Flexibility in Real Estate Project Development

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Cover Story

- Case investigates two general questions in real estate development strategy:
 - Is it worth investing upfront to acquire particular infrastructures (e.g. a park, utilities, etc.)?
 - Is flexibility in expansion timing worthwhile?
- Particular instance studied:
 - Development of apartment units around a park



Apartment Development Project

- Development of apartment units around a park
 - Five phases, 24 months each, start in 2007
 - All units 1000 SF each
 - Market value represents NPV (at 9% discount rate) of all revenues once phase is completed

| Phase | Type | SF | Units | Net Acreage | Start | Completion |
|-------|------|---------|-------|-------------|-------|------------|
| I | APT | 50,000 | 50 | 1.15 | 1/07 | 1/09 |
| II | APT | 80,000 | 80 | 1.84 | 1/08 | 1/10 |
| Ш | APT | 90,000 | 90 | 2.07 | 1/09 | 1/11 |
| IV | APT | 110,000 | 110 | 2.53 | 1/10 | 1/12 |
| V | APT | 100,000 | 100 | 2.30 | 1/11 | 1/13 |
| Total | | 430,000 | 430 | 9.87 | | |

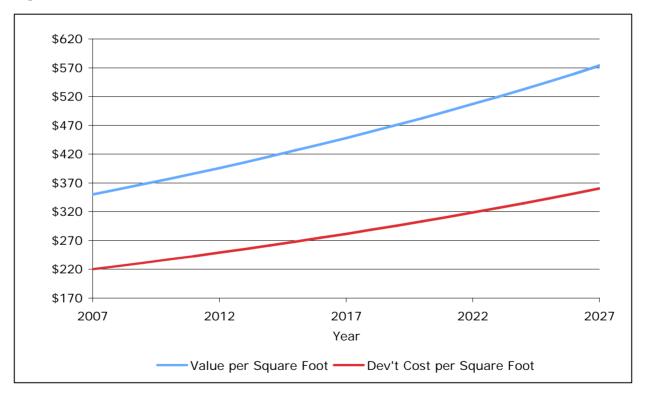


Key Assumptions

- Deterministic forecast for market value of built property currently evaluated at \$350/SF -- increases at 2.5% per year
- Deterministic forecast for development costs currently evaluated at \$220/SF -- increase with inflation at 2.5% per year
- Development of all phases in a row benefits from cost reductions of 2.5% due to economies of scale
- <u>Discount rate</u> for market value of built apartment property (r_V) is 9%, and for construction costs (r_C) is 6%, close to currently prevailing risk-free rate



 Market value of built property and development cost models





Assumptions:

- Park area is about 50,000 SF (1 acre) and costs \$1M to develop along with the five development phases.
- Cost is distributed to each phase as \$200,000 each.
- Land acquisition costs \$15M, paid when phase I begins
- Infrastructure development, which includes site grading, paving, utilities, and landscaping is estimated at \$29/SF



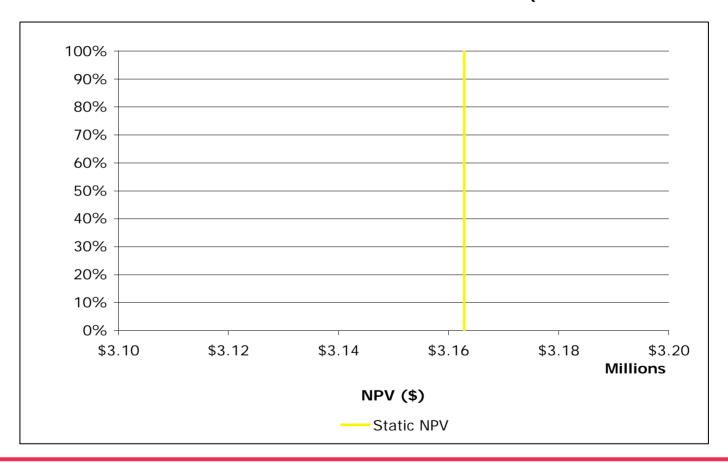
Pro Forma Discounted Cash Flow (DCF) model

| Year | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 |
|----------------------------------|-----------------------|---------------|--------------|--------------|--------------|--------------|--------------|
| Period | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Built property value per SF (\$) | \$350 | \$359 | \$368 | \$377 | \$386 | \$396 | \$406 |
| Dev't cost per SF (\$) | \$220 | \$226 | \$231 | \$237 | \$243 | \$249 | \$255 |
| Phase I value | \$0 | \$0 | \$18,385,938 | \$0 | \$0 | \$0 | \$0 |
| Phase I dev't cost | \$10,725,000 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase II value | \$0 | \$0 | \$0 | \$30,152,938 | \$0 | \$0 | \$0 |
| Phase II dev't cost | \$0 | \$17,589,000 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase III value | \$0 | \$0 | \$0 | \$0 | \$34,770,106 | \$0 | \$0 |
| Phase III dev't cost | \$0 | \$0 | \$20,282,316 | \$0 | \$0 | \$0 | \$0 |
| Phase IV value | \$0 | \$0 | \$0 | \$0 | \$0 | \$43,559,216 | \$0 |
| Phase IV dev't cost | \$0 | \$0 | \$0 | \$25,409,234 | \$0 | \$0 | \$0 |
| Phase V value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$40,589,270 |
| Phase V dev't cost | \$0 | \$0 | \$0 | \$0 | \$23,676,787 | \$0 | \$0 |
| Acquisition cost | \$15,000,000 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Infrastructure cost | \$1,426,911 | \$2,340,134 | \$2,698,467 | \$3,380,580 | \$3,150,086 | \$0 | \$0 |
| Park development cost | \$195,000 | \$199,875 | \$204,872 | \$209,994 | \$215,244 | \$0 | \$0 |
| Value of built property | \$0 | \$0 | \$18,385,938 | \$30,152,938 | \$34,770,106 | \$43,559,216 | \$40,589,270 |
| Total cost | \$27,346,911 | \$20,129,009 | \$23,185,655 | \$28,999,808 | \$27,042,116 | \$0 | \$0 |
| Net value | -\$27,346,911 | -\$20,129,009 | -\$4,799,717 | \$1,153,130 | \$7,727,990 | \$43,559,216 | \$40,589,270 |
| PV of built property | \$115,903,253 | | | | | | |
| PV total cost | \$11 <u>2,740,370</u> | - | | | | | |
| NPV | \$3,162,873 | ← | | | | | |
| Return ever initial cost (incl. | 4 20/ | _ | | | | | |
| discount rate) | 12% | | | | | | |



No Pre-Investment in Park

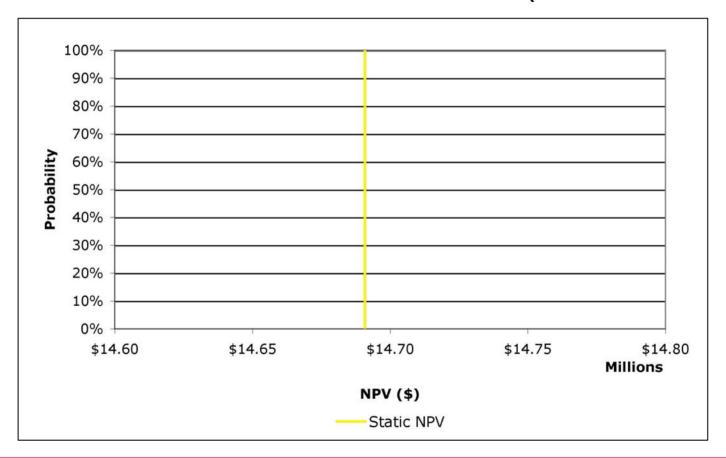
Value At Risk and Gain curve (VARG curve)





With Pre-Investment in Park

Value At Risk and Gain curve (VARG curve)



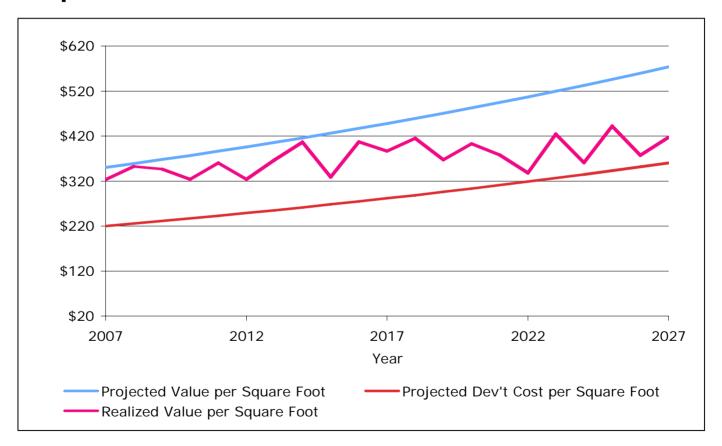


- Recognize reality of uncertainty
 - Only around market value of built property
 - Initial value within \pm 50% of projection
 - Annual growth factor also \pm 50% of projection
 - 15% volatility around each annual growth value
 - Otherwise same DCF model

- Use Monte Carlo: 2000 simulations each
 - Easy, routine no special program

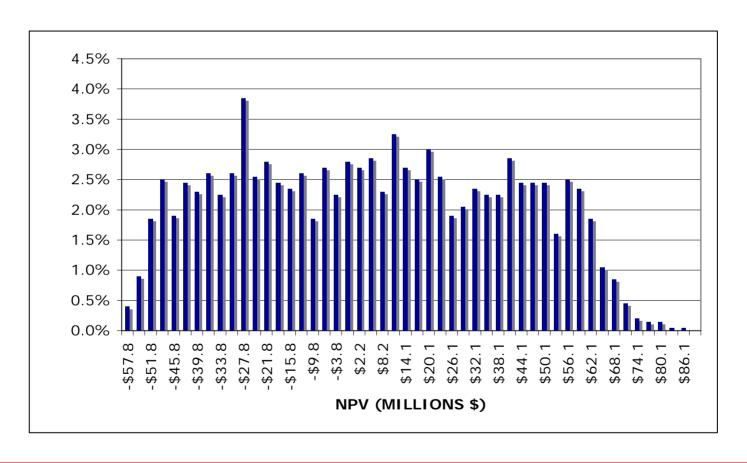


Example of Uncertain Market Value Pattern





NPV distribution for 2000 simulations



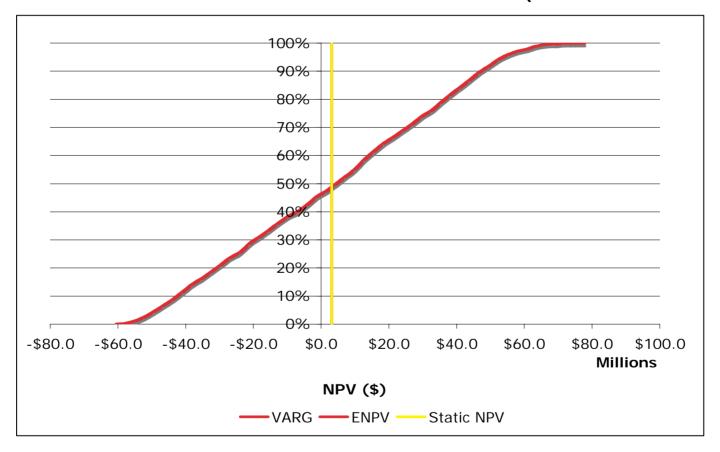


Observations

- Expected NPV is \$3.17M, compared to \$3.16M for static case
 - Relatively similar due to unconstrained nature of problem
 - Situation different from parking garage
- Now deal with a <u>distribution</u> of outcomes rather than single value
 - Expected NPV (ENPV) rather than NPV



Value At Risk and Gain curve (VARG curve)

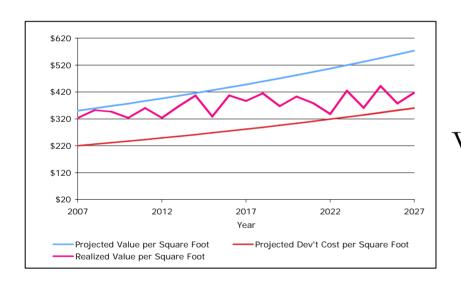




- Sources of flexibility
 - Develop park upfront all at once for \$1M
 - Increases market value of built property by 10% ONLY when market value was increasing in previous year
 - Expand at strategic times
 - When market value of built property is 50% higher than development cost
 - Abandon
 - At year 20, when phase is undeveloped



 If recognize uncertainty, what are possible outcomes from managerial perspective?





Value > Cost all time

Value < Cost most times



Positive NPV Scenario

| Year | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 |
|------------------------------------|--------------------------------|-------------|--------------|---------------|--------------|---------------------|---------------------|--------------|
| Develop? Abandon? Wait? | Develop | | | | | | | |
| Dev't value criteria | \$7,200,000 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase I value | \$0 | \$0 | \$23,276,250 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase I abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase I dev't cost | \$11,000,000 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Develop? Abandon? Wait? | | Wait | Develop | | | | | |
| Dev't value criteria | | \$8,602,270 | \$18,751,000 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase II value | \$0 | \$0 | \$0 | \$0 | \$38,233,990 | \$0 | \$0 | \$0 |
| Phase II abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase II dev't cost | \$0 | \$0 | \$18,491,000 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Develop? Abandon? Wait? | | | Wait | Develop | | | | |
| Dev't value criteria | | | \$21,094,875 | \$17,481,349 | \$0 | \$0 | \$0 | \$0 |
| Phase III value | \$0 | \$0 | \$0 | \$0 | \$0 | \$44,531,643 | \$0 | \$0 |
| Phase III abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase III dev't cost | \$0 | \$0 | \$0 | \$21,322,434 | \$0 | \$0 | \$0 | \$0 |
| Develop? Abandon? Wait? | | | | Wait | Develop | | | |
| Dev't value criteria | | | | \$21,366,093 | \$25,859,464 | \$0 | \$0 | \$0 |
| Phase IV value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$59,004,637 | \$0 |
| Phase IV abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase IV dev't cost | \$0 | \$0 | \$0 | \$0 | \$26,712,272 | \$0 | \$0 | \$0 |
| Develop? Abandon? Wait? | | | | | Wait | Develop | | |
| Dev't value criteria | | | | | \$23,508,604 | \$24,588,622 | \$0 | \$0 |
| Phase V value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$52,649,235 |
| Phase V abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase V dev't cost | \$0 | \$0 | \$0 | \$0 | \$0 | \$24,890,981 | \$0 | \$0 |
| Phase(s) completed | | | Phase I | Phase I | Phase II | Phase III | Phase IV | Phase V |
| Project abandoned? | 0.15 000 000 | Φ0 | • | | Φ0 | | | 40 |
| Acquisition cost | \$15,000,000 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 \$0 | \$0 |
| Infrastructure cost | \$1,463,499 | \$0 | \$2,460,141 | \$2,836,850 | \$3,553,943 | \$3,311,629 | \$0 \$0 | \$0 |
| Park development cost | \$1,000,000 | \$0 \$0 | \$0 | \$0 \$0 | \$0 | \$0 \$44.534.643 | \$0 \$50,004,637 | \$0 |
| Value of built property Total cost | \$0 | \$0 \$0 | \$23,276,250 | \$0 | \$38,233,990 | \$44,531,643 | \$59,004,637 \$0 | \$52,649,235 |
| Net value | \$28,463,499 | \$0 \$0 | \$20,951,141 | \$24,159,285 | \$30,266,215 | \$28,202,609 | * - | \$0 |
| 1 | -\$28,463,499 \$130,603,060 | \$0 | \$2,325,109 | -\$24,159,285 | \$7,967,775 | \$16,329,033 | \$59,004,637 | \$52,649,235 |
| PV of built property | \$139,603,060 | | | | | | | |
| PV total cost NPV | \$112,442,848 \$27,160,211 | | | | | | | |
| | | | | | | | | |
| rate) | 95% | | | | | | | |
| rate) | | | | | | | | |



Positive NPV Scenario

| Year | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 |
|--------------------------------|-------------------------|-------------|--------------|---------------|--------------|--------------|--------------|--------------------------|
| Develop? Abandon? Wait? | Develop | | | | | | | |
| Dev't value criteria | \$7,200,000 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase I value | \$0 | \$0 | \$23,276,250 | \$0 | \$0 | \$0 | \$0 | \$0 \$0 |
| Phase I abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase I dev't cost | \$11,000,000 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Develop? Abandon? Wait? | | Wait | Develop | | | | | |
| Dev't value criteria | | \$8,602,270 | \$18,751,000 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase II value | \$0 | \$0 | \$0 | \$0 | \$38,233,990 | \$0 | \$0 | \$0 \$0 \$0 \$0 |
| Phase II abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase II dev't cost | \$0 | \$0 | \$18,491,000 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Develop? Abandon? Wait? | | | Wait | Develop | | | | |
| Dev't value criteria | | | \$21,094,875 | \$17,481,349 | \$0 | \$0 | \$0 | \$0 |
| Phase III value | \$0 | \$ | \$0 | \$0 | \$0 | \$44,531,643 | \$0 | \$0 \$0 \$0 |
| Phase III abandonment value | \$0 \$0 | | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase III dev't cost | \$0 | | \$0 | \$21,322,434 | \$0 | \$0 | \$0 | \$0 |
| Develop? Abandon? Wait? | | | | Wait | Develop | | | |
| Dev't value criteria | | | | \$21,366,093 | \$25,859,464 | \$0 | \$0 | \$0 |
| Phase IV value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$59,004,637 | \$0 |
| Phase IV abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| \$920 _T | | \$0 | \$0 | \$0 | \$26,712,272 | \$0 | \$0 | \$0 \$0 \$0 |
| | | | | | Wait | Develop | | |
| \$820 | | | | | \$23,508,604 | \$24,588,622 | \$0 | \$0 |
| \$720 | | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$52,649,235 |
| ~ / | \sim | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| \$620 | | \$0 | \$0 | \$0 | \$0 | \$24,890,981 | \$0 | \$0 |
| \$520 | | | Phase I | Phase I | Phase II | Phase III | Phase IV | Phase V |
| \$420 | | _ | | | | _ | _ | . |
| | | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| \$320 | | \$0 | \$2,460,141 | \$2,836,850 | \$3,553,943 | \$3,311,629 | \$0 | \$0 |
| \$220 | | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| | | \$0 | \$23,276,250 | \$0 | \$38,233,990 | \$44,531,643 | \$59,004,637 | \$52,649,235 |
| \$120 | | \$0 | \$20,951,141 | \$24,159,285 | \$30,266,215 | \$28,202,609 | \$0 | \$0 |
| \$20 | | \$0 | \$2,325,109 | -\$24,159,285 | \$7,967,775 | \$16,329,033 | \$59,004,637 | \$52,649,235 |
| 2007 2012 2017 | 2022 2027 | | | | | | | |
| Year | | | | | | | | |
| | 't Cost per Square Foot | | | | | | | |
| Realized Value per Square Foot | | | | | | | | |
| | | | | | | | | |



Negative NPV Scenario

| Year | 2007 | 2021 | 2022 | 2023 | 2024 | 2025 | 2026 | 2027 |
|-----------------------------|------------------------------|---------------|--------------|--------------|-------------|-------------|--------------|-------------|
| Develop? Abandon? Wait? | Wait | Develop | | | | | | |
| Dev't value criteria | \$2,500,000 | \$7,817,520 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase I value | \$0 | \$0 | \$0 | \$23,695,280 | \$0 | \$0 | \$0 | \$0 |
| Phase I abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase I dev't cost | \$0 | \$15,542,712 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Develop? Abandon? Wait? | | Wait | Wait | Wait | Wait | Wait | Wait | Abandon |
| Dev't value criteria | | \$12,508,032 | \$11,698,569 | \$11,785,149 | \$6,601,009 | \$5,958,301 | \$12,838,556 | \$5,854,494 |
| Phase II value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase II abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$2,498,636 |
| Phase II dev't cost | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Develop? Abandon? Wait? | | Wait | Wait | Wait | Wait | Wait | Wait | Abandon |
| Dev't value criteria | | \$14,071,536 | \$13,160,890 | \$13,258,292 | \$7,426,135 | \$6,703,088 | \$14,443,376 | \$6,586,305 |
| Phase III value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase III abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase III dev't cost | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Develop? Abandon? Wait? | | Wait | Wait | Wait | Wait | Wait | Wait | Abandon |
| Dev't value criteria | | \$17,198,544 | \$16,085,532 | \$16,204,579 | \$9,076,387 | \$8,192,663 | \$17,653,015 | \$8,049,929 |
| Phase IV value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase IV abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase IV dev't cost | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Develop? Abandon? Wait? | | Wait | Wait | Wait | Wait | Wait | Wait | Abandon |
| Dev't value criteria | | \$15,635,040 | \$14,623,211 | \$14,731,436 | \$8,251,261 | \$7,447,876 | \$16,048,195 | \$7,318,117 |
| Phase V value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase V abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase V dev't cost | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Phase(s) completed | | | | Phase I | Phase I | Phase I | Phase I | Phase I |
| Project abandoned? | | | | | | _ | | Abandoned |
| Acquisition cost | \$0 | \$21,194,607 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Infrastructure cost | \$0 | \$2,067,885 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Park development cost | \$0 | \$1,412,974 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Value of built property | \$0 | \$0 | \$0 | \$23,695,280 | \$0 | \$0 | \$0 | \$2,498,636 |
| Total cost | \$0 | \$40,218,178 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| Net value | \$0 | -\$40,218,178 | \$0 | \$23,695,280 | \$0 | \$0 | \$0 | \$2,498,636 |
| PV of built property | \$6,413,958 | | | | | | | |
| PV total cost | \$17,700,500 \$44,074,504 | ← | | | | | | |
| NPV | -\$11,374,581 | | | | | | | |

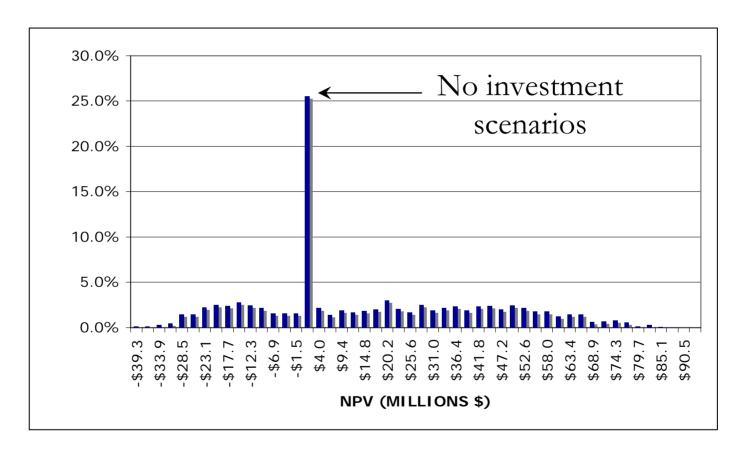


Negative NPV Scenario

| | Year | 2007 | 2021 | 2022 | 2023 | 2024 | 2025 | 2026 | 2027 |
|----------------|-------------------------------|-------------------|---------------|--------------|----------------|-------------|-------------|--------------|-------------|
| | Develop? Abandon? Wait? | Wait | Develop | | | | | | |
| | Dev't value criteria | \$2,500,000 | \$7,817,520 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| | Phase I value | \$0 | \$0 | \$0 | \$23,695,280 | \$0 | \$0 | \$0 | \$0 |
| | Phase I abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| | Phase I dev't cost | \$0 | \$15,542,712 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| | Develop? Abandon? Wait? | | Wait | Wait | Wait | Wait | Wait | Wait | Abandon |
| | Dev't value criteria | | \$12,508,032 | \$11,698,569 | \$11,785,149 | \$6,601,009 | \$5,958,301 | \$12,838,556 | \$5,854,494 |
| | Phase II value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0_ | \$0 |
| | Phase II abandonment value | \$0 | \$0 | \$0_ | \$0 | \$0 | \$0 | \$0 | \$2,498,636 |
| | Phase II dev't cost | \$0 | \$0 | \$ | \$0 | \$0 | \$0 | \$0 | \$0 |
| | Develop? Abandon? Wait? | | Wait | Wait | Wait | Wait | Wait | Wait | Abandon |
| | Dev't value criteria | | \$14,071,536 | \$13,1 | \$13,258,292 | \$7,426,135 | \$6,703,088 | \$14,443,376 | \$6,586,305 |
| | Phase III value | \$0 | \$0 | | \$0 | \$0 | \$0 | \$0 | \$0 |
| | Phase III abandonment value | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| | Phase III dev't cost | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| | Dovolon? Abandon? Mait? | | \Mait | Wait | Wait | Wait | Wait | Wait | Abandon |
| 0 | | | | \$16,085,532 | \$16,204,579 | \$9,076,387 | \$8,192,663 | \$17,653,015 | \$8,049,929 |
| | | | | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| o | | | | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| | | | | \$0 | \$0 | \$0 | \$0 | \$0 | \$0 |
| | | | | Wait | Wait | Wait | Wait | Wait | Abandon |
| 1 | | | | \$14,623,211 | \$14,731,436 | \$8,251,261 | \$7,447,876 | \$16,048,195 | \$7,318,117 |
| • | \sim | _ | | \$0 | \$0 \$0 | \$0 \$0 | \$0 \$0 | \$0 | \$0 |
| 0 | | | | \$0 \$0 | \$0 \$0 | \$0 \$0 | \$0 \$0 | \$0 \$0 | \$0 \$0 |
| | | | | \$0 | * * * | \$0 | * * * | \$0 | T - |
| o | | | | | Phase I | Phase I | Phase I | Phase I | Phase I |
| | | | | 60 | ሶ ለ | ¢ 0 | | | Abandoned |
| , ——— | | | | \$0 | \$0 \$0 | \$0 \$0 | \$0 \$0 | \$0 \$0 | \$0 |
| ~ | | | | \$0 | \$0 \$0 | \$0 \$0 | \$0 \$0 | \$0 \$0 | \$0 |
| | | | | \$0 | | \$0 \$0 | \$0 \$0 | \$0 \$0 | \$0 |
| 20 + | 0010 | | | \$0 | \$23,695,280 | \$0 \$0 | \$0 \$0 | \$0 \$0 | \$2,498,636 |
| 2007 | 2012 2017 | 2022 | 2027 | \$0 | \$0 | \$0 \$0 | \$0 \$0 | \$0 \$0 | \$0 |
| | Year | | | \$0 | \$23,695,280 | Φ0 | \$0 | \$0 | \$2,498,636 |
| -Projected Va | alue per Square Foot ——Projec | ted Dev't Cost pe | r Square Foot | | | | | | |
| Realized Val | ue per Square Foot | | | | | | | | |
| | | | | | | | | | |

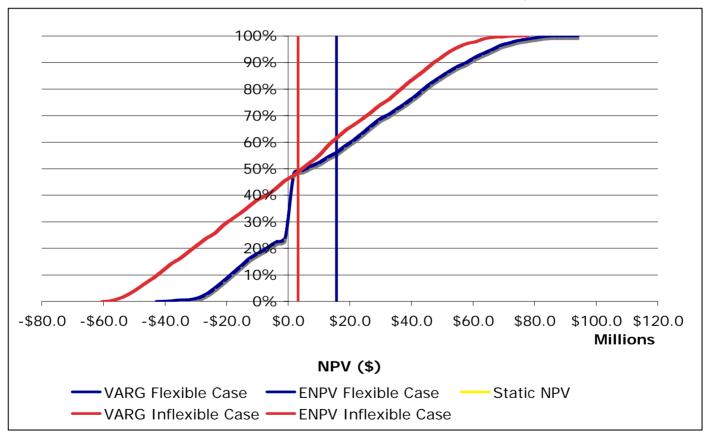


NPV distribution for 2000 simulations



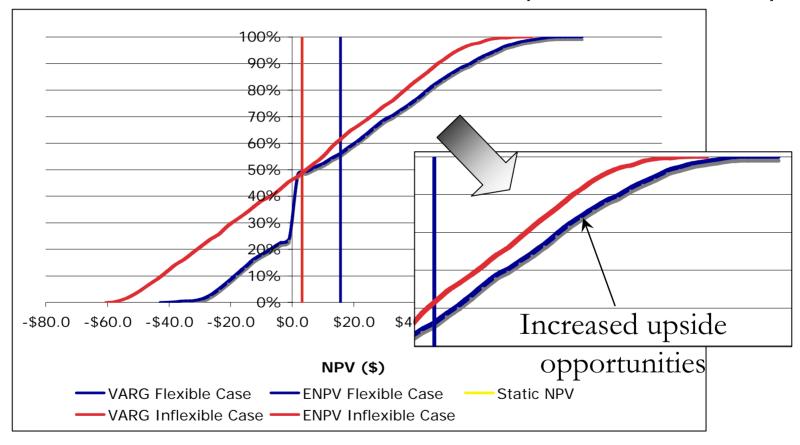


Value At Risk and Gain curve (VARG curve)





Value At Risk and Gain curve (VARG curve)





- About distribution
 - Downside reduced, upside increased
- Increased Expected NPV = ENPV
- Consequences of recognizing uncertainty
 - sometimes wait
 - sometimes do not invest
 - sometimes abandon after starting the project
- Sudden jump near zero NPV
 - Scenarios where do not invest compared to scenarios where profit is barely made over initial costs



- Static, "certain" case: <u>no pre-investment</u> in park
 NPV = \$3.16M
- Inflexible case: no pre-investment in park
 ENPV = \$3.17M
- Flexible case: pre-investment and timing flexibilities
 ENPV = \$15.67M
- Value of flexibility due to combination of strategic timing and pre-investment in park:

$$E[V_{Flexibility}] = MAX(O, E[NPV_{Flex.}] - E[NPV_{Inflex.}])$$

= \$12.5M



Other valuation attributes, in Millions \$:

| | Inflexible | Flexible | Better? |
|-----------------------------|------------|----------|--------------|
| Expected initial investment | \$27.35 | \$21.35 | Flex. Better |
| Expected NPV | \$3.17 | \$15.67 | Flex. Better |
| Minimum NPV | -\$60.56 | -\$42.94 | Flex. Better |
| Maximum NPV | \$77.91 | \$94.19 | Flex. Better |
| DR) | 12% | 73% | Flex. Better |



- Static, "certain" case: with pre-investment in park
 NPV = \$14.69M
- Inflexible case: with pre-investment in park
 ENPV = \$10.21M
- Flexible case: pre-investment and timing flexibilities
 ENPV = \$15.67M
- Value of flexibility due to strategic timing: $E[V_{Flexibility}] = \$(15.67 - 10.21)M = \$5.46M$
 - Deduced value of pre-investment in park:

$$12.5M - 5.5M = 7.0M$$



Other valuation attributes, in Millions \$:

| | Inflexible | Flexible | Better? |
|-------------------------------------|------------|----------|----------------|
| Expected initial investment | \$28.13 | \$21.35 | Flex. Better |
| Expected NPV | \$10.21 | \$15.67 | Flex. Better |
| Minimum NPV | -\$57.79 | -\$42.94 | Flex. Better |
| Maximum NPV | \$95.46 | \$94.19 | Inflex. Better |
| Return over initial cost (incl. DR) | 36% | 73% | Flex. Better |



Take-Aways

- Easy method to deal with uncertainty
 - No special software required
- Important to recognize uncertainty
 - It's reality!
- Recognizing uncertainty leads to flexible design and management
- Improve expected NPV and other desirable attributes



Questions/Comments?

