# Guided Study Program in System Dynamics <br> System Dynamics in Education Project <br> System Dynamics Group <br> MIT Sloan School of Management ${ }^{1}$ 

## Solutions to Assignment \#11

December 17, 1998

## Reading Assignment:

Please read the following:

- Introduction to Computer Simulation, ${ }^{2}$ by Nancy Roberts et al., Chapter 16

Also read the following:

- World Dynamics, ${ }^{3}$ by Jay W. Forrester, Chapters 1, 2, and 4


## Exercises:

1. Introduction to Computer Simulation, Chapter 16: Representing More Complex Causal Relations

Chapter 16 of Introduction to Computer Simulation builds the same model as chapter 5 of the Study Notes, but with a slightly different approach. Please read over chapter 16 up to the end of Exercise 11. In your assignment solutions document, include answers to parts b., c., and d. of Exercise 11: Comparing Gap and Multiplier Formulations for Urban Growth, the model diagram, documented equations, and graphs of model behavior.

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## Model diagram:



## Model equations:

aging rate $=$ Industrial Structures $/$ LIFETIME OF INDUSTRIAL STRUCTURES Units: structure/year
The number of structures that are demolished every year because of aging.
gap $=$ GOAL FOR INDUSTRIAL STRUCTURES - Industrial Structures
Units: structure
The gap between the desired number of industrial structures and the actual number of structures.

GOAL FOR INDUSTRIAL STRUCTURES $=100$
Units: structure
The desired number of industrial structures.
industrial construction = INDUSTRIAL GROWTH FACTOR * gap
Units: structure/year
The number of industrial structures built every year.
INDUSTRIAL GROWTH FACTOR $=0.1$
Units: 1/year
The growth factor of industrial structures.
Industrial Structures $=$ INTEG (industrial construction - aging rate, 10)
Units: structure
The number of industrial structures.
LIFETIME OF INDUSTRIAL STRUCTURES $=20$
Units: year
The average lifetime of an industrial structure.

Exercise 11, part c.:

Industrial Structures, industrial construction, and aging rate


Industrial Structures : land —— structure industrial construction : land- structure/year aging rate : land $\longrightarrow$ structure/year

The behavior of the goal-gap model is indeed very different from the behavior of the model using the multiplier formulation. The model no longer generates $S$-shaped growth. S-shaped growth is impossible in this model because the feedback structure of the goalgap model is linear and is simply trying to close the gap between the desired number of industrial structures and the actual number of structures. Because the model contains only linear relationships, the loop dominance can never shift, and the negative feedback loop is dominant throughout the simulation.

## Exercise 11, part d.:

The asymptotic behavior of the model in part c . is not very realistic, especially because it shows a very rapid increase in new industrial buildings in the initial stages of the simulation. Very often, the early growth of a system is quite slow. The initial growth is driven primarily by the existing stock; the maximum potential of the system is not very important. Therefore, exponential growth dominates in the early stages of growth.
With time, however, the restrictions on growth become apparent as the system starts to approach its full potential. Growth starts to slow down as the system approaches its limit. This part of growth is driven by negative feedback, goal-seeking behavior.
In certain systems, such as the one in question, this sort of S-shaped behavior makes more sense than expecting growth to be extremely rapid in the very early stages of growth.

## 2. Independent Modeling Exercise

A. Please do parts a. through e. of Exercise 12: Jobs and Migration from Chapter 16 of Introduction to Computer Simulation. In your assignment solutions document, include the model diagram, documented equations, graphs of any lookup functions you might use, and graphs of all simulations that the exercise asks you to run. Also answer all the questions in parts $c ., d .$, and $e$.

Exercise 12, part a.:
Model diagram:


Exercise 12, part b.:

## Model equations:

inmigration $=$ Population $*$ inmigration fraction
Units: person/Month
The number of people migrating into the city each month.
inmigration fraction $=$ INMIGRATION NORMAL $*$ inmigration multiplier Units: 1/Month
The actual inmigration fraction changes as the number of jobs per worker changes.
inmigration multiplier $=$ inmigration multiplier lookup(jobs per worker / NORMAL
JOBS PER WORKER)
Units: dmnl
The inmigration multiplier is a function of the number of available jobs per worker.
inmigration multiplier lookup([(0,0) - (2,4)], (0,0), $(0.1,0.01),(0.2,0.01),(0.3,0.02)$, (0.4,0.03), (0.5,0.04), (0.6,0.07), (0.7,0.1), (0.8,0.2), (0.9,0.5), (1,1), (1.1,1.5), (1.2,2.2), $(1.3,2.8),(1.4,3.2),(1.5,3.5),(1.6,3.7),(1.7,3.8),(1.8,3.9),(1.9,3.95),(2,4))$

Units: dmnl
The lookup function for the inmigration multiplier.
INMIGRATION NORMAL $=0.015$
Units: 1/Month
The normal inmigration fraction.
JOBS $=15000$
Units: job
The number of jobs in the city.
jobs per worker = JOBS / Population
Units: job/person
The number of jobs per person in the city.
NORMAL JOBS PER WORKER = 1
Units: job/person
A person is expected to have, on average, one job.
outmigration $=$ Population $*$ OUTMIGRATION NORMAL
Units: person/Month
The number of people migrating out of the city each month.
OUTMIGRATION NORMAL $=0.015$
Units: 1/Month
The normal outmigration fraction.
Population $=$ INTEG (inmigration - outmigration, 5000)
Units: person
The number of people in the city.

## Graph of the "inmigration multiplier lookup" function:

jobs
inmigration multiplier lookup


## Exercise 12, part c.:

The model is in equilibrium when "Population" equals the number of available "JOBS." The number of "jobs per worker" is then 1 , resulting in an output value of 1 for the "inmigration multiplier." The "inmigration fraction" then equals the "INMIGRATION NORMAL," so the rate of "inmigration" equals the rate of "outmigration," and "Population" is in equilibrium. With the given equations, the equilibrium value for "Population" is 5000 people.

Starting the model in equilibrium, we obtain the following behavior:

## Population, inmigration, and outmigration



Exercise 12, part d.:
Change the following equation:
JOBS $=4500$

Population, inmigration, and outmigration, JOBS $=4500$


Population : jobs=4500
 person inmigration : jobs=4500 $\qquad$ person/Month outmigration : jobs=4500 $\qquad$ person/Month

The behavior is now characteristic of a simple goal-gap negative-feedback loop, where "Population" adjusts to equal the lower number of "JOBS."

Exercise 12, part e.:
A 100-percent step increase in jobs:
Change the following equation:
JOBS $=10000$

Population, inmigration, and outmigration, jobs increase $100 \%$


Population : jobs increase 100\% inmigration : jobs increase $100 \%$ $\qquad$ outmigration : jobs increase $100 \%$ person/Month
$\qquad$ person/Month

A 200-percent step increase in jobs:
Change the following equation:
JOBS $=15000$

Population, inmigration, and outmigration, jobs increase 200\%


Population : jobs increase 200\% inmigration : jobs increase $200 \%$ outmigration : jobs increase $200 \%$ $\qquad$

The criterion for what step increase in number of jobs is necessary to create an S-shaped curve depends on how you constructed the table function. One way to see whether the behavior is in fact $S$-shaped is to look at a graph of the behavior of the net flow into "Population," the difference between "inmigration" and "outmigration":


An increasing net flow means that the stock is growing faster and faster in an exponential manner. As the graph above shows, the "net flow" initially increases in both simulations, indicating that the "Population" initially grows exponentially. The declining net flow later in the simulation indicates slowing growth, resulting in the system approaching equilibrium. Thus, the system generates S -shaped growth.

## B. Please replace part $f$. of the exercise with the following:

Add the effect of jobs on out-migration. Draw reference modes for the system behavior. Then simulate the model. Does the model generate the behavior you expected? Why or why not? Explain any differences in behavior between this model and the original model from part A. In your assignment solutions document, include the model diagram, documented equations, graphs of any lookup functions you might use, and a graph of the new model behavior.

## Model diagram:



## Model equations:

inmigration $=$ Population $*$ inmigration fraction
Units: person/Month
The number of people migrating into the city each month.
inmigration fraction $=$ INMIGRATION NORMAL * inmigration multiplier Units: 1/Month
The actual inmigration fraction changes as the number of jobs per worker changes.
inmigration multiplier = inmigration multiplier lookup(jobs per worker / NORMAL JOBS PER WORKER)
Units: dmnl

The inmigration multiplier is a function of the number of available jobs per worker.
inmigration multiplier lookup $([(0,0)-(2,4)],(0,0),(0.1,0.01),(0.2,0.01),(0.3,0.02)$, (0.4,0.02), (0.5,0.04), (0.6,0.07), (0.7,0.1), (0.8,0.2), (0.9,0.5), (1,1), (1.1,1.5), (1.2,2.2), $(1.3,2.8),(1.4,3.2),(1.5,3.5),(1.6,3.7),(1.7,3.8),(1.8,3.9),(1.9,3.95),(2,4))$

Units: dmnl
The lookup function for the inmigration multiplier.
INMIGRATION NORMAL $=0.015$
Units: 1/Month
The normal inmigration fraction.
JOBS $=5000$
Units: job
The number of jobs in the city.
jobs per worker $=$ JOBS $/$ Population
Units: job/person
The number of jobs per person in the city.
NORMAL JOBS PER WORKER = 1
Units: job/person
A person is expected to have, on average, one job.
outmigration $=$ Population $*$ outmigration fraction
Units: person/Month
The number of people migrating out of the city each month.
outmigration fraction $=$ OUTMIGRATION NORMAL * outmigration multiplier
Units: 1/Month
The actual outmigration fraction changes as the number of jobs per worker changes.
outmigration multiplier = outmigration multiplier lookup(jobs per worker / NORMAL
JOBS PER WORKER)
Units: dmnl
The outmigration multiplier is a function of the number of jobs per worker.
outmigration multiplier lookup $([(0,0)-(2,4)],(0,4),(0.2,3.95),(0.4,3.8),(0.6,3.5)$,
(0.8,2.8), (1,1), (1.2,0.5), (1.4,0.3), (1.6,0.2), (1.8,0.1), (2,0.05))

Units: dmnl
The outmigration multiplier lookup function.
OUTMIGRATION NORMAL $=0.015$
Units: 1/Month

The normal outmigration fraction.
Population $=$ INTEG (inmigration-outmigration, 3000)
Units: person
The number of people in the city.
Graphs of lookup functions:
jobs
inmigration multiplier lookup

jobs2
outmigration multiplier lookup


## Model behavior:

Population, inmigration, and outmigration

$\begin{array}{ll}\text { Population : jobs2 } \\ \text { inmigration : jobs2 } & \begin{array}{c}\text { person } \\ \text { outmigration }: \text { jobs2 }\end{array} \sim \\ \text { person/Month } \\ \text { person/Month }\end{array}$

Because both "inmigration" and "outmigration" now depend on the number of "jobs per worker," the system adjusts to employment conditions through both flows and takes a shorter time to reach equilibrium than the simulation in part A .

## 3. World Dynamics, Chapter 1: Introduction

Please read Chapter 1 and then answer the following questions:
A. Refer to Figure 1-1 on Page 3. What does the growth fraction of population have to be in order to give the results shown in Figure 1-1?

The doubling time is 50 years. Using the well-known formula relating the doubling time and the growth fraction, the growth fraction is 0.7 / 50 years, or 0.014 per year.
B. Refer to the numbered assertions on pages 11 to 13 . Choose one and explain, in feedback terms, why you would intuitively agree or disagree with the assertion.

Item number 8 on page 13 is particularly relevant to world sustainability. Efforts to industrialize developing countries may have seemed wise to some in 1971, but evidence today, more than 25 years since the book was written, indicates that the ultimate results may indeed prove to be the destruction of developing country economies.

The large population of poor people in most developing countries serves as a ready pool of laborers and customers for new (though cheap) products. In these
economies, there is no need to wait for the population to grow to create additional sales. With the kick start provided by foreign capital the vast pool of cheap labor can quickly be converted to a huge, basics level, customer base. This provides the potential for a rapid positive feedback, where even limited increases in salaries creates a collective wealth that can support further industrial development.

This rapid growth has additional consequences. The rapid growth is largely uncontrolled by legal constraints because legal and institutional development seemingly require a longer period to evolve than does the new industrial system. Consequently, pollution standards, for example, if they exist, go unenforced, and corrupt arrangements allow poorly planned industrial complexes to be built.

Also, since developing world economies started their transition toward industrialization at a time when pressures on their resources (from overpopulation and external exploitation) was already high, the pollution levels (for example) rapidly rise above acceptable standards, and resource stocks drop rapidly below critical levels (e.g. forest lands in Indonesia), which they were already approaching anyway.

Rapid growth is made possible by a ready pool of labor that is converted into consumers. The rapidity of the growth creates a situation where normal constraints are limited. In addition, the rapid growth coupled with a lack of constraints causes pollution and other danger indicators to quickly exceed acceptable values that were already being approached.

## 4. World Dynamics, Chapter 2: Structure of the World System

Please read Chapter 2 and then answer the following questions:
A. Page 19 lists the five levels that were chosen as the most important levels for the structure of the world system. If you were to present a list of SIX levels, which additional level would you choose? If you would like, you can also replace some of the original five levels, but your final list should have six levels. Please give reasons for your choices.

The list of five is very good, but the obvious sixth stock would be Technology. From 1970 to 1997, mankind has "progressed" at a higher rate than at any other time in history. The invention of the home computer. The creation of new ways to fight diseases. The founding of MTV. All of these events, and many others, are important, and all have had great effects on our lives and the environment around us. Computing technologies have made workplaces more efficient, the medical field is saving more lives then ever, and MTV, well that is just plain cool (I guess).

Along with the five levels, I would add the level "education." This level would represent the general level of culture, science, and/or ability of people. I think education is a very important stock for human societies and greatly influences several sub-systems.
B. Chapter 2 lists four factors that limit the growth of population: crowding, food supply, pollution, and natural resources. Describe some factors that you think represent "crowding" and explain how they might act as a limit to population growth.

Factors that describe crowding I believe would include 'Number of people per square mile' and 'Amount of food available per person'. Notice that each of these factors is based on a ratio of some amount versus the population. As these factors move towards critical levels, they would start to affect the ecological balance. As the factor 'number of people per square mile' increases it would cause either migration away from the area or consumption of valuable farmland, which would decrease the factor 'amount of food per person'. Out migration would cause increases in the 'Number of people per square mile' at their new location and affect the ecological balance in the new area. As one can quickly grasp, the system is very interconnected and solutions may not only not solve the problem but may actually enhance the problem.

## 5. World Dynamics, Chapter 4: Limits to Growth

Please read Chapter 4 and answer the following questions:
A. Now that you have studied in-depth the way crowding, food supply, pollution, and natural resources hinder population growth, briefly discuss which one of these limitations has had the greatest effect. Relate your discussion to the text as well as personal knowledge and experience.

Economist Herman Daly has noted on several occasions that trends in recent decades suggest we are likely to poison the planet before we run out of nonrenewable resources. I am inclined to agree with his assessment. Any number of symptoms of this phenomenon point to the growing impact of human pollution: accelerated loss of biodiversity (though natural resource extraction and population growth certainly are important contributors here), global warming trends, deteriorating air quality in urban areas, the expanding range of carcinogens and their disbursement into our surroundings (dioxin comes to mind as but one example here). I'm not sure these consequences have made a significant difference in population growth in the recent past, but the role of pollution as a constraint in the foreseeable future seems very important to me.
B. Section 4.2 (page 69) discusses Natural Resources Depletion as a limit to growth. In particular, natural resources fall fastest when usage rate is highest. Is it likely and realistic for natural resources to be completely depleted? If not, what is likely to be the equilibrium value of natural resources, if any? (A qualitative explanation will suffice.)

As modeled, natural resources would have to approach zero as an equilibrium value because there is no inflow, only an outflow, to the stock. Even if natural resources are used at a very low constant rate, the stock will continue to decline toward zero.

Potential natural resources are vast if one is willing to include the whole mass of the earth. Energy, and technology, needed for extraction might be seriously limiting however, especially under stressed economic situations. Additionally, some materials can be recycled, and others are renewable (e.g. energy from wood).

Thus the model might be better formulated with a low level of recycled resources (a fixed fraction of what is used) and with an extremely large but decreasingly available store of "resources." With this approach the resource use would always be declining, but would not reach zero except at some time in the indefinite future.
C. The simulation output on Page 75 in Chapter 4 shows behavior of the stocks from 1900 to 2100 with a pollution crisis. How do you think each of the stocks will behave from 2100 to 2200? Justify your hypotheses.

Absent wholesale rethinking in human institutions and economic practices, I think the next century of the model (2100-2200) would witness:
(1) continued decline in natural resources (following the same downward trend?),
(2) overshoot and collapse in human population (though achieving a lower peak than in the prior century),
(3) collapse of quality of life followed by oscillation,
(4) mild oscillation in capital investment (operating beneath the natural resource constraint), and
(5) dampened oscillation in the pollution level.

The oscillatory behavior of several levels seems clear to me, and I would expect this to continue. I also sense the declining level of natural resources serves as a constraint on the degree to which the other levels fluctuate.


[^0]:    ${ }^{1}$ Copyright © 1998 by the Massachusetts Institute of Technology. Permission granted to distribute for non-commercial educational purposes.
    ${ }^{2}$ Roberts, Nancy, David Andersen, Ralph Deal, Michael Garet, and William Shaffer, 1983. Introduction to Computer Simulation: A System Dynamics Approach. Waltham, MA: Pegasus Communications. 562 pp.
    ${ }^{3}$ Forrester, Jay W., 1971. World Dynamics, (1973 second ed.). Waltham, MA: Pegasus Communications. 144 pp.

