Review the rules for complex numbers:

$$\begin{aligned}
\text{Definition:} \quad & i = \sqrt{-1} && (1)\\
& i^2 = -1 && (2)\\
\text{Addition:} \quad & (a+bi)+(c+di) = (a+c)+(b+d)i && (3)\\
\text{Multiplication:} \quad & (a+bi)(c+di) = (ac-bd)+(bc+ad)i && (4)\\
\text{Division:} \quad & \tfrac{a+bi}{c+di} = \frac{a+bi}{c+di}\frac{c-di}{c-di} && (5)\\
& = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2} && (6)\\
\text{Magnitude } (z=a+bi): \quad & |z|^2 = a^2+b^2 && (7)\\
\text{Phase:} \quad & \angle z = \tan^{-1}\frac{b}{a} && (8)
\end{aligned}$$

Manipulating numbers numerically

Take these two numbers (they are called a conjugate pair):

$$z_1 = 1 + i\sqrt{3}$$
$$z_2 = 1 - i\sqrt{3}$$

- Firstly, what are the magnitudes and phase angles?

  $|z_1| =$ $|z_2| =$

  $\angle z_1 =$ $\angle z_2 =$

- How about their sum?

$$z_1 + z_2 =$$

- And the product?
  Use either the individual components, or the magnitude and phase:
  $|z_1 z_2| = |z_1||z_2|$, $\angle(z_1 z_2) = \angle z_1 + \angle z_2$

$$z_1 z_2 =$$

Algebraic expressions

- If A is a constant, find:

$$1 - \frac{1}{1+iA} =$$

- If ω, R, L and C are constants, and:

$$z_1 = R + i\omega L \qquad \text{and} \qquad z_2 = \frac{1}{i\omega C}$$

  Find the following:

$$\frac{z_1 z_2}{z_1 + z_2} =$$

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