Forces in Equilibrium

· Static Equilibrium
- Objects completely at rest

$\vec{v}=0 \quad \vec{\omega}=0 \Big\}$ All times

$\vec{a}=0 \quad \vec{\alpha}=0$

· Dynamic Equilibrium

$\vec{v}$ = constant $\quad \vec{a}=0$

$\vec{\omega}$ = constant $\quad \vec{\alpha}=0$

Particles: Equilibrium if

$\Sigma F = 0$

Rigid Bodies: Sum of torques about any axes must also be zero.

Condition 1:

$\Sigma \vec{F}_e = \frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P}_{cm}$ = constant

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma F_z = 0$

· Resultant force acting on body must vanish.

· Linear acceleration $\vec{a}_{cm}=0$ in an inertial reference frame.

(1)

Condition 2:

$\Sigma \vec{\tau}_o = \frac{d\vec{L}}{dt} = I\vec{\alpha} \equiv 0 \quad \vec{L}$ = constant.

$\Sigma \tau_x = 0$ Sum of all external

$\Sigma \tau_y = 0$ torques must be zero.

$\Sigma \tau_z = 0$ i.e. $\vec{\alpha}=0$

No angular acceleration!

General Case

· 3 Force Eqtn's

· 3 Torque Eqtn's

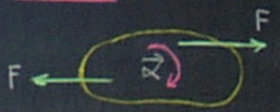
Special Case - Coplanar Forces

· 3 Scalar Eqtn's

$\Sigma F_x = 0 \quad \Sigma \tau_z = 0$

$\Sigma F_y = 0$ (Any Origin)

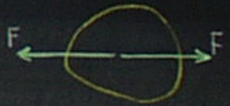
Case I: 2-Forces



$\Sigma \vec{F} = 0$

$\Sigma \vec{\tau} \neq 0$

No!



$\Sigma \vec{F} = 0$

$\Sigma \vec{\tau} = 0$

2-Forces

· Must be equal & opposite

· Must have same line of action

Yes

(2)

Lecture 26 Blackboard #1

Condition 2:

$\sum \vec{\tau}_i = \frac{d\vec{L}}{dt} = I\vec{\alpha} \equiv 0$ $\quad \vec{L}$: constant.

$\sum \tau_x = 0$ Sum of all external

$\sum \tau_y = 0$ torques must be zero.

$\sum \tau_z = 0$ i.e. $\vec{\alpha} = 0$

No angular acceleration!

General Case

· 3 Force Eqtn's

· 3 Torque Eqtn's

Special Case - Coplanar Forces

· 3 Scalar Eqtn's

$\sum F_x = 0 \quad \sum \tau_z = 0$

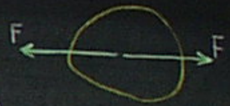
$\sum F_y = 0 \quad$ (Any Origin)

Case I: 2-Forces

$\sum \vec{F} = 0$

$\sum \vec{\tau} \neq 0$

No!



$\sum \vec{F} = 0$

$\sum \vec{\tau} = 0$

2-Forces

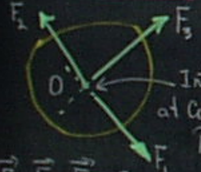
· Must be equal & opposite

· Must have same line of action

Yes

(2)

Case II 3-Forces



$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$ (given)

$\sum \vec{\tau}_0 = 0$

Torque Axis

$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots \equiv 0$

Origin at O:

$\vec{r}_1$: point of application of $\vec{F}_1$

etc.

$\sum \vec{\tau}_0 = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \cdots$

Let Axis O' = distance $\vec{r}'$ from O.

$\vec{r}_1 - \vec{r}'$: Pt. of application of $\vec{F}_1$ rel. to O'

Torques about O':

$\sum \vec{\tau}_{O'} = (\vec{r}_1 - \vec{r}') \times \vec{F}_1 + (\vec{r}_2 - \vec{r}') \times \vec{F}_2 + \cdots$

$= \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \cdots - \vec{r}' \times (\vec{F}_1 + \vec{F}_2 + \cdots)$

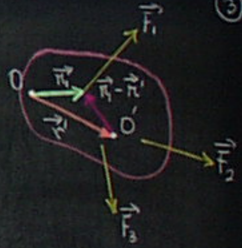
$\Downarrow$

$\equiv 0$ Equilibrium

$\sum \vec{\tau}_{O'} = \sum \vec{\tau}_O$

If an object is in translational equilibrium and the net torque about one point is zero, it must be zero about any other point on or off the object.

(3)



Lecture 26 Blackboard #2

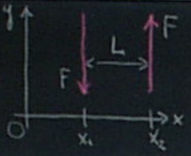
Couples/Torques.

- Pair of Forces
- Equal in magnitude
- Opposite directions

$\Sigma \vec{F} = \vec{F} - \vec{F} = 0$

$\Sigma \vec{\tau}_0 = F x_2 - F x_1$

$= F(x_1+L) - F x_1 = FL$



Same value for any axis $\perp$ plane of forces.

Problem Solving

1. Make a sketch.
2. Draw free body diagram showing only forces acting ON body.
   No other forces!
   Guess direction if necessary.
3. Choose coord axis. Pick sign for rotations. Forces → components.

Write out all equations:

$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$

$\Sigma \tau_x = 0 \quad \Sigma \tau_y = 0 \quad \Sigma \tau_z = 0$

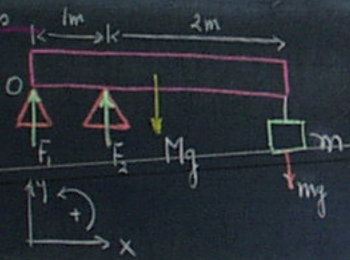
Choose convenient axis for all torque eq.
Often best if one or more forces pass through axis!
May need several axis to find all unknown forces.

④

Example: Pier Forces

M: 50 kg (Plank)
m: 20 kg (Load)

$F_1 = ?$
$F_2 = ?$



$\Sigma F_x = 0$ (No Horizontal Forces)

$\Sigma F_y: F_1 + F_2 - Mg - mg = 0$

$F_1 + F_2 = (M+m)g = (50+20) \times 9.8 = 686\,N$

$\Sigma \tau_0 = 0 \quad F_1(0) + F_2(1) - Mg(1.5) - mg(3) = 0$

$F_2 = 50 \times 9.8 \times 1.5 + 20 \times 9.8 \times 3 = 1323\,N$

$F_2 = 1323\,N$

$F_1 = 686 - 1323 = -637\,N$

F_1 = down; opp direction
F_1 - tension, pulling
F_2 : compression

⑤

Lecture 26 Blackboard #3

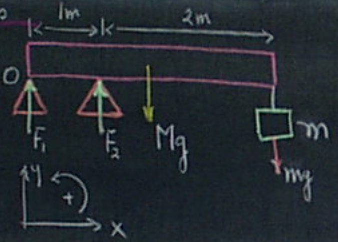
Example: Pier Forces

$M = 50\,kg$ (Plank)

$m = 20\,kg$ (Load)

$F_1 = ?$

$F_2 = ?$



$\Sigma F_x = 0$ (No Horizontal Forces)

$\Sigma F_y: F_1 + F_2 - Mg - mg = 0$

$F_1 + F_2 = (M+m)g = (50+20)\times 9.8 = 686\,N$

$\Sigma \tau_O = 0 \quad F_1(0) + F_2(1) - Mg(1.5) - mg(3) = 0$

$F_2 = 50\times 9.8\times 1.5 + 20\times 9.8\times 3 = 1323\,N$

$F_2 = 1323\,N$

$F_1 = 686 - 1323 = -637\,N$

F_1 = down; opp direction

F_1 = tension, pulling

F_2 = compression

⑤

Example: Ladder and Wall

Friction: Ladder and wall

Ladder and floor

μ_s = coeff of static friction

$\Sigma F_x = 0 \quad f_1 - F = 0$ ①

$\Sigma F_y = 0 \quad N - Mg + f_2 = 0$ ②

$\Sigma \tau_O = 0 \quad LF\sin\theta - \frac{1}{2}LMg\cos\theta + Lf_2\cos\theta = 0$ ③

①,② → ③ $\quad f_1\sin\theta - \frac{Mg}{2}\cos\theta + (Mg - N)\cos\theta = 0$ ④

$2f_1\tan\theta + Mg - 2N = 0$

$f_1 = \mu_s N \quad f_2 = \mu_s F$ (at pt. of slipping)

$\tan\theta = \frac{2N - Mg}{2f_1} = \frac{N - Mg/2}{\mu_s N}$

$N = Mg - f_2 = Mg - \mu_s F = Mg - \mu_s f_1$

$= Mg - \mu_s^2 N$

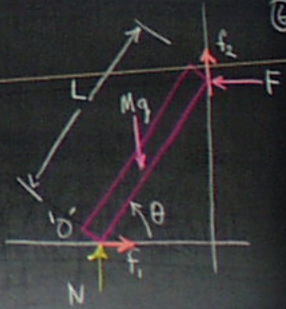
$\therefore N = \frac{Mg}{1+\mu_s^2}$

$\therefore \tan\theta = \frac{1 - \frac{1}{2}(1+\mu_s^2)}{\mu_s}$

$\tan\theta = \frac{1-\mu_s^2}{2\mu_s}$

μ_s	$\tan\theta$	θ
0.1	4.95	78.6
0.2	2.40	67.4
0.3	1.52	56.6
0.4	1.05	46.4
0.5	.75	36.9
0.6	.53	28.1

⑥



Lecture 26 Blackboard #4

Example Levitating Cart:

7

$M = 1000g$

Assume:

$\vec{N} \equiv 0$

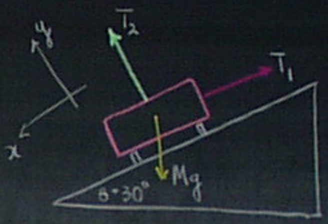
No normal force!

$\Sigma F_x = -T_1 + Mg\sin\theta = 0$

$\Sigma F_y = T_2 - Mg\cos\theta = 0$

$T_1 = Mg\sin\theta = 1000 \times 9.81 \times \frac{1}{2} = 490g$

$T_2 = Mg\cos\theta = 1000 \times 9.81 \times 0.866 = 870g$



Example Cylinder and Step

8

Cylinder: Mass. $Mg = 500\,N$

$R = 0.8\,m$

Step $H = 0.3\,m$

What is $\vec{F}$ to raise cylinder?

What is reaction at P?

Cylinder ready to raise when normal force at $Q \to 0$!

$d = \sqrt{R^2 - (R-h)^2} = \sqrt{2Rh - h^2}$ Moment arm of W about P.

Torques about P: $Wd - F(2R-H) = 0$

$$F = W\frac{\sqrt{2Rh - h^2}}{2R-H}$$

$\Sigma F_x = 0 \quad F - N\cos\theta = 0$

$\Sigma F_y = 0 \quad N\sin\theta - W = 0$

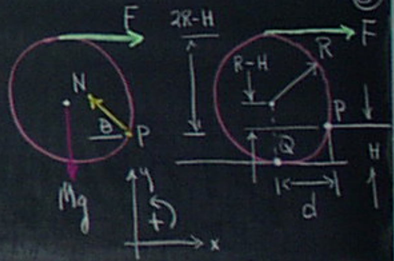
$\tan\theta = \frac{W}{F} \quad N = \sqrt{W^2 + F^2}$

Solving:

$F = 385\,N < Mg$

$\theta = 52.4°$

$N = 631\,N$



Lecture 26 Blackboard #5

Example Cylinder and Step

Cylinder Mass: $Mg = 500\,N$

$R = 0.8\,m$

Step $H = 0.3\,m$

What is $\vec{F}$ to raise cylinder?

What is reaction at P?

Cylinder ready to raise when normal force at $Q \to 0$!

$d = \sqrt{R^2 - (R-h)^2} = \sqrt{2Rh - h^2}$ Moment arm of W about P.

Torques about P: $Wd - F(2R-H) = 0$

$$F = W\frac{\sqrt{2Rh-h^2}}{2R-H}$$

$\Sigma F_x = 0 \quad F - N\cos\theta = 0$

$\Sigma F_y = 0 \quad N\sin\theta - W = 0$

$\tan\theta = \dfrac{W}{F} \quad N = \sqrt{W^2+F^2}$

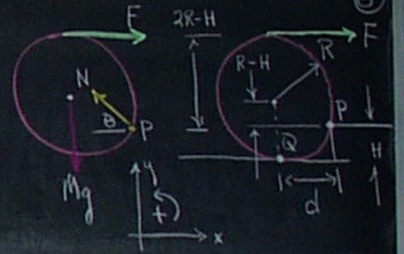
Solving:

$F = 385\,N < Mg$

$\theta = 52.4°$

$N = 631\,N$

8



Example: Pulling a Cylinder.

$M = 5\,kg$

$\mu_k = 0.42$

What is F?

What is β

$\vec{a} = 0$

$\vec{v}$ = constant!

$\Sigma F_y = 0 \quad N - Mg\cos 33° = 0$ (1)

$\Sigma F_x = 0 \quad F - f - Mg\sin 33° = 0$ (2)

$f = \mu_k N$ (3)

(1)+(2) $F = Mg(\sin 33 + 0.42\cos 33)$

$F = 43.95\,N$

Torques about C:

N, Mg No contribution!

$\Sigma \tau_c = 0 \quad R(F\cos\beta) - fR = 0$

$\cos\beta = \dfrac{f}{F} = \dfrac{\mu_k Mg\cos 33°}{F}$

$= 0.393$

$\beta = 66.9°$

9