ANDREW LO: I hope you all had a good Columbus Day weekend. The stock market certainly did. Any questions from last time? No? OK. So what I want to do today is to continue talking about futures and forward contracts. Today we're going to finish up on these interesting instruments, with a couple of examples, and then with a specific method for pricing forward and futures contracts. So let me refresh your memory, it's been a week, so I know.

So we're going to go back and look at a specific futures contract. And I'm going to take this contract and then try to talk a bit about how you might use contracts like this in hedging your risks, as well as in making certain kinds of bets, if you will. So remember that this contract is a contract that was issued on July 27, 2007-- so it was the middle of the summer-- for oil to be delivered in December. And there's a specific date in December where all oil futures contracts of this type will settle-- that is will come to maturity-- where the date is going to be specified in advance and everybody knows it. And so in July, when you buy this contract at a price of $75.06 per barrel, and each contract is for 1,000 barrels.

When you quote "buy the contract," what that means is that you are agreeing today, July 27-- you are agreeing today, that come December you are going to buy 1,000 barrels at a price of $75.06 per barrel. So that's the agreement. And the party who is selling you the contract, the counterparty, is agreeing to provide you with that oil at that price in December. So the futures price is $75.06. And as we said last time, the current market price on July 27, 2007, that's called the spot price. The spot price may be higher or lower than the futures price, depending on what expectations are for what's going to happen with oil over the subsequent six month period.

Now the initial margin, as I mentioned last time, was $4,050. The maintenance margin, the margin that you need to maintain. So that if the initial margin goes down in value, you have to actually put money back into your brokerage account, your margin account. And if you fall below that $3,000 threshold, you'll get a phone call, which is known as a margin call. Several weeks ago, somebody told me that they've been getting lots of phone calls all from the same person, a person called margin. And you know that can happen when markets go awry.

Now again no cash changes hands today, because the value of the contract when it's struck is a zero NPV transaction. And how do you know it's zero NPV? Again, because if it's positive for

one party, it's coming from the other party. So which party would you like to be? You'd like to be the party receiving that positive NPV, nobody wants to be the party that is losing the NPV. So the futures price will adjust, in order to make it zero NPV. In fact, that's what we mean when we say that it's zero NPV. It is the futures price that makes it so.

So I'll give me an example. If it turns out that somebody suggests a futures price of $60 a barrel on that day. Lots and lots of people are going to want to buy that contract, because that's a good deal relative to where oil really should be. And that means lots of people are going to want to buy, but nobody's going to want to sell at that price of $60. So if everybody wants to buy and nobody wants to sell, what has to happen to the price? Exactly, it goes up. And it keeps going up until the number of buyers equals the number of sellers. That's the point at which it's a zero NPV transaction.

Now let's take a look at what the payoff is of such a contract on day zero, in this case July 27, 2007, the contract is worth nothing. But if the futures price moves tomorrow, then the contract could actually have value. And a diagram of how that works is something like this. If the futures price-- not the spot-- the spot obviously will move also. But I'm talking about the futures price, because the futures contract is specified so that every day's worth of gains or losses in the futures contract relative to its price, you will have to either get paid, or you will have to pay.

So this blue line shows you the payoff if you're holding a long position in one of these contracts-- if you bought a contract. If you sold the contract, then your payoff diagram is the dotted line. Now the blue line basically says that if the futures price goes above $75.06, then you make money. If the futures price goes below $75.06, you lose money. So when you buy a contract like this, it is as if you actually bought the oil. But you haven't really bought the oil, all you've done is to buy the right and obligation to purchase the oil in the future.

Let me let me give you another example that will make this even more concrete. The only way to understand this-- because this is not a natural security for most of us-- stocks and bonds you might find natural, futures contracts are weird in that they have zero investment today, and so they're worthless today. But they're not worthless after the initial date when you enter into that agreement.

So let's do an example Yesterday, you bought 10 December live cattle contracts on the Chicago Mercantile Exchange at a price of $0.7455 per pound. OK, so you basically bought some cows. And the contract size is 40,000 pounds of cow. I don't know how much cow that is,

but even if you're on the Atkins diet that's plenty of cow. [LAUGHTER] And so what you have though in this contract is not the cows, but rather you have the obligation to buy the cows in December for a price of $0.7455 per pound, and there is 40,000 pounds of it. So the value of your position is the size of the contract, multiplied by the futures price, multiplied by the number of contracts. So it's $298,200.

That's the size of your position, or sometimes that's also called the notional size. You've heard that term over the last few weeks-- notional. Well, this is an example of a notional. So you don't actually own $298,200 of anything, because of course, we've said that the contract is zero NPV when you enter into it. All you've done is to agree to buy 40,000 pounds of cow in December at a particular price. So the idea is that you control the notional amount of $298,200, and what you do specifically get is the profits and losses from that notional.

So let's do an example. That was the position yesterday. No money changed hands. You got some initial margin that you had to put down, but that's really your money in a brokerage account. You're not giving it to anybody. It's safety money, it's collateral. Now today what happens? Let's suppose that today the futures price closes at $0.7435. All right, it's just gone down by 2/10 of a cent. The value of cattle has gone down. Your holding long this cattle contract, maybe you're a restaurateur, you have a chain of steakhouses, and so you need to buy cattle on a regular basis.

So the price of cattle just went down. Did you make money or lose money? Yeah, you lost, if you're long. On the other hand, if you're a cattle farmer and you sold the contract, you did a good thing, because you locked in the price of $0.7455, and now the price went down. So let's calculate what the value of the notional size of the position is. It's $297,400. That yields a loss of $800. So you know what happens today? Today, your broker will deduct $800 from your account, from your margin account, and take that $800 and put it into the cattle farmer's account. So now he has the $800.

Now, if the day after, if tomorrow, it turns out that the price of cattle goes up by 2/10 of a cent, it goes back to $0.7455. You know what happens? You get $800 back. Now the cattle farmer loses that $800 and gives it back to you. You see this way you always settle up every day. So if for some reason the cattle farmer ends up going bankrupt, and isn't able to deliver any cattle to you, then you're at out at most one day's worth of movements.

And that's one of the reasons why futures markets and futures brokers are so careful about

closing down accounts that don't meet their margin requirements. It's because they don't want to have credit risk lingering, growing, and unknown. The first moment that you do not make a margin call-- you do not deposit the requisite margin-- the first time that happens, they have the right, which they exercise always, to close down your position. You're out of the game, and that's the end.

So it reduces dramatically, the amount of credit risk that either counterparty has. I don't have to trust you that three months from now you're going to actually have 40,000 pounds of cow for me. All I need to do is to make sure that this contract settles every day. And the uncertainty then gets resolved day by day, but your credit risk is very well managed, and mine is too.

So this is a very important innovation. It's very different from a forward contract, in the sense that forward contracts contain enormous amounts of credit risk right. Because once we enter into a forward, that's just like a futures contract, but the difference is that we don't exchange any money ever until the settlement date. And by that point you could be so far out of the money, you could be so far in debt to me, as well as to other creditors, that you just can't afford to pay. And so I'm stuck with this piece of paper that says you're going to give me 40,000 pounds of cattle, and you can't even afford to buy me a steak dinner. That's a problem.

So this futures exchange is a beautiful thing. It reduces credit risk. It also encourages liquidity, it encourages trading. Why? Because at any point in time, on any given day between now and December, if you decide that you want to get out of the restaurant business and you don't want this contract any more, you can get out of it. Poof. You just get out of it by doing an opposite transaction.

So if you bought a contract for December, you know what you do when you want to get out of it? You sell a contract for December. You literally sell. So it's actually duplicated transaction, but it's of the opposite sign. And so they cancel out, because you're going to get delivery, and you will provide delivery, and those will cancel out. Yeah, Justin.

AUDIENCE: The price of oil has been going down lately. So let's say I had a long position in oil, and then I found out that I was going to really lose half of that money, and I decided just to forego my margin. What else would I have to pay?

ANDREW LO: Well, first of all, you are liable for all of the losses, not just the margin. So the margin account is not meant to be a non-recourse loan. They will go after your assets. Now you could declare bankruptcy, personal bankruptcy, and get protection under Chapter 7. But that will hurt your

credit ratings and all other nasty things will happen to you if that occurs.

AUDIENCE: So when you're saying that they close out your account, when your margins down. So they close it out, but if your losses are higher than your margin was anyway, so you're still liable for those in addition to [INAUDIBLE].

ANDREW LO: So you make a good point. Is it generally possible that your losses are greater than the amount of margin? So in that case, who gets left holding the bag? You know who gets left holding the bag? That blue box in the middle, the Futures Clearing corporation. But the reason that they establish a particular level of margin is to be able to ensure that that's a very unlikely event. And it goes back to what are the likely daily swings in the futures price.

If you put enough margin in your account, so that I can be sure that 99% of the time you can cover the daily swing, then I don't have to worry. Now, of course, if we had a day like last Friday, or on Monday, you know that's pretty outrageous. That's one of the reasons why a number of futures exchanges have increased their margin levels. It's because the daily swings have gotten much bigger. But as long as they can cover the one day movement, they don't have to go after your home or your other assets. Yeah, Dennis.

AUDIENCE: You said if I bought a contract now, then I just have to sit on the same contract. What happens if I bought at a $1.00, it's at $0.50 now, I can't sell at a $1.00.

ANDREW LO: Oh, no. You certainly cannot sell at the $1.00, you have to sell at $0.50. Says

AUDIENCE: So I'm not really out of the position.

ANDREW LO: You are out of the position, because you don't have a claim, or you don't have a commitment to enter into that trade in December. That's what I mean when I say you're out of the position. You also happen to be out of money in your example. [LAUGHTER] In other words, you lost $0.50. That's not coming back. But what it means to sell is that you bought a contract that says in December you're going to buy 40,000 of cattle-- you're committed to doing that.

Now if on the other hand, the next day you decide you want to get out of that commitment-- the way to get out of it is not to try to contact the counterparty and say, would you mind canceling my trade. The way to do it is to simply sell a commitment at 40,000 pounds of cattle for December. So your commitment to buy and your commitment to sell, basically cancel each other out. So on settlement date, the Futures Clearing corporation will net out all of these buys

and sells, and the net amount will be transacted between the providers of the cattle and the buyers of the cattle.

AUDIENCE: So this means that there's no physical delivery then.

ANDREW LO: That's right. So that's an example where if you bought and sold, then you would be netted out and you would not have a physical delivery. Yeah, [INAUDIBLE].

AUDIENCE: So if the margin is just a fund for exchanging commodities, what does the Futures Clearing corp-- what do they make? Is it a percentage of-- how do they make money?

ANDREW LO: Well, their job is really not to make money, but to create an exchange for its members. So many exchanges are not for profit. Some of them are for profit, but the objective of the Futures Clearing corporation is really not to make a lot of money. What they're trying to do is just create a market and let people who want to trade with each other, trade freely and efficiently. They will charge perhaps a small transaction fee, that you have to pay in order to support the operations. But they're not trying to make tons of profits off of that necessarily.

Now they may be trying to make profits off of other activities, but the objective of the Clearing Corporation itself is not to make tons of profit, It's really just to provide a stable environment where people can transact. And in some cases, the members of the exchange own the Clearing Corporation. So it's their own dollars that support the actual physical operations of the organization.

AUDIENCE: And just going back to that point you had about no physical delivery. Two or three weeks ago, the price of oil spiked, [INAUDIBLE] I think the way I read in the papers, was people were trying to sell it, not buy it, because otherwise they would get physical delivery. So do you recall that?

ANDREW LO: I recall the spike. I certainly don't recall the logic about physical delivery. I mean it could be that there are a number of people who are long the oil contracts that basically want it to be cash settled. And the way that they have it cash settled at a particular point in time, before settlement date, is they close out their positions. And so by closing out their positions, they basically reverse the trade, and that would actually push down the oil price. So maybe the reverse argument, a lot of short sellers were trying to argue that oil was going to go down, and they wanted to cover their position, so they bought. In any case, you don't have to take physical delivery if you specify to your broker that you want all of this to be cash settled. Yeah?

AUDIENCE: Two part question. Would you say that the credit risk involved in a forward contract is somewhat similar to the credit risk in credit default swaps? And if so, is there something analogous to a credit default swap that's similar to a futures contract?

ANDREW LO: So the answer to your first question is yes, because a credit default swap contract is basically a kind of a forward contract. It does involve intermediate payments, but if it turns out that the credit changes dramatically, those intermediate payments either may be too much or not enough to cover the underlying value of the contract. And after you strike a credit default swap, it will take on value. As for an exchange, what a wonderful idea. That is exactly what's being proposed now. It hasn't been done yet, but there have been a number of proposals to set up exactly a structure like this for credit default swaps.

In order to do that, you have to standardize those contracts, and you have to be able to do the paperwork in a relatively efficient manner. And so that's actually being discussed, debated, and I think that there's a proposal by the Chicago Mercantile Exchange to start doing that. If you do start doing that, you will see that market growing even bigger than it is today, and at the same time, the risks are actually going to decrease. Because with daily settlement of credit default swaps, just like with futures contracts, all you need is one day margin in order to eliminate 99% of the problems.

AUDIENCE: [INAUDIBLE]

ANDREW LO: So let me-- that's a good question. Let me now talk about how to price futures and we'll take in interest rates explicitly. So the question is what determines either a forward or a futures price? You now know what a futures price is, right? It's the price at which you're willing to do a future transaction. What determines that price? We say supply and demand and the market, but is there some logic that we can give to this process? And the answer is yes, we're going to use the exact same argument that we use for pricing everything else. We're going to come up with two identical cash flows. And two assets that have identical cash flows have to have the same what?

AUDIENCE: Price.

ANDREW LO: Price, value, right. So for now, I'm going to actually ignore the difference between futures and forwards. The only difference is the back and forth amount of money that we give to each other, and therefore, the accumulated interest or foregone interest that we pay when we put

our money back and forth into each other's accounts. So let me abstract from that and-- you know if you're interested, you can actually see the derivation of that in recitation. I want to focus on the bigger question of how these things are priced with respect to other prices.

So let me start with some notation. I've got a particular contract, let's say a futures or a forward contract. And I've also got the spot price of the asset at a point in time. So I'm going to let St denote my spot price. I'm going to let F of little t big T determine the forward price. And H of little t big T, the futures price. And for now, I'm going to just assume that H and F are pretty close.

Now notice that when I write down a futures price or a forward price, I've got two sub-indexes. I've got little t and big T. The reason I need two is that for every forward or futures contract, there are two dates you need to worry about. The date at which you are pricing the contract, namely today, and the settlement date. So you need to have those two indexes. So right away we know that this contract is a little bit more complicated than say a stock, where there is no settlement date.

So I want to go back to a comment that was made by one of you when we first started talking about futures and forwards. And the comment was why go to the trouble of using these contracts, when you could just buy the asset itself and hold onto it. If you need oil in December, in order to make sure you have oil in December, why don't you just buy it in October and hold it for two months. Then you have it in December. And in fact, that's exactly what we're going to do to figure out what the appropriate price is of the specific futures or forward contract. So here we go.

I'm going to do my exact same analysis that I've done many times before, when we tried to price bonds, and stocks, and other basic securities. The left hand column here is going to be the cash flows associated with a typical forward contract. So a forward contract is one way. You enter into the contract, let's say at date zero. And you pay nothing for the contract right, this is a zero NPV transaction. And you are long the forward contract, with the forward price F of 0,T.

The only cash flow that occurs with a forward contract is on settlement date. And on settlement date, you've agreed to pay F of 0,T for delivery of whatever it is that you bought the forward contract on. So the only cash flow that comes out of a forward contract is this F right here. Everybody see that? Nothing up my sleeve, it's very simple calculation.

Now, I want you to look at the right hand column, which is going to be less simple. The right hand column, I want you to imagine doing the following. I want you to imagine buying the commodity at date zero. However, I don't want you to use any money. I want you to buy it with no money down. That's the start of a scam, it sounds like it, but I promise you it's not. So the way you're going to buy the commodity is you have to pay the price, the spot price. And the spot prices is S sub 0 You don't have S sub 0, so borrow it.

Now, I'm going to abstract from credit risk, which I know is on everybody's minds today. But let's suppose that you're all good credits, so I'm not worried about loaning you the money at the risk free rate. So now you've borrowed S sub 0 dollars, and then you spent it right away buying the asset. So as of date zero, in the right hand column, you own the asset.

Now you have to wait T periods, and while you wait you may have some costs. For example, if the asset that you bought is gasoline, well you've got to store it in just the right way. You probably don't want to put it next to your furnace in the basement. You probably want to put it in a cool place, isolated, and so on and so forth. On the other hand, if what you bought is pork bellies, you probably want to put that in a freezer compartment, as opposed to in your garage. So you might have to pay costs for storing.

And at the end of that time T, you have to pay interest on your loan. So you borrowed S sub 0 dollars, you don't get that for free, you got to pay interest on it. This is a question about interest, so you've got to pay interest on that money. And so you have to pay back at this point T-- you have to pay back the money you borrowed-- S sub 0, 1 plus R to the capital T, plus whatever your storage costs are. But I'm going to allow that having the asset around might be kind of convenient.

There might be a benefit to having the asset around-- a convenience yield. Maybe if you need to use it sooner, you have it there. And having it there saves you a little bit of trouble in order to be able to get whatever it is you need to get done with that underlying asset. So I'm going to deduct from my cumulative storage costs any convenience yield-- that's future speak for any kind of benefits that you get from holding onto the physical asset. So your net storage costs are given here-- that's what you pay at the end of T periods.

I argue that these two cash flows give you the exact same value of the asset. In other words, in both cases you happen to have the asset at the time T. So these two contracts have to have the same value because they offer the same set of cash flows, in terms of the underlying

commodity. You get the commodity in both cases. So another way of thinking about it is if your objective is to have 40,000 pounds of cattle in December, both of these will get you to the exact same point.

Both of these costs you nothing on date zero. And therefore, if they cost you nothing, and they give you the same outcome at the end, they've got to sell for the same price. So this has to equal this. That's it. That's the simple argument. And the counter argument or proof that this has to be true is-- let's assume it's not. Let's assume that this is a lot bigger than this. Well, if this is bigger than this, then what should you do? What?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Right. Which one? Which one? Sell the forward contract, and then buy this thing, whatever it is, do this. Now what if it's the reverse? What if this is bigger than this? Then buy the forward, and then do the opposite of this. Flip it around. Short sell the asset if you can, and then take the money and lend it out at interest rate r, and dot dot dot, you follow the logic.

So that gives us a relationship between the forward price and other stuff. And what is the other stuff? The forward price has to be related to the spot price, the interest rate, the time to settlement, and any other weird things about the commodity that may affect the value of it. Like the storage costs or the convenience yield-- you've got to factor that in. So this is the relationship that tells you how to price a forward contract.

Now a futures contract is almost like a forward. The only difference is the interest differential on a daily basis, where you actually are moving money back and forth into our accounts. But the cumulative sum is going to end up being approximately the same. So for the purposes of this class, I'm going to assert that this is approximately the same. In fact, you can show that there's another relationship that looks at the interest rate per period. And it's a little bit more complicated, but not much more complicated. You can see that in your textbook, if you're interested. But for now, I want to just focus on this relationship. This relationship tells us how to price futures and forwards.

And now if I divide by 1 plus r, f to the T, then what I've got is that the forward price divided by the interest rate, that calculates the current value of that forward price, has got to equal the spot price plus the present value of the net storage costs. This is the relationship that we've been looking for, and you guys have been struggling for the last couple of lectures. You've been asking well, gee, doesn't the interest rate belong in there, and what about having the

asset, wouldn't it be nice to have it, and so on and so forth.

All of those considerations are summed up in this one expression. A very nice expression. Very intuitive. What you pay at date T, when you take the present value of it, that has to be equal to what the thing is worth today plus any benefits for having the thing, as opposed to not having the thing between now and settlement. That's it. Now this is for the very beginning when you strike the contract.

What about at an arbitrary point in time between 0 and T? Well, all of these arguments work exactly the same way when you're looking at two dates t and T, as opposed to 0 and T. So the relationship that I showed you, it's a little bit more complicated now, because you've got to take into account the fact that the time to settlement is not capital T, it's capital T minus where you are today. But that's the only change. Other than that, everything is the same. And you have to make sure that you accumulate the future value of all the net storage costs, so that you actually move all of the costs to the end, and then you bring it back to time T.

Now, let's take this out for a spin. Let's see how this works. Let's take a look at gold. Gold is easy to store. There's no storage costs really. I mean gold is relatively compact, a little heavy, so you're going to have to lift it and put it in your vault, as some of you, I'm sure, are doing nowadays. [LAUGHTER] But the bottom line is that the storage costs are negligible. There are no dividends. Gold does not pay out dividends. There are no real benefits either, there is no convenience yield. It's not like you need a little piece of gold every once in a while for your pleasure, and so you want to scrape that off and enjoy it. [LAUGHTER] It just sort of sits there.

So if that's the case, you factor that into that relationship that I showed you, and that last term, the PV of net storage costs is nothing. And so the relationship is really simple. The forward slash futures price today is just equal to what the current spot price is multiplied by 1 plus the risk free rate of interest between today and a settlement date. If this relationship is violated-- when you look at gold futures, and gold spot, and you see that this relationship was violated, that's a sign that there's an arbitrage. You can make money off of that.

So that really is the way to make a million dollars with no money down-- is to try to find violations of this arbitrage relationship. It's going to be hard, because there are a lot of people that are looking at it all the time. And so when there is an inequality of some sort, it's probably not going to be very big, and it probably won't last very long. But to the person who found it first, they might actually be able to make a little bit off of that discrepancy, by either buying or

selling gold, and transacting these markets quickly.

Let me do another example. So this is gold. What about gasoline? Gasoline it turns out, is very different from gold. First of all, it's a pain in the neck to store safely. So if you don't want to be blown up in the middle-- and this is what I really mean by blowing up right, gasoline-- you want to prevent that from happening, you've got to pay a storage cost. On the other hand, there is a convenience yield. If you've got the gasoline, you can actually use it along the way. You have to replenish it, in order to get the same level of stock at the end, but it's convenient that you have it, instead of having to go get it. Because getting it involves trouble and costs. So that's the convenience yield.

So if you factor that in, then what you get is the futures or forward price is equal to 1 plus r,f plus a plus a storage cost per period, minus a convenience yield per period, and then raised to the T minus t power, multiplied by the current spot price. If this is violated, then you're going to want to do one of two things. Either you're going to want to buy your own gasoline and store it, or you're going to want to short it and do the opposite. After Hurricane Katrina hit, we had violations from this for a period of time, which suggested that it was actually worthwhile for you to go out and build your own storage facilities, because the storage facilities were destroyed.

Now it's only if you had the technology to build those storage facilities that you could actually profit from it. But there are periods of time where market dislocation can occur, and the discrepancy between futures prices and spot prices-- that gives you valuable information about what's happening in markets, and in some cases, in non-financial contexts, like commodities. Whether there's a shortage or whether there's a glut. Weather impacts these commodities. And so by looking at this relationship you gather very valuable information.

Here's another example. Another example is financials. I'm going to take this example as the last one that I want to focus on, because I want to now talk about how to use this for your own purposes. A financial future is a futures contract on an index like the S&P 500. So there, all of those contracts are cash settled, there is no physical delivery.

Although, you can easily imagine a situation where you could have physical delivery. Somebody literally delivers 500 shares of stocks, 500 stocks with a certain number of shares for each, in order to get the S&P 500. But that's a pain, and that defeats the purpose of the futures market, which is to try to make things simple and to make it more efficient. So a stock index future is really a pure bet on an underlying index. And it gives you, the investor or the

hedger, a way to get exposure or get out of exposure of that underlying in a very direct way.

Now in this case, there's no real convenience yield, but there is a dividend that gets paid by the particular set of securities. So if you're holding the S&P 500 portfolio, then you're going to be getting paid dividends for individual stocks in that portfolio. And so you'd want to factor in in your futures arbitrage relationship the fact that you're getting a benefit, like a convenience yield, that you have to subtract off of this relationship.

So you don't have a cost of storage, because this is a financial futures, but you do have a convenience yield, in terms of a payment if you held the physical shares of the S&P 500. So that's the difference between futures. Futures, you don't get that dividend, so you got to take that out in order to do the calculation. That tells you what the futures price is relative to the spot.

So now if I give you an exam question that says, today's spot price is such and such, and the risk free interest rate over the next three month period is such and such, you should be able to tell me what the no arbitrage futures price should be today. Or vice versa, if I told you what the futures price is, and I told you what the interest rate is, you should be able to infer from that what the spot price is going to be.

On October 19, 1987, the morning before the New York Stock Exchange opened, there was a very big discrepancy between the spot price and the futures price. That discrepancy caused arbitrageurs to rub their hands and say, oh my god, this is Christmas coming early, I'm going to take advantage of this. And so what they ended up doing-- it turned out because the relationship was violated in one specific way-- they ended up buying the futures and shorting the stocks. That was the beginning of the October, 1987 crash, that within a day dropped the market by about 20%. Nowadays, that's no big deal, we're used to that. [LAUGHTER] But back then, it was really something.

So now that you have examples of how these prices are determined, let me take this out for a different kind of a spin. I want to show you how you use one of these things. And the way I'm going to do that is with the S&P 500. What I skipped were more numerical examples that I would encourage you to go through on your own. But this is an example that's important, so I want to take you through it carefully, make sure everybody understands.

Suppose that you've got $1 million to invest in the stock market, and you've decided that you want to invest it in the S&P 500. You don't want to invest it in any other individual stocks. You

want a broadly diversified investment, and the S&P looks like a pretty good thing. So there are several ways of doing this, I'm going to focus just on two. One of them is you could put your money in 500 different stocks. And you have to spend a little bit of time figuring out what the proportions are, because if you want to replicate the S&P, the S&P is a value weighted index, it's not equal weighted. It's weighted by market capitalization.

So you've got to actually go through and figure out how big each company the S&P is, and then calculate those weights. And then you've got to give this order to your broker, and $1 million dollars isn't what it used to be, so I suspect that that would generate some pretty tiny trades. You got 500 securities, and you've got a bunch of different odd lot trades. Good luck finding a broker that's willing to do it at a reasonable price. It's a pretty long list. Or you can buy a futures contract. In particular, you can buy a contract on the S&P 500 futures.

So I want to go through and show you what that involves. Now let's take your $1 million and let's deposit it at the Futures Brokerage account. So the money is sitting there, earning whatever interest they pay you on that account. Which is not much, it's probably akin to a money market return. So what you do is you want to buy futures contracts and you want to have the equivalent exposure of $1 million invested in the S&P 500.

Now the way that the S&P 500 futures contract works, is that the value of the contract, the notional amount of the contract, is 250 times the index. Whatever the index is worth, they just make up a number, like I don't know 250, and multiply that by the value of the index. And they say that is what your exposure is for one contract. So what is that? Let's suppose the the S&P index is now at a 1,000. So the value of the futures contract is 250 times that and that's going to be $250,000.

In order for you to have the equivalent of $1 million in the S&P, you need four of those contracts. Four times a notional of 250 is equal to $1 million. Now what does this say? This says that you are essentially agreeing that you're going to buy the S&P 500 whenever it settles. But you're not really buying the S&P 500, you're buying a pure bet that is equivalent to 250 times the S&P 500.

So let's take a look at what that means. Suppose that the S&P index fluctuates, bounces around, then it turns out that you'll see that your cash portfolio-- the portfolio fluctuations if you had put $1 million into the S&P directly-- it fluctuates in exactly the same way that your futures portfolio fluctuate. If the S&P goes down to 900, the notional value of your portfolio with four

contracts is $900,000. So you've actually lost $100,000, and that's going to be deducted from your account.

If on the other hand, the S&P goes up by 100, then your cash portfolio will be worth $1,100,000, and your futures portfolio will be worth the same. You will now get $100,000 deposited into your account. By holding this futures contract, it's as if you were actually invested in the S&P. What you're getting is the daily fluctuations. But you actually don't own the security, you simply agreed to buy this so-called index on the maturity date. And by doing so, and because that contract value is so closely tied to the S&P 500 index, it moves in lockstep with the cash portfolio. Any questions about this? Yeah.

AUDIENCE: Does someone own those shares behind you or it's just--

ANDREW LO: No.

AUDIENCE: --an agreement that we're going to wait on this--

ANDREW LO: Exactly. Right. So you and I, we're just going to agree, we're going to bet. We're going to bet and we're going to agree on a particular price for S&P 500 three months from now. And if it goes up, and I bought the contract, then I win. If it goes down, then you sold the contract, then you win. But it's a pure bet between you and me.

AUDIENCE: In the middle is the Futures--

ANDREW LO: The Futures Clearing corporation sits in the middle to make sure that you and I don't run away.

AUDIENCE: Why do they do this?

ANDREW LO: Why do they do this?

AUDIENCE: I mean why do they [INAUDIBLE] days of sunlight.

ANDREW LO: Well, first of all, in some cases they do. So for example, you could buy a contract on the number of degree days of a certain amount in Florida. Now why would you want to do that? It turns out that one of the largest crops of oranges are grown in Florida. And it turns out that the output of oranges groves is very closely tied to temperature. So if it goes up to 39 degrees or below 32 degrees, you can actually have very different kind of crops.

And so you can bet on that, and at some point you can actually trade on it. I don't know if you can trade on it now, but there are markets for some of the wildest things. And the reason that you have these markets is because when two mutually consenting adults have opposite views and they want to express them, then you want to be able to let them do that, and allow them to basically either hedge their risks, or take on risks that they're able to do. So this is an example of that.

You're an investor. You want to buy stocks, but you don't want to buy 500 little stocks one by one. You want to get the exposure right away. Now of course, there's another way to do this, you can put it in a mutual fund. But the problem with the mutual fund is that it only gets priced once a day, whereas this thing gets priced every second of the day when the futures exchange is open. Of course, nowadays, you can buy an ETF, an Exchange Traded Fund. So that's another way of getting exposure. But the S&P futures was around long before ETFs and allowed people to do all sorts of hedging transactions.

Now I'm going to give you a second example that I think will make it a little bit more clear, and actually will answer a question that was asked, I think two lectures ago. When I first started this lecture, I said that maybe a company would only want to hedge 25% of its risk. And somebody asked well, what does that mean 25%? And I said, I'll answer that question. Well, so I'm going to answer that question now.

So suppose now as a different example, you have a diversified portfolio of large cap stocks worth $5 million. So you already own the stocks, and it's currently worth $5 million, but you don't have any confidence that the market is going to stay where it is. You think it's going to go down. And so you want to hedge some of that risk. You don't want to hedge all of it, because you do have faith that over time markets will do well, but you just want to be able to dampen a little bit of the downward spiral if it does occur.

So you might consider selling 25% of your portfolio. Getting rid of 25% of it and putting that in cash. That's one way to do it. But the problem as you know is that it's not that easy to sell 25% of 500 stocks, because you have to again, slice the portfolio, stock by stock. You're going to have a trade list of 500 stocks, which comprise 25% of your portfolio. So it's a pain.

But here's an easier way to do it. You can short sell five S&P contracts. And I'm arguing that that will do the exact same as if you just liquidated 25% of your portfolio. Now let's see if that's right. So let's go through the exact same table. The cash portfolio-- let's see what happens to

the cash portfolio if the S&P goes up or down by 100 points. If it goes up by 100 points, then you've made money. You've got $5.5 million. If it goes down by a 100 points, you've lost money. You've lost to $4.5 million.

Now let's see what happens if you don't do anything with the cash portfolio, but you simply short sell five S&P futures contracts. If you do that then obviously if the S&P doesn't change, then nothing happens to your portfolio. But if the S&P goes up, then you're going to make some money. Sorry. So yeah, if the S&P goes up, you're going to lose money in the sense that what's going to happen is that your short positions are going to cost you $125,000. How did I get $125,000? Anybody work through the math for me?

The S&P 500 goes up by a 100 points. The futures price goes up by 250 times 100 points. My position, I've got five of these contracts, I've just lost $25,000 per contract. I've got five of these contracts, I lost $125,000. Now what about the downside? The downside if the S&P goes down by 100, then the price goes down by $25,000. I'm short, so I make $25,000 per contract. I've got five contracts, I've made $125,000.

So look what happens. In this case, when the S&P goes up, I don't make as much, because my hedge works against me. On the other hand, when the S&P goes down, I don't lose as much, because the hedge is working for me. Because I've only taken out 25% of my portfolio with this hedge, it's dampening, but not eliminating that kind of fluctuation. Yeah?

AUDIENCE: I think that this an obvious question, but why do you do that, versus just putting it in cash. Because you can make the argument that if you had 25%, and had it earning interest, and so you'd still be up too.

ANDREW LO: Well, that's the argument that I gave earlier, which is that you'd have to sell 25% of your portfolio. This is a way of doing it. And not only that, if you did it this way, it would be a lot cheaper to implement in the sense that you don't have to do 500 transactions, you do one transaction. So the transactions cost is a lot cheaper, and it's also easier to keep track of. You don't have to figure out what the price of 500 securities are. You've got the price of just one security to worry about. Yeah.

AUDIENCE: And I think also you're not losing out on what you could've had in cash in terms of interest, because that interest is factored in to the futures.

ANDREW LO: That's right. Remember we used that interest equation so all the foregone interest is in there.

OK, so the meaning of I want to hedge 25% means I'm going to use the futures contract, so that the notional exposure is 25% of the current value of my portfolio. So if you're Merck pharmaceutical company that has a certain percentage of their revenues in foreign denominated currencies, you can hedge half of the risk of those exchange rate fluctuations by taking half of the revenue stream-- let's say it's $10 billion-- and buying or selling, depending on which way you're going, the amount of futures or forwards to get rid of that exposure. Yeah.

AUDIENCE: In this example, we put our million in the margin account, but we only should put as much as [INTERPOSING VOICES].

ANDREW LO: That's right. You don't have to put $1 million in the margin account, because typically the margin is going to be something like in this case 7% or 8% of the notional exposure. So you could take the rest of that money and go to Las Vegas if you like. Although, some would say this is better than Las Vegas. Yeah.

AUDIENCE: This is the main reason why we buy futures and not ETFs. You can leverage your bet as much as you want.

ANDREW LO: That's right with an ETF, if you want $1 million of exposure, you got to put $1 million into the ETF. With the futures contract, if you want to put $1 million of exposure on, you need 7%. And the reason is obvious, it's because of that daily mark to market. So ETFs have not killed the futures market, but it does provide another vehicle for retail investors who may not want the leverage, who may not need to leverage, to not have to worry about the leverage.

This leverage-- leverage is a scary thing, as I said before. This is the chain saw that you don't want to be giving your eight-year-old as a toy. Because when prices move quickly, you're going to have very big swings in the underlying value of your margin account. So if you've got only 7% margin in an account, think about it, that means that if the prices go down by 7%, you are wiped out. Your entire margin account is gone.

When futures brokers take your money, they assume that you know what you're doing. And so they assume that the margin that you're putting down is margin that you can afford to lose, and that you understand that what you're getting is much bigger exposure that presumably is either for speculative purposes, in which case you won't over leverage, or for hedging purposes, in which case you've got some other assets that are counterbalancing these swings. Like in this case. You know obviously, when you look at the fluctuations in your positions, they

are extraordinarily big relative to the margin.

Let's do a quick back of the envelope calculation. Let me tell you what I mean. Suppose that you put 5% margin down. You buy a contract, put 5% margin down, and let's suppose that the price of the futures contract drops by 2.5%. What is the rate of return on the amount of money you've put down as margin, if that's your initial investment? You can think about it as an investment, because that's the only way a futures broker will let you buy a contract. If you put down $100,000 and the futures price goes down by $50,000, what's the rate of return on your investment? Yeah, it's minus 50%, that's a big move. That's a huge move in a day.

So when you deal with margin, you have to be extraordinarily careful. You have to have very, very tight risk controls. You have to understand what the swings can be, and you have to manage that risk very, very carefully, on an intradaily basis in some cases, because these futures prices can swing a lot even within a day. Any other questions? Well, that's it for futures and forwards. You now know how to price them. You now know how to use them for hedging purposes.

And there are all sorts of other kinds of futures and forwards-- interest rate, bond, currency, single stock futures now exist. In fact, there are even futures contracts on the VIX, there's futures contracts on electricity usage, there's futures contracts on the presidential election. If you go to the Iowa Electronic Markets, the University of Iowa, they created a futures exchange that has two contracts. One that pays $1 if McCain gets elected, and the other that pays $1 if Obama gets elected. And by looking at the prices, you can actually see what the folks that are trading these futures contracts are thinking, in terms of who's got the edge.

So the futures prices contain an enormous amount of information. But keep in mind the information is only as good as you are. By you, I mean the market. If the market is comprised of knuckleheads, the prices you get will be knucklehead prices. If the market contains really smart sharp sophisticated individuals, you'll get extremely informative prices.

So prices, while they are the best thing that we have as a guide for the future, they're clearly not perfect. And there are periods of time when the market prices are less perfect than others. And as I told you before, for the next three weeks, finance theory is going to be on vacation in the US stock market, because all the uncertainty that has been building up over the last several years are now focused on the next three weeks. Markets will be swinging back and forth pretty wildly, and it's because people are reacting emotionally, not necessarily with their

full logical capabilities.

That's it for futures and forwards, and now what I'm going to turn to is options. These are the last set of securities that I want to go through with you that are not like the securities that we've done before. And let me just pull up the lecture notes for options. I want to start with a little bit of an introduction for how to motivate options.

I think most of you know what options are. Their name is quite apropos, because they do give you options. Futures and forwards require you to engage in a transaction, but options don't. They give you the right, but not the obligation. So you have the option of not entering into that final transaction at settlement date.

I'm going to start with some motivation, then go through some payoff diagrams, go through options strategies, and then I'm going to talk very briefly about valuation of options. I have to talk to you guys about Black-Scholes. You can't leave MIT without hearing about Black-Scholes. [LAUGHTER] So I've got to do a little bit of that. But really the derivation is quite a bit more sophisticated, and that's why you might want to take 15.437 Options and Futures, where the entire course is devoted to these instruments. They are that complex and that important.

So let me first describe exactly what an option is. An option actually is a specific example of something that you now know of more generally as a derivative. A derivative security gets its name because the value of the security is derived from yet another security. It's derivative, as opposed to I guess fundamental or primary. And examples of derivatives are warrants versus options. Options are securities that you can think of as pure bets between two parties. Warrants are options that are issued by a company on its own shares. So the net supply of options is zero, but the net supply of warrants is not zero, it's issued by companies.

And there are two different kinds of options, calls and puts. A call option is a piece of paper that says the holder of this piece of paper is allowed to buy a security on or possibly before a particular date, usually called the exercise date or maturity date. And the difference between being able to exercise early versus exercising at the maturity only, is the difference between an American and a European option. An American option is one where you can exercise it early. And a European option is one where you can only exercise it on a specific date, the maturity date or the expiration date.

And puts are the opposite of calls. Instead of giving you the right to buy, it gives you the right to sell or to put the stock to somebody else. And the prices at which you get to either buy in

the case of calls, or sell in the case of puts, is called the strike price or the exercise price.

All right. So I'm going to define a little bit of notation. Stock prices is S sub t. Strike price is K. Notice that K does not have a time subscript, because it's fixed at the time the options are issued and it doesn't change throughout the life of that option, it's part of the contract terms. And then the call price is C,t. Put price is P,t.

And the value of these contracts at maturity is actually pretty simple. If today a particular stock is trading at $60 a share, and you purchase an option to buy that stock at $70 a share in three months, does that piece of paper have any value? The current price is $60, this piece of paper gives you the right to buy it at $70 in three months. Is that worthless? Why not? The price is at $60, you can get it at $60 today. So why would you want it at $70 three months from now?

Exactly the price may go up. The reason the piece of paper is not worth zero today is that there is a chance, no matter how small you might think it is, there is still a chance that something wonderful might happen in the next three months, and then the price will go up to $80. And if it goes up to $80, you'll be very happy that you have the right to buy it at $70. How happy will you be? You'll be $10 per share happy. [LAUGHTER] That's what that expression says.

On the expiration date, you get to buy the shares-- if you're holding a call option, you get to buy it for K dollars, but in fact the market has determined that the price at that time is really S,T dollars. So if you're holding this piece of paper, this is your profit-- S,T minus K per share. Now if it turns out that you get to buy it for $60, and it ends up trading at $40, well then you're not going to exercise that right. You're going to let the option expire, and when it expires it'll be worthless if this number is negative. It can be negative of course, but you're not obligated to buy it.

On the other hand, if this were a futures contract, you certainly are obligated to buy it and then you'd get a negative return. But an option is a wonderful thing, in that the payoff is never negative. It's either zero or it's S,T minus K. That's for a call. Now a put option, it's exactly the reverse. If you get to sell the stock, then your profit is what you get to sell it at versus what it's really trading at. And so you actually hope that it's really trading at a very low price. Because if you get to sell it at a high price, but it's trading at a low price, you profit the difference. So the payoff for a put option is exactly the reverse, maximum of zero and K minus S.

Now, it's very important that you understand this asymmetry, because that asymmetry is going

to lead to all sorts of interesting things about these instruments. And before we go and talk about that kind of asymmetry, I want to give you another way of looking at options. Which is to look at options as a kind of insurance contract, because actually all insurance contracts are a form of an option.

So let me give you an example. Suppose that you want to insure the value of a particular stock that you're holding. You're holding General Electric and it's trading at $20 a share, and you'd like to make sure that it never goes below $18 a share. You want to buy insurance that if it goes below $18 a share, you will get paid $18 a share. Well, the way you do that is you buy a put option. A put option on General Electric where the strike price is $18 a share. Because if it goes below $18 a share, you get to sell General Electric for that $18. So you'll get the $18, regardless of whether it goes to $10, or $5, or who knows what.

It turns out that the put option is exactly like insurance, and let's take a look and see why. These are the typical terms of an insurance contract. What's the asset that you're insuring? General Electric. What's the current asset value? $20 a share. What's the term of the policy? How long do you have the policy for? It's the time to maturity. What's the maximum coverage? What are you covered for? $18 a share, that's right. That's what you bought the coverage for, that's what you're going to get if it goes below.

What's the deductible? How much could you lose before the insurance kicks in? $2 a share, exactly. That's the deductible. And finally, what does it cost you to buy this insurance, what's the insurance premium? Exactly, the price of the put. That's it. Beautiful thing. A put option is nothing more than an insurance contract on the value of a stock. And it's going to turn out that a call option will be intimately tied to what a put option is. So every call option can be converted into a portfolio that includes a put.

So all options you can think of as insurance contracts, but there are a few differences. The difference between an option is that you can exercise it early. So for example, for whatever reason, if you decide that you want to buy General Electric at $18 a share, when it's trading at $17.50, and you still have one month to go. But you want to get paid that $18 now, you can do that. You can't do that with your car insurance, right? I guess you could, you could ram it into a post, and I want to get paid now, so let's-- [LAUGHTER] But that's not really considered a proper thing to do.

So early exercise is one difference. Second difference is marketability. If at some point you

don't want the insurance anymore, you can't get rid of it and give it to somebody else. You can't transfer your auto insurance to your friend, if you decide you don't need it anymore. But you can transfer the insurance policy here. You can sell the option, you can sell it.

And also there are dividends that are being paid on the stock that you have to worry about with options, whereas with a typical insurance contract a car doesn't necessarily pay dividends. And the reason that's important is when it pays dividends, the value goes down, and so you have to make adjustments for that with an option. You have to protect an option for dividend payments. You don't need to do that for insurance, because typically you don't assume that the insurance value, the value of the asset goes down that much over time. Yep?

AUDIENCE: When they buy the put option, they also eliminated the chance to enjoy it, from the prices are going to go up, with the futures we'd have a higher value.

ANDREW LO: Well, no that's actually not true. With the put option, it gives you the right to sell the stock. If you buy the stock and you hold onto it, and you also buy a put, that protects the downside. But the upside, that's all yours. Because as the stock goes up, what happens to the value of the put?

AUDIENCE: It's going to zero.

ANDREW LO: Exactly, it stops at zero. So as the stock goes up, the put doesn't have any value anymore. It becomes worthless, worth less and less. And on the date of expiration, if the stock is way above the value of the strike, then it expires worthless. It doesn't go negative. If it went negative, if you had a futures position, then you'd be right, you've actually capped your gains. But this doesn't. See with this you get the best of both, or so it seems. You get the upside, but it protects the downside.

And as you all probably know, insurance is not cheap. So it sounds good, but you've got to pay for this. And so you bet that the price of a call option or put option is not zero when you strike it. Unlike a futures contract that's worthless, an option is not worthless on day zero. It's worth a lot.

For example, right now what's really expensive-- and if you want to check this, you could take a look for fun. If you want to buy insurance on the S&P 500-- now we've had a great rally on Monday, the S&P was up 1,000 points. If you want to buy insurance on the S&P 500 index, you can do that. There are options on the index.

So you might say, OK let's say that the S&P is at 1,000 today, I would like to buy protection that over the next month it doesn't go down by more than 100 points, 10%. So what do you do? You buy a put option on the S&P with the strike price of what? 900, right. OK, for a month. That's what you want to buy.

Go out and calculate that price. You're going to be shocked at how expensive it is, to get that insurance for four weeks. Four weeks, that's all. It's really expensive today. I think it's approximately 10 times more expensive today, than it was a year ago. The implied volatility is up by at least an order of magnitude. So if you want that insurance, it's available, but you have to pay for it.

So the question in all of these things is is it worth it? In order to decide whether it's worth it, you've got to do two things. First look into the inner most workings of your own soul and ask how frightened you truly are. And the second thing you got to do is look at the market. And is the market functioning reasonably well, or is the market reflecting all of these kinds of crazy fears.

In order for us to be able to talk about it intelligently, we need a way to price it. We need the kind of logic that I showed you with futures contracts. And we're going to get that logic. I'm going to show you how to price these things using a very, very simple model that is incredibly powerful, but we're not there yet. Before we do that, I want to make sure you understand what these contracts can do for you in terms of changing your payoff profiles of your portfolio. Yeah?

AUDIENCE: So wouldn't European option be similar to a futures, since you have that you can only exercise on maturity date?

ANDREW LO: Well, no, that's not what makes it similar to a futures. Because while you cannot exercise it early, you never have to exercise it at all. So a European option gives you only one date where you are able to exercise, but even on that date you never have to exercise it. With the futures contract, you have to exercise it on that day. You've made a commitment.

AUDIENCE: But it would have a net present value of zero.

ANDREW LO: No, no, it won't, because still on that date you have a positive amount of protection. Like the example I gave you. Let's suppose that I bought a European S&P option for the day after election day, Wednesday, November 3rd is it. That will have positive value today.

In other words, I'm going to have to pay money in order for you guys to sell it to me, because you're going to be providing me with some protection that if the wrong thing happens on Tuesday, the world is not going to blow up on Wednesday. I'm not telling you what the wrong thing is, I'm neutral in all of this. But that's an example where that insurance really has value. So you're not going to give it to me for free, and I'm willing to pay for it.

All right, since we're out of time, I'm going to just leave you with this diagram that shows you the difference between a call option and a futures contract. Remember the futures contract what that looked like-- that was a straight line. Right Exactly. This is not a straight line, this is kinked-- very kinky security. And so we're going to talk next time about how to price kinky securities, and how to combine them, and engage in even more kinky kinds of payoffs. [LAUGHTER]