Hello and welcome to the second lecture on the binomial theorem.

In this lecture, we will look at the binomial theorem and some of the related results which can be derived from the binomial theorem.

First, recall the definition of nC_r, which is the number of ways of choosing r objects from n distinct objects, and which can be written as $n!/[(n-r)!$

$* r!]$.

So the binomial theorem given by this result: $(x+y)^n = nC_0 x^n y^0 + nC_1 x^{n-1} y^1 + ... + nC_{n-1} x^1 y^{n-1} + nC_n x^0 y^n$, where nC_r is the number of ways of choosing r objects from n distinct objects. This can be more succinctly written as the summation $\sum_{r=0}^{n}{nC_r x^{n-r} y^r}$.

The binomial theorem can be taken to be an identity in x and y. The binomial theorem is applicable for x and y complex, and n being a positive integer.

At this point, I would like to mention that generalizations of the binomial theorem for the case when n is any real number also exist. In the definition/in the expression of the binomial theorem, we take x^0 to be equal to 1 for all x which are complex numbers, i.e., irrespective of the value of x, we define x^0 to be equal to 1.

Notice that there are n+1 terms in the binomial theorem, and there are NOT n terms but n+1 terms in the binomial theorem. You notice that there is a sort of pattern to the terms in the binomial theorem, and that is well captured by the general term of the binomial theorem. In general, the (r+1)th term in the expansion of $(x+y)^n$ can be written as $nC_r x^{n-r} y^r$. To give you an example of the application of the binomial theorem, let's look at $(x+y)^1$.

From the binomial theorem, we get that this is nothing but $1C_0 x^1 y^0 + 1C_1 x^0 y^1$ which gives us $x+y$, which is what we expect. We can also derive for the case when we have $(x+y)^2$, which is nothing but $2C_0 x^2 y^0 + 2C_1 x^1 y^1 + 2C_2 x^0 y^2$. Since $2C_0$ is 1, we have x^2; $2C_1$ is nothing but 2, we have $2xy$; and $2C_2$ is again 1 so we have y^2. This is the well known result $(x+y)^2 = x^2 + 2xy + y^2$.

Similarly, you can derive that $(x+y)^3$ is $x^3 + 3x^2 y + 3x y^2 + y^3$, and you can derive this result from the binomial expansion as well.

So one way in which you can interpret the binomial theorem is as follows: to derive $(x+y)^n$ is equal to this expression, just consider $(x+y)^n$ as nothing but $(x+y)*(x+y)*...*(x+y)$ for a total of n terms, and the interpretation of this expansion can be given as follows.

If you choose n x's, then you choose an x from each of these, uh, each of these parentheses here, and so there is only one way of choosing x's from each of these parentheses. There is only one way of choosing n x's and 0 y's, and that gives you the coefficient of $x^n y^0$. If you want to choose $(n-1)$ x's from this expression, then you have to choose $(n-1)$ x's from the n possible x's, and you have to choose 1 y from the n possible y's, and that can be done in n choose 1 ways, and that gives you the coefficient of $x^{n-1} y^1$ (and so on). In this way, you can derive the/you can provide a combinatorial interpretation of the binomial expansion.

So, as I mentioned previously, there are generalizations for the binomial theorem for the case when this exponent n is not necessarily a positive integer, but when it can be any real number.

This is the general result of the binomial theorem, and you can derive several related results by playing around with the general result of the binomial theorem. For instance, you could replace y by -y in the theorem and, upon doing so, you would get an expression for $(x-y)^n$. This is going to be $nC_0 x^n (-y)^0 + nC_1 x^{n-1} (-y)^1 + ... + nC_n x^0 (-y)^n$, and this can be more simply written as summation $\sum_{r=0}^{n}{(-1)^r nC_r x^{n-r} y^r}$. This result is very similar to the result we derived for the binomial theorem. A second result which we can derive is by replacing y by 1, in which case we can derive an expansion for $(x+1)^n$ as $nC_0 x^n 1^0 + nC_1 x^{n-1} 1^1 + ... + nC_n x^0 1^n$. This can be written as $nC_0 x^n + nC_1 x^{n-1} + ... + nC_n x^0$. Now, to arrive at a more simpler expression of this result, use the fact that nC_r is nC_{n-r} and you can derive this to be equal to $nC_n x^n + nC_{n-1} x^{n-1} + ... + nC_0 x^0$. This last expression can simply be written as summation $\sum_{r=0}^{n}{nC_r x^r}$. This is the binomial expansion of $(x+1)^n$.

Similarly, we can also derive this result from the binomial expansion: we can derive an expansion for $(x+y)^n + (x-y)^n$. If you apply the binomial expansion for these two separate expressions, you will get $2*[nC_0 x^n y^0 + nC_2 x^{n-2} y^2 + ...]$. The alternate terms will cancel out and it will leave you with this result.

So that's it for this lecture. Hope you had fun listening to the binomial theorem and some of its related results. In the next lecture, we will be looking at some examples of the usage of the binomial theorem and some possible problems you will face in an exam situation.

Thank you.