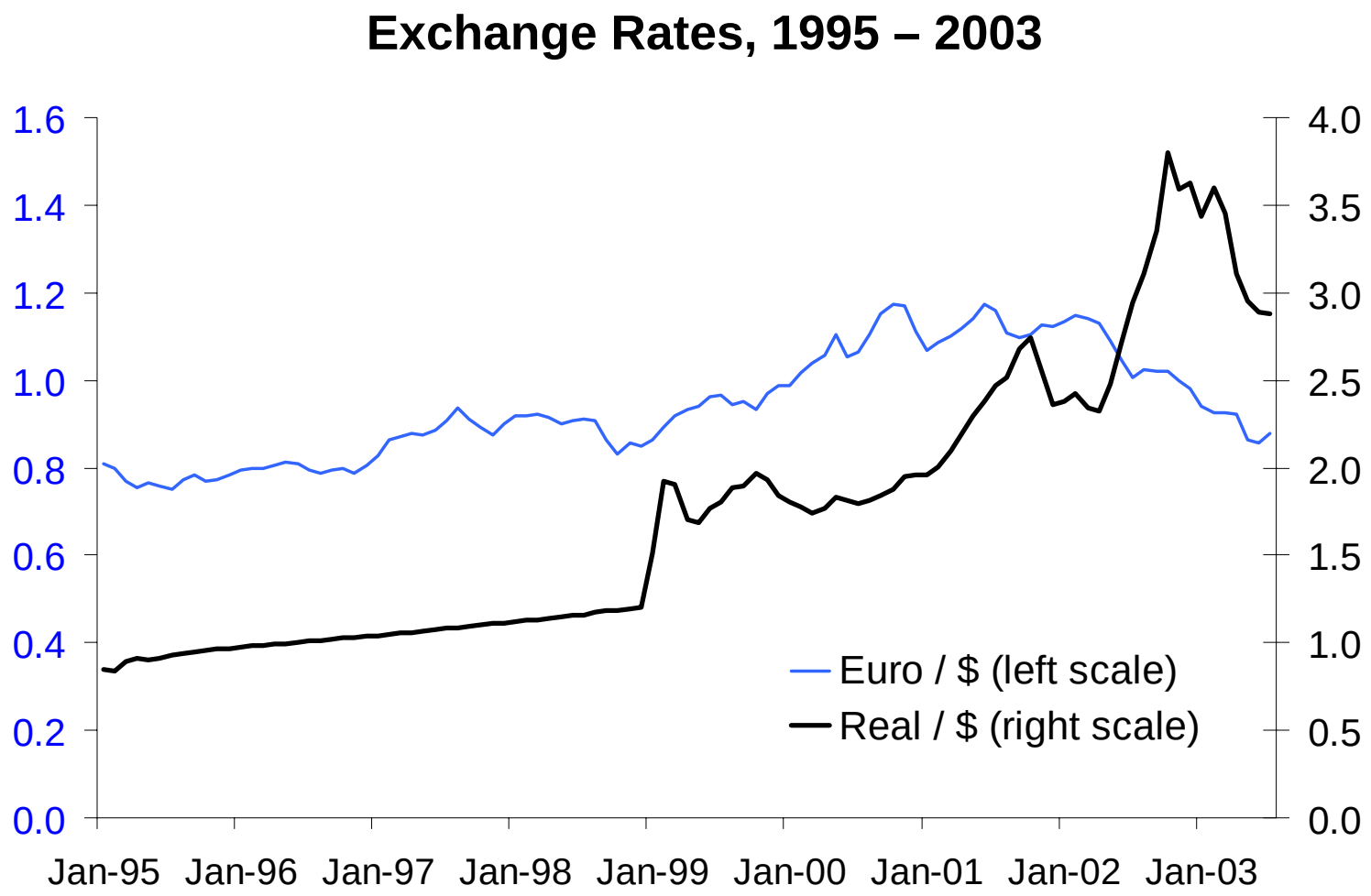
Motivation

- Your company, based in the U.S., supplies machine tools to customers in Germany and Brazil. Prices are quoted in each country's currency, so fluctuations in the € / $ and R / $ exchange rates have a big impact on the firm's revenues. How can the firm reduce (or 'hedge') these risks?

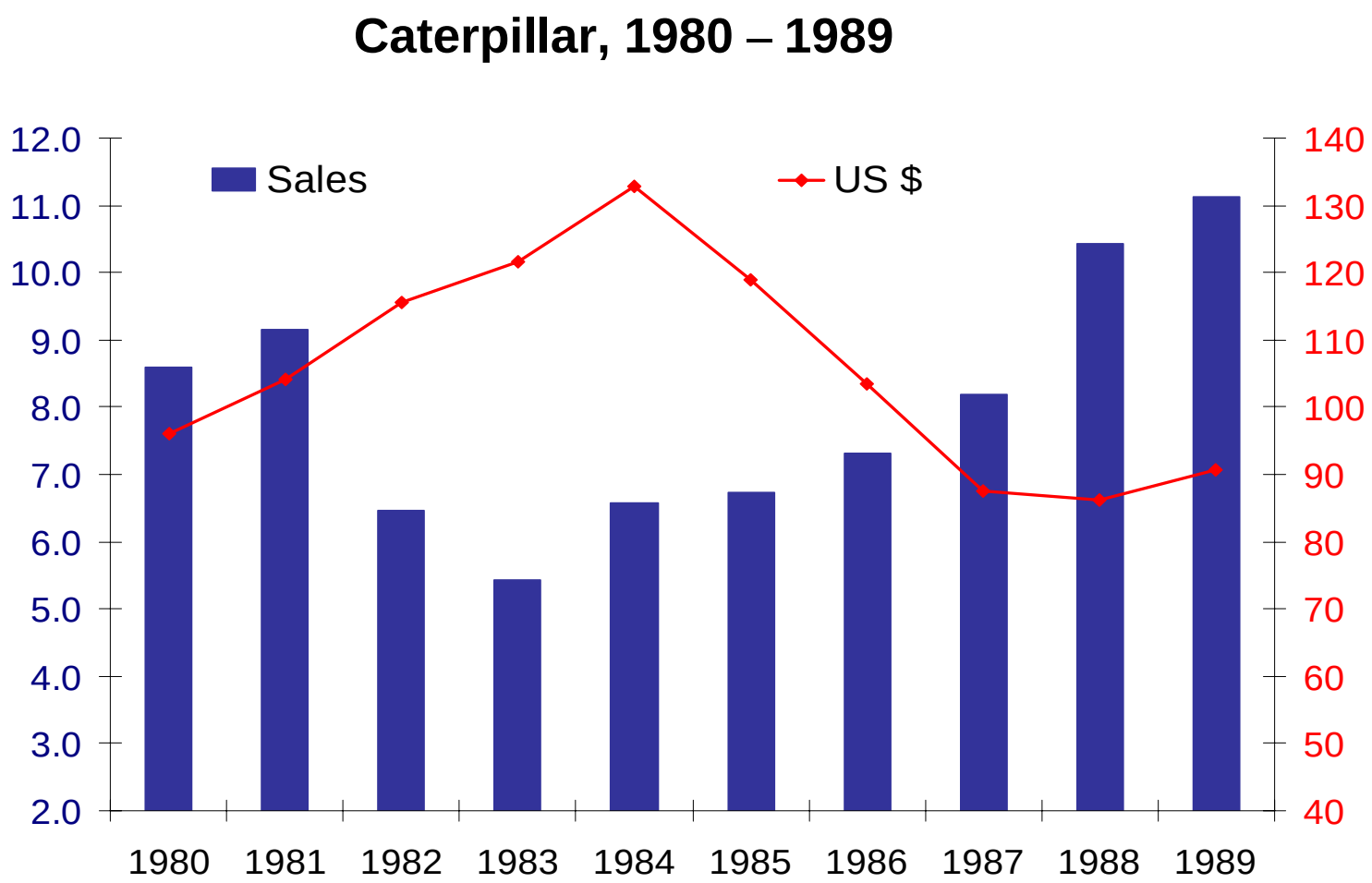
- Your firm is thinking about issuing 10-year convertible bonds. In the past, the firm has issued straight debt with a yield-to-maturity of 8.2%. If the new bonds are convertible into 20 shares of stocks, per $1,000 face value, what interest rate will the firm have to pay on the bonds?

- You have the opportunity to buy a mine with 1 million kgs of copper for $400,000. Copper has a price of $2.2 / kg, mining costs are $2 / kg, and you can delay extraction one year. How valuable is the option to delay? Is the mine a good deal?

Motivation



Motivation



Motivation

Hedging or Speculation?

Alternative Tools?

- Futures, forwards, options, and swaps
- Insurance
- Diversification
- Match duration of assets and liabilities
- Match sales and expenses across countries (currency risk)

Should Firms Hedge With Financial Derivatives?

- “Derivatives are extremely efficient tools for risk management”
- “Derivatives are financial weapons of mass destruction”

Motivation

View 1: Hedging is irrelevant (M&M)

- Financial transaction, zero NPV
- Diversified shareholders don't care about firm-specific risks

View 2: Hedging creates value

- Ensures cash is available for positive NPV investments
- Reduces need for external finance
- Reduces chance of financial distress
- Improves performance evaluation and compensation

Examples:

- Homestake Mining
  Does not hedge because "shareholders will achieve maximum benefit from such a policy."
- American Barrick
  Hedges aggressively to provide "extraordinary financial stability… offering investors a predictable, rising earnings profile in the future."
- Battle Mountain Gold
  Hedges up to 25% because "a recent study indicates that there may be a premium for hedging."

Motivation

Evidence*

- Random sample of 413 large firms
- Average cashflow from operations = $735 million
- Average PP&E = $454 million
- Average net income = $318 million

57% of Firms Use Derivatives In 1997

- Small derivative programs
- Even with a big move (3σ event), the derivative portfolio pays only $15 million and its value goes up by $31 million

* Guay and Kothari, *Journal of Financial Economics*, 2003

Motivation

Basic Types of Derivatives

- **Forwards and Futures**

  A contract to exchange an asset in the future at a specified price and time.

- **Options** (Lecture 10)

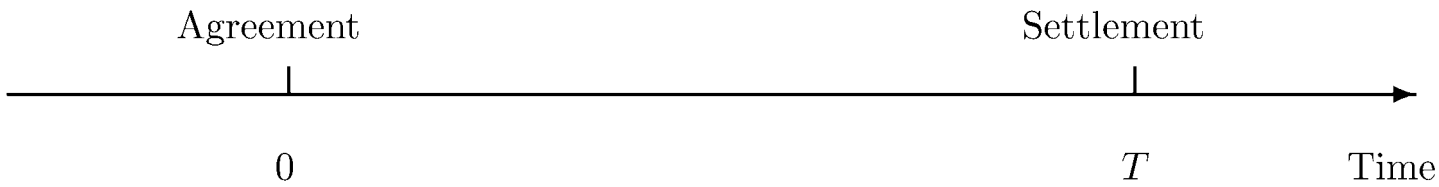
  Gives the holder the right to buy (call option) or sell (put option) an asset at a specified price.

- **Swaps**

  An agreement to exchange a series of cashflows at specified prices and times.

Forward Contracts

Definition: A **forward contract** is a commitment to purchase at a future date a given amount of a commodity or an asset at a price agreed on today.



- The price fixed now for future exchange is the **forward price**
- The buyer of the underlying is said to be "**long**" the forward

Features of Forward Contracts

- Customized
- Non-standard and traded over the counter (not on exchanges)
- No money changes hands until maturity
- Non-trivial counterparty risk

Forward Contracts

Example:

- Current price of soybeans is $160/ton
- Tofu manufacturer needs 1,000 tons in 3 months
- Wants to make sure that 1,000 tons will be available
- 3-month forward contract for 1,000 tons of soybeans at $165/ton
- Long side will buy 1,000 tons from short side at $165/ton in 3 months

Futures Contracts

Forward Contracts Have Two Limitations:

- Illiquidity
- **Counterparty risk**

Definition: A **futures contract** is an exchange-traded, standardized, forward-like contract that is **marked to market** daily. This contract can be used to establish a long (or short) position in the underlying asset.

Features of Futures Contracts

- Standardized contracts:
  - Underlying commodity or asset
  - Quantity
  - Maturity
- Exchange traded
- Guaranteed by the **clearing house**—no counter-party risk
- Gains/losses settled daily (marked to market)
- **Margin** required as collateral to cover losses

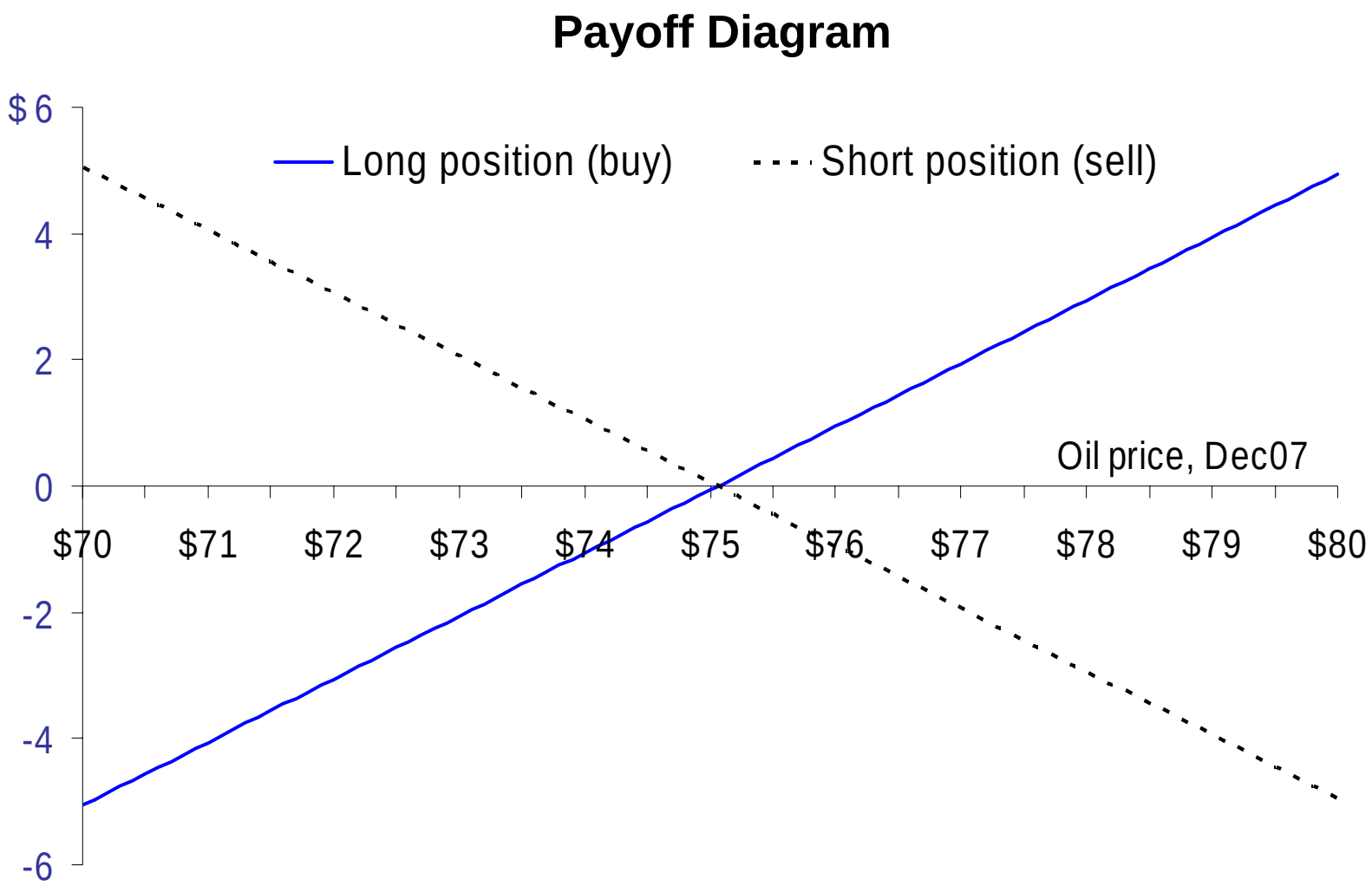
Futures Contracts

Example:

NYMEX crude oil (light) futures with delivery in Dec. 2007 at a price of $75.06 / bbl. on July 27, 2007 with 51,475 contracts traded

- Each contract is for 1,000 barrels
- Tick size: $0.01 per barrel, $10 per contract
- Initial margin: $4,050
- Maintenance margin: $3,000
- No cash changes hands today (contract price is $0)
- Buyer has a “long” position (wins if prices go up)
- Seller has a “short” position (wins if prices go down)

Futures Contracts



Futures Contracts

Example. Yesterday, you bought 10 December live-cattle contracts on the CME, at a price of $0.7455/lb

- Contract size 40,000 lb
- Agreed to buy 40,000 pounds of live cattle in December
- Value of position yesterday:

$$(0.7455)(10)(40{,}000) = \$298{,}200$$

- No money changed hands
- Initial margin required (5%–20% of contract value)

Today, the futures price closes at $0.7435/lb, 0.20 cents lower. The value of your position is

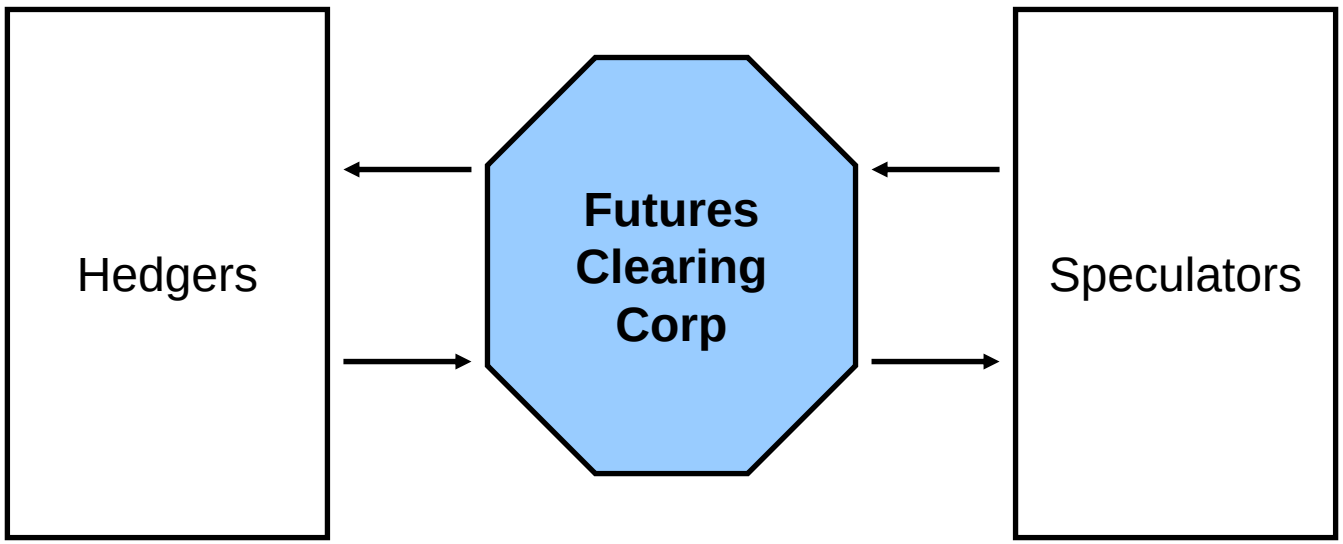
$$(0.7435)(10)(40{,}000) = \$297{,}400$$

which yields a loss of $800.

Futures Contracts

Why Is This Contract Superior to a Forward Contract?

- Standardization makes futures liquid
- Margin and marking to market reduce default risk
- Clearing-house guarantee reduces counter-party risk



Valuation of Forwards and Futures

What Determines Forward and Futures Prices?

- Forward/futures prices ultimately linked to future spot prices
- Notation:

Contract	Spot at t	Forward	Futures
Price	S_t	$F_{t,T}$	$H_{t,T}$

- Ignore differences between forward and futures price for now

$$F_{t,T} \approx H_{t,T}$$

- Two ways to buy the underlying asset for date-*T* delivery
  1. Buy a forward or futures contract with maturity date *T*
  2. Buy the underlying asset and store it until *T*

Valuation of Forwards and Futures

Date	Forward Contract	Outright Asset Purchase
0	▪ Pay \$0 for contract with forward price $\$F_{0,T}$	▪ Borrow $\$S_0$ ▪ Pay $\$S_0$ for Asset
T	▪ Pay $\$F_{0,T}$ ▪ Own asset	▪ Pay back $\$S_0(1+r)^T$ ▪ Pay cumulative storage costs (if any) ▪ Deduce cumulative "convenience yield" (if any) ▪ Own asset
Total Cost at T	**$\$F_{0,T}$**	**$\$S_0(1+r)^T$ + net storage costs**

$$F_{0,T} \approx H_{0,T} = (1+r_f)^T S_0 + \text{FV}_T(\text{net storage costs})$$

$$\frac{F_{0,T}}{(1+r)^T} \approx \frac{H_{0,T}}{(1+r)^T} = S_0 + \text{PV}_0(\text{net storage costs})$$

Valuation of Forwards and Futures

Date	Forward Contract	Outright Asset Purchase
t	▪ Pay \$0 for contract with forward price $\$F_{t,T}$	▪ Borrow $\$S_t$ ▪ Pay $\$S_t$ for Asset
T	▪ Pay $\$F_{t,T}$ ▪ Own asset	▪ Pay back $\$S_t(1+r)^{T-t}$ ▪ Pay cumulative storage costs (if any) ▪ Deduce cumulative "convenience yield" (if any) ▪ Own asset
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$$\frac{F_{t,T}}{(1+r)^{T-t}} \approx \frac{H_{t,T}}{(1+r)^{T-t}} = S_t + \text{PV}_t(\text{net storage costs})$$

Valuation of Forwards and Futures

What Determines Forward/Futures Prices?

- Difference between the two methods:
  - Costs (storage for commodities, not financials)
  - Benefits (convenience for commodities, dividends for financials)
- By no arbitrage (**Principal P1)**, these two methods must cost the same

Gold

- Easy to store (negligible costs of storage)
- No dividends or benefits
- Two ways to buy gold for T
  - Buy now for S_t and hold until T
  - Buy forward at t, pay $F_{t,T}$ at T and take delivery at T
- No-arbitrage requires that

$$F_{t,T} \approx H_{t,T} = (1+r_f)^{(T-t)} S_t$$

Valuation of Forwards and Futures

Gasoline

- Costly to store (let *c* be percentage cost per period)
- Convenience yield does exist (let *y* be percentage yield per period)
- Not for long-term investment (like gold), but for future use
- Two ways to buy gasoline for *T*
  - Buy now for S_t and hold until *T*
  - Buy forward at *t*, pay $F_{t,T}$ at *T* and take delivery at *T*
- No-arbitrage requires that

$$F_{t,T} \approx H_{t,T} = (1 + r_f + c - y)^{(T-t)} S_t$$

Valuation of Forwards and Futures

Financials

- Let underlying be a financial asset
  - No cost to store (the underlying asset)
  - Dividend or interest on the underlying
- Example: Stock index futures
  - Underlying are bundles of stocks, e.g., S&P, Nikkei, etc.
  - Futures settled in cash (no delivery)
  - Let the annualized dividend yield be *d;* then:

$$F_{t,T} \approx H_{t,T} = (1 + r_f - d)^{(T-t)} S_t$$

Valuation of Forwards and Futures

Example:

Gold quotes on 2001.08.02 are

- Spot price (London fixing) $267.00/oz
- October futures (CMX) $269.00/oz
- What is the implied interest rate?

$$F = S_0(1+r_f)^{2/12}$$

$$r_f = (F/S_0)^6 - 1 = 4.58\%$$

Valuation of Forwards and Futures

Example:

Gasoline quotes on 2001.08.02:

- Spot price is 0.7760
- Feb 02 futures price is 0.7330
- 6-month interest rate is 3.40%
- What is the annualized net convenience yield (net of storage costs)?

$$0.7330 = (0.7760)(1 + 0.0340 - y)^{6/12}$$

$$y = 1.0340 - \left(\frac{0.7330}{0.7760}\right)^2 = 14.18\%$$

Valuation of Forwards and Futures

Example:

- The S&P 500 closed at 1,220.75 on 2001.08.02
- The S&P futures maturing in December closed at 1,233.50
- Suppose the T-bill rate is 3.50%
- What is the implied annual dividend yield?

$$\begin{aligned} d &= \left[1 + r_f - (F/S_0)^{12/4}\right] \\ &= \left[1 + 0.0350 - (1233.50/1220.75)^3\right] = 0.33\% \end{aligned}$$

Applications

Index Futures Have Many Advantages

- Since underlying asset is a portfolio, trading in the futures market is easier than trading in cash market
- Futures prices may react quicker to macroeconomic news than the index itself
- Index futures are very useful for:
  - Hedging market risk in block purchases and underwriting
  - Creating synthetic index fund
  - Portfolio insurance

Applications

Example:

You have $1 million to invest in the stock market and you have decided to invest in the S&P 500. How should you do this?

- One way is to buy the S&P 500 in the cash market:
  - Buy the 500 stocks, weights proportional to their market caps
- Another way is to buy S&P futures:
  - Put the money in your margin account
  - Assuming the S&P 500 is at 1,000 now, number of contract to buy: (value of a futures contract is $250 times the S&P 500 index)

$$\frac{\$1{,}000{,}000}{250 \times 1{,}000} = 4$$

Applications

Example (cont):

- As the S&P index fluctuates, the future value of your portfolio (in $MM) is given by the following table (ignoring interest payments and dividends):

S&P 500	Cash Portfolio	Futures Portfolio
900	\$0.90	\$0.90
1,000	\$1.00	\$1.00
1,100	\$1.10	\$1.10

- Suppose you a diversified portfolio of large-cap stocks worth $5MM and are now worried about equity markets and would like to reduce your exposure by 25%—how could you use S&P 500 futures to implement this hedge?
  - (Short)sell 5 S&P 500 futures contracts (why 5?)

Applications

Example (cont):

- Compare hedged and unhedged portfolio (in $MM):

S&P 500	Cash Portfolio	Cash Plus Futures Portfolio
900	\$4.50	\$4.50 + \$0.125 = \$4.625
1,000	\$5.00	\$5.00
1,100	\$5.50	\$5.50-\$0.125=\$5.375

- Fluctuations have been reduced
- As if 25% of the portfolio has been shifted to cash

Extensions and Qualifications

- Interest-rate, bond, and currency futures are extremely popular
- Single-stock futures are gaining liquidity
- Volatility futures recently launched (VIX)

Key Points

- Forward and futures contracts are zero-NPV contracts when initiated
- After initiation, both contracts may have positive/negative NPV
- Futures contracts are “marked to market” every day
- Futures and forwards are extremely liquid
- Hedging and speculating are important applications of futures/forwards

Additional References

- Brealey, R., Myers, S., and F. Allen, 2006, *Principles of Corporate Finance*. New York: McGraw-Hill Irwin.
- Guay, W. and S. Kothari, 2003, "How Much Do Firms Hedge with Derivatives?," *Journal of Financial Economics* 70, 423–461.
- Siegel, D. and D. Siegel, 1990, *The Futures Market: Arbitrage, Risk Management, and Portfolio Strategies*. Hinsdale, IL: Dryden Press.

15.401 Finance Theory I
Fall 2008