PROFESSOR: Hi. Welcome to Excitatory Topics in Physics, lesson 2. I hope you all had a good week, good 4th of July. I was asleep for about half of it, so I don't really know what happened. But I'm here now, and-- yeah. Let's begin.

Last class I started talking about special relativity, which is relativity where we assume there's negligible gravity. To deal with effects where gravity is important, you have to use general relativity, but special relativity is a lot simpler, so we'll start with special relativity first.

Special relativity all derives from two basic postulates-- the two basic postulates of special relativity, two postulates which are axioms of the theory-- unprovable axioms. And I introduced them last class. The first was that the laws of physics are the same for all inertial observers, and the second is that the speed of light is the same for all inertial observers.

And I defined inertial observer as simply somebody that makes measurements-- an observer is just somebody that makes measurements, and an inertial observer is somebody that makes measurements while moving at a constant speed-- or constant velocity, to be more precise. I'll just keep things simple. I'll just say speed. So if you're accelerating, then you'd be a non inertial observer. For non-inertial observers to make measurements, you have to use general relativity, because there's actually a deep connection between acceleration and gravity. But I'll get to that eventually.

Now, to demonstrate to you-- the first postulate was pretty easy to accept. It simply says that nature is fair. The laws of physics are the same for everybody. The second postulate is actually very surprising and mysterious, perhaps disturbing, because it goes completely against what we'd expect to be true.

Last class I did experiment with someone-- a student, Life-- where we both walked across the class and Life measured a speed for me and you measured a speed for me. They happened to be two different speeds, and there is a simple way of relating them. There's a simple way of relating all the speeds. We simply subtracted one from the other. You measured four miles per hour for me, two for Life, and he measured two for me and zero for himself. And it's easy to get to. You just take two from four. Simple subtraction of velocities.

Well, I told you at the end of last class that that rule actually doesn't apply for light-- for some reason, some mysterious reason. But we don't attempt to explain that. We simply take that as a given. And we wouldn't take that as a given if we knew for a fact that it was wrong. But in fact, numerous experiments have been conducted testing the validity of the second hypothesis, and they've all confirmed it. They've all confirmed it to be true.

Just to remind you what it means, it means that if I pointed a laser-- if I pointed a light at the wall and started walking, then I'd measure a certain speed for light. I'd measure about 186,000 miles per second for light. And you'd also measure the same speed, even though I'm walking with it. You don't add my speed to it. And that's crazy. It's completely counter to what we'd expect.

So actually, you might suspect that maybe the subtraction of velocities that we were doing before-- the subtraction of speeds that we were doing before, four minus two-- maybe that's just an approximation. And it turns out that it actually is an approximation. And I'll give you the real formula. Oh, you have a question? OK. I'll give you the real formula to deal with subtraction of velocities. I keep saying velocities. Speeds. I won't-- I'll just be doing things in one direction. So if I say velocity, it's either going to be negative or positive. So don't worry about that. Velocity is just speed with direction, for any of you that aren't familiar with the word.

Now I'm going to show you something. Where's-- yeah, this is the chalk I was using. The way that we're used to dealing with relating different speeds is, we simply add them or we subtract them. If I'm on a train and I'm walking, and you're-- I'm just giving you another example. If I'm on a train and I'm walking a few miles per hour, and you observe the train going at 40 miles per hour, then you expect to find 40 plus three, or 43, miles per hour for me, if I'm walking in the direction of the train. If I walk in the opposite direction, then you'd expect to find 40 miles three miles per hour, 37 miles per hour.

That rule only works for small speeds. And that was actually known for a long time. Nowadays we refer to this rule as the Galilean addition of velocities. You make a Galilean transformation. I won't get into the details of these transformations, but call it Galilean addition of velocities. And you simply take the two speeds and you add them. I'll write up here. Galilean addition of velocities. Hi.

Suppose I'm traveling at a speed of v relative to you all. Suppose I'm traveling at a speed of v. Actually, let me-- let me start over. Let me start over. Let's say I'm on a train. Let's say I'm on a train that's traveling at a speed of v relative to you. And relative to the train, I'm traveling at a speed of u. So u equals-- I'll start with v. v equals speed of train, and u equals speed of me relative to the train.

Then what's the speed of me relative to you? It's just u plus v-- or so you would think, and so everybody thought for a long time. Everybody since Galileo's time thought it would just be u plus v. So u plus v-- I'll put this in quotation marks, jest to encode the meaning of the speed of me relative to you. So u plus-- I could write it in words. I was going to put it in quotation marks, but I'll just write it in words so that you know what I'm talking about. So u plus v equals the speed of me relative to you. Or maybe I should say class. I'll say class. I should also specify here, too. Speed of the train relative to the class. Yes?

AUDIENCE: It's relative to the class if the class is surrounding me?

PROFESSOR: Oh, OK. Well, the class isn't moving as a whole to any appreciable extent. So-- yeah. That's important, though. That's an important point. It's an important point. OK. To use special relativity at all, to use it at all, we have to consider only inertial observers-- only observers that are at rest or moving at a constant speed. If you're accelerating, you simply can't use special relativity. Special relativity is not designed to deal with problems that arise for non-inertial observers. You need general relativity, and that's much more difficult to use. But I'll let you know-- I'll talk about general relativity-- maybe at the end of the class, maybe next class. Well, definitely next class if I don't get to it today.

But that's an important point. And it's always important to specify what you're measuring-- well it's always important to specify the observer that-- OK. This is a hard sentence to make. It's a hard sentence to make. So I'll just demonstrate with this. Speed of the train relative to you. Speed of me relative to you. It's always important to specify-- to make that specifier, relative to. Relative to me, relative to you. There's no single speed. There's no single speed. There's no absolute speed. There's no correct speed. There's no best speed. They're all equal. We're all created equal. All inertial observers are equal. But some are more equal than others. Speed of train relative to the class.

But yeah, this is really relativity at its simplest, though-- the relativity of speed. Some people measure one speed, other people measure another speed. They're all equally good. One's not better than the other. OK. So this is the Galilean addition of velocities. You simply take the speed of me relative to you, and you take the speed of the train relative to you, and you add them. u plus v. Simple as that. But as I said, that doesn't work for light. Oh, yes. Question?

AUDIENCE: But are you moving? Are you on the train?

PROFESSOR: Oh, yeah. I'm on the train. I'm on the train. I'm on the train, and I'm walking at some constant speed, or running at some constant speed. I'm being chased by somebody and I'm running at some constant speed. But forget about the person that's chasing me. Say he's invisible and he's undetectable, because I'm crazy.

The u plus v is this totally familiar formula. Now, as I said, it's incorrect. So I'll give you the correct formula, which is much more complicated than that but, you'll be able to appreciate it, because-- well, you'll see. Here's the correct formula. I'm just telling you this so that you can have some power in doing some calculations-- in doing some correct calculations.

Relativistic addition of velocities. Or the Einsteinian addition of velocities. I'll use the same variables that I used up there, u and v. And now the speed of me relative to the class-- now the speed of me relative the class is actually u plus v divided by 1 plus uv over c squared, where c is the speed of light.

And if you'd like NKS-- if you like SI units, using meters, kilograms, and seconds, then that's actually 299,792,458 meters per second-- exactly, actually. The meter is actually defined so that the speed of light-- I won't get to this. But this is actually the exact speed of light, and there's a little bit of interesting history that has to go with that. And I can tell you-- if any of you have questions after class, I can talk about it.

OK. So c is the speed of light. And-- yeah. This is the correct formula to use. It's actually approximately equal to this formula up there for small speeds-- so if u and v are both much, much less than the speed of light. So if u and v are everyday speeds-- certainly, if I'm walking, that would be an everyday speed.

But even if I'm running or even if I'm in a car, even if I'm on an airplane, even if I'm on a rocket, all those speeds are so much less than the speed of light. The speed of light is immensely huge. It's immensely huge. And so pretty much all speeds that we can think of, all speeds that we can get to-- at least travel ourselves-- all those speeds are much, much less than light. So you can use that formula with relative confidence. Yes?

AUDIENCE: Do you know how fast a speed man has ever traveled?

PROFESSOR: The fastest speed man has ever traveled?

AUDIENCE: Yeah.

PROFESSOR: Oh.

AUDIENCE: Whether it be in an airplane or rocket.

PROFESSOR: Right, right. I was going to-- I was actually going to look it up before class, because I was going to do a simple calculation later on in the class. But it's definitely-- definitely less than 10% of the speed of light. I'm pretty sure-- yeah, definitely less than 1% of the speed of light. Probably something like-- maybe a 1/10,000 the speed of light, or 1/1,000 the speed of light.

Does anybody actually know what it is? Well, I'll look it up, and I'll send you all email about that, because I was going to do a calculation later on where I do use that speed. But I'm not going to that calculation now, because I forgot to look it up. But we can appreciate it. But it's much, much-- yeah, much, much less than the speed of light.

But we haven't actually-- we haven't accelerated ourselves to that fast. But we've accelerated particles very, very fast-- something like 0.99999 times the speed of light. 99.999 times speed of light we've actually accelerated particles to. And it's much easier to do that with particles, because they're much lighter than we are, so it doesn't require as much energy. Do you have a question?

AUDIENCE: Do we know why that equation is correct?

PROFESSOR: Oh, yeah. Oh, yeah, yeah. This equation--

AUDIENCE: Can you repeat question?

PROFESSOR: The question is, do we know why this equation is correct? Yeah, we do know why. You can actually derive this equation with the principles-- with the two postulates of relativity, along with the definition of velocity. But I don't plan to go through mathematical derivations in this class, because I told you from the beginning-- I promised you from the beginning that there wasn't going to be a lot of math.

It's pretty simple to derive it though. And I can show you how to do it, if you're interested. If anyone else is interested, I can show you how to derive it. But it certainly agrees with experiment. I could say that somebody noticed it happens to agree with experiment. But no, you can actually derive it. You can actually prove it, prove that it's true. Dot. Relative.

I basically put that there for you to enjoy. Put in some numbers, have fun with it. Something that's fun to do with this equation is to use it wrong. And that's probably not the best way to show how to use an equation, using it wrong, but there's something funny-- there's something that I think is pretty funny that you can do with this equation. You can misinterpret it.

This is probably a-- well, you can misinterpret this equation, but it's-- you'll see why it's wrong. I just want to show you something that I notice which is kind of funny about the equation. It allows you to prove, for example, that 1 plus 1 isn't equal to 2. It's approximately equal to 2. It's actually equal to 1.99. And I'll show you how you can get at that conclusion.

Let's say I'm walking-- well I'll show you-- OK. Let's say I'm walking on this train, which is moving-- let's say the train is moving at 1 miles per hour for you. And I'm walking on the train 1 miles per hour relative to the train. So I can now ask you, how fast am I moving relative to you? I told you this is the wrong formula, but it worked pretty well for small speeds. But this is the correct formula, which works for all speeds. And so I like to use this formula in getting at the speed. Yes?

AUDIENCE: If you do it with the same speed and you put it into both equations, will-- if it's a small speed, would it come out the same?

PROFESSOR: The same as-- the question--

AUDIENCE: A small speed like 40 miles per hour the train is moving. [INAUDIBLE] be the same?

PROFESSOR: The question is, if you put in small speeds for this equation-- if you let u and v both be really small, then is the result the same as this result?

AUDIENCE: Yeah.

PROFESSOR: Yeah. It's approximately the same. Yeah. It's very, very close. And you can prove-- for those of you who are more mathematically minded, you can show that if you make the approximation that u and v are much less than c, much less than the speed of light, then you can show that is approximately the same as that, because u times v is both then much less than the speed of light. A small number over a really big number is approximately 0, and then 1 plus approximately 0 is 1. And so then u plus v of approximately 1 is approximately u plus v. Yes?

AUDIENCE: Why is light involved?

PROFESSOR: Why is light involved? The question is, why is light involved? Well, I told you there are two postulates to special relativity. So you might suspect that light's involved in all these equations, because it's right there in the postulates. I can't tell you precisely why it's involved, because I would have to get mathematic-- I would have to get more mathematical.

But I can give a plausibility argument for why it's involved, for why this isn't-- I can give a plausibility argument for why this equation is not totally wrong, because-- it's actually because of what I said before. If u and v are really small, then you expect this to be true, because we find this to be true in our everyday life to an incredible accuracy. And so since the speed of light is much faster than everyday speeds, this is a reasonable formula, at least. It's a reasonable formula.

OK. Now I want to do that calculation. The train is moving 1 miles per hour relative to you, and I'm moving 1 miles per hour relative to the train, so how fast am I moving relative to you?

AUDIENCE: Two.

PROFESSOR: Approximately two, because I'm moving much less than the speed of light, but not exactly 2. And if any of you have a calculator, you can go with me as I do this. So u equals-- actually, if you're actually going to calculate it, then we have to use consistent units. And I was talking about miles per hour, and I don't remember what the speed of light is in miles per hour.

So let's say that I'm just moving at 1 miles-- let's say I'm moving at 1 meters per second relative to the train, and the train is moving 1 meters per second relative to you. So let's say u equals 1 meters per second, and v equals 1 meters per second. Then u plus v is 1 meters per second plus 1 meters per second. And then the denominator is 1 plus 1 meters per second times 1 meters per second over approximately 300 million meters per second squared.

And those of you who have a calculator can plug in these numbers, and you'll find it's something like-- if you're actually doing it, then you can tell me in a second what it is. But it's something like 1.99999-- give me another nine-- meters per second. So we've just shown-- wink-- you've just shown that 1 meters per second plus 1 meters per second equals 1.99999 meters per second. So if you divide by meters per second, then you get 1 plus 1 equals 1.99999. So it's approximately equal to 2. Yes?

AUDIENCE: Isn't technically an infinite nine. The same as--

PROFESSOR: OK. It's not an infinite nine. I don't know if anyone is calculating it, but actually it's 999-- there was something like six or seven nines something like that, and then you get a four and a three. How many nines is it?

AUDIENCE: Eight nines.

PROFESSOR: Eight nines. OK. I'll fix it. One, two, three, four, five, six, seven, eight.

AUDIENCE: But how do you get a three [INAUDIBLE]

PROFESSOR: Oh, how do you get a three? It's just arithmetic. The calculator is doing it for you. I'll fix this. 1.999999993-- sorry, 3. Therefore 1 plus 1 is approximately 2, and that's wonderful.

AUDIENCE: Yay.

PROFESSOR: Of course-- of course, I haven't really proven anything, because I'm not really saying 1 meters per second plus 1 meters per second. That's the interpretation of what I'm doing, kind of, but it's not precisely the interpretation of it. I'm actually not dealing with numbers when I say the speed of-- well, I'm not dealing purely with numbers when I say the speed of me relative to you. And that's why this is wrong. Of course, it is true that 1 plus 1 is approximately equal to 2, because 1 plus 1 is equal to 2, and 2 is approximately 2. So you get the right answer for the wrong reason. I love when that happens. I love when that happens. OK. No, let's-- I'll leave this here. OK.

This demonstrates that speed is relative, and you know for every day experiment-- everyday experiments. Maybe you do experiments everyday. But you know from everyday experience that speed is relative. Everybody notices this, that speed is relative. But there are some things that you wouldn't expect to be relative. One of the most striking relative things is simultaneity, and by that I mean that-- by that I mean that if I observe two events to be simultaneous-- two events occurring at the same time-- if I measure them as being simultaneous events, then that doesn't necessarily mean that they'll be simultaneous for you, and that's unexpected. And I can show you very simply why simultaneity is relative.

Again, let's say that I'm on a train, because I like trains. Say I'm on a train. Let's say I'm on a train. I'm sitting inside the train. I'm sitting in the center of the train. And I turn on a flashlight.

This is supposed to be a train.

[LAUGHTER]

Yeah. I say train because in Einstein's time, that was pretty much the fastest vehicle they had, I guess. All of his experiments-- well, all of his thought experiments, because he did thought experiments-- all of his thought experiments involve trains. And so in his tradition, I will continue to talk about trains.

Let's say I'm sitting in the middle of a train and I turn on a flashlight. And the light rays go in all directions. They go in all directions. And eventually they'll hit the walls of the train. They'll hit-- they'll hit this point. One ray of light we'll hit this point, and another ray of light will hit this point. And if I'm sitting in the center of the train, then since both of these points are equidistant from me, I would measure them as-- I would measure the light hitting both points at the same time. So I would observe the two events to be simultaneous.

Let me be-- I keep talking about events. By event I just mean something that happens. Lightning strikes here, I punch a guy here. I don't know why that was the second thing to come to my mind. Events are just things that happen-- things that happen at a specific place, a specific location, and a specific time.

So you actually need four numbers. You need four numbers to specify an event. You need the location, which is three numbers, because there are three dimensions of space-- this direction, this direction, and this direction. And you measure the time of the events. So events are specified by four numbers. And it's for this reason that light-- not light. It's for this reason that time is often considered as the fourth dimension, because you simply need it to specify events. And there are other reasons too, much deeper reasons, but I don't know if I'll get to those.

Anyway, getting back getting back to the train, which I love. I'm sitting in the center, and I observe these two events to be simultaneous, and it's as simple as that. Let's say now that I ask you, which do you observe to occur first? Do you observe the light hitting at this edge to occur first, or the light hitting this edge to occur first?

But before you answer that, you have to know which way the train is traveling. So I'll tell you that the train is traveling in this direction. Yeah. The train is traveling in this direction. So who thinks that event are still simultaneous for you guys? One, two, three, four. Anybody else? Five. OK, some of you think-- six, seven. OK. Seven. OK.

Who thinks-- I guess the rest of you think it's not simultaneous. I'll assume that. Who thinks that-- well, if they're not simultaneous, then obviously one occurs before the other. One event occurs before the other. Now, who thinks that in the event of light hitting this wall occurs before the event of light hitting this wall? Several of you. And I guess the rest of you think it hits this wall first. Well--

AUDIENCE: [INAUDIBLE]

PROFESSOR: What's that?

AUDIENCE: Where are we relative to?

PROFESSOR: Oh, you're just sitting here. Where are you relative to? The train. You're just sitting here. And I'm sit-- suppose I'm sitting on a train, and I hold out a flashlight, and it's going in all directions. The train is going this why. The train is going this way. I'm sitting down in the middle of it, and I turn on a flashlight, and the light goes in all directions.

AUDIENCE: Yeah, but what way are you pointing?

PROFESSOR: Oh, which way-- OK. I'm-- OK. Maybe a candle would be better than a flashlight. I went to the light to go in all directions. The light to go in all directions. So the light will hit-- the light will hit the top walls too, and it'll hit a lot of walls, but I'm just focusing on the events of light hitting these two walls. OK? Now, the answer is that the events aren't simultaneous to you, even though they are simultaneous to me. And I kind of hinted at that, because I told you from the outset that simultaneity is relative, which is a wonderful phrase, I think. Simultaneity is relative. I like saying it.

OK. Now, how could we determine which one occurs first, which event occurs first? Well, we've got these two postulates here. The first one doesn't really seem to be useful. OK, yeah, the laws of physics are the same for everybody. cool. The speed of light is the same for everybody. Well, that's actually the useful one, because you'll observe the same speed of light that I'll observe while sitting on the train.

Light has to travel from here to here and from here to here. And if the train is moving, then actually, the distance the light has to travel when going from here to here is less than the distance the light has to travel when going from here to here, because the train is coming this way and so the wall gets closer. The wall gets closer to the light coming to it, so the light actually travels a shorter distance than it has to travel when going to this wall.

As a result, since the speed of light is just a number-- since the speed of light is a constant, you can get the time that it takes to get from the center of the train to the walls just by using the simple formula distance equals rate times time. You can say distance divided by speed is time. And you get that the time interval between when I turn on the light and when it hits here, the time interval that is less than the time interval between the two events of my turning on the light and the light reaching this wall. And so you all will observe the events to occur at a different order. That's the simple proof that simultaneity is relative.

And it's really interesting to think about, because we often think of things happening at the same time. We often think of-- you're on the phone talking to somebody, and you both see something happen, say, outside, like there's lightning. And you'd expect them-- well, if someone says, hey, I just saw lightning, and you say, yeah, I saw it too, you'd expect to think that they happened at the same time. And, well, just as before, for everyday phenomena it is approximately true that simultaneity is not relative. For everyday speeds-- if the train was traveling at a speed much less than light, then the two events would be approximately simultaneous. So the relativity of simultaneity is only apparent at high speeds, again.

Special relativity reduces, once again, to simple Newtonian mechanics for low speeds. And for that reason, Newton wasn't completely wrong. He wasn't completely wrong. He was just incomplete. Newtonian mechanics just deals correctly with speeds that are much less than light. And you can actually prove that rigorously. You can prove that rigorously using the modified laws and mechanics that you get from the two postulates of special relativity. You can prove rigorously that Newtonian mechanics reduces as a limiting case of special relativity.

Now we know of two things that are relative. We know about at least two things that are relative. We know simultaneity is relative and we know that speed is relative. We know that the speed of light's not relative. The speed of light is invariant. That's the word that we use, invariant. Not relative.

Oh, by the way, I just remembered-- you're all good with the word relative by now, I'm sure, because I've been using a whole lot. But I realized I didn't really know what the word relative meant until I started reading some books on relativity. I just had never-- I never used it in the context of speed relative to this, or measurements relative to this observer. I don't know. I guess-- maybe I was so illiterate or something, or maybe I still am really illiterate. But if any of you've found yourself thinking that that's a strange word, well, I was in the same situation. And so I learned-- I learned what the word relative meant by reading popular books on relativity. I also learned with the word dumbbell meant. You know, a dumbbell? I learned what the word dumbbell meant by learning about p orbitals.

[LAUGHTER]

Yeah. I-- I had no idea what a dumbbell was before I learned about p orbitals. For those of you who haven't heard about p orbitals-- I'm just giving you more evidence to my illiteracy. OK?

Now that you can see that simultaneity is relative, you're probably-- or hopefully you're wondering a bit about time. What is it about time that-- well, there might be something about time that's a little bit different than what we expected, something a little bit different from what we're used to.

And it's true. Time is actually very mysterious once you invoke special relativity. And the meaning of space and time, once Einstein introduced his relativity, was completely contrary to what Newton had conceived of the meaning of space and time.

According to Newton, everybody-- every observer measures the same time for events, and they measure the same coordinates of events. Well, they measure the same lengths of-- actually, it's probably best if I don't get to space yet, because I haven't told you of the strange things about space yet. I'll leave-- OK. I'll leave this in a second. I'll talk about time. I'll talk about time first. Remind me to get to space. Well, I won't forget, but-- let me just talk a little bit about time first.

For the simultaneity of-- from the relativity of simultaneity, we suspect something is mysterious about time, and that's in fact true. And relativity of simultaneity in fact isn't-- well, it's shocking, but I don't think it's as shocking as the relativity of time intervals. Let me explain this now.

It turns out that-- OK, well-- sorry. I want to be-- sorry. I want to be precise. They're related. Relativity of simultaneity and relativity of time intervals are related, because you measure some time interval, and I measure a different time interval. They couldn't be the same if you measured something different.

But now I'd like to just be a bit more precise about this relativity of time intervals. OK. Let's say I'm on a train again, because trains are so much fun to go on. Let's say measure the time interval between two events. Let's say I take out a watch, and I write down the time of the first event happening. And then I write down the time of the second event happening, and then I just subtract the two times to get the time interval. So I measure the time interval between two events while I'm on the train.

So I write down some time interval. You could do the same thing. Everybody observes events happening. Reality isn't relative. Reality isn't relative. The same-- at least according to special relativity, reality isn't relative. Events will happen regardless of-- the same events will happen regardless of who's observing the events happening. It's just the numbers that we associate to the events that are relative. But the same events will still happen, so in that sense reality is not relative. Reality is invariant, special relativistically invariant. Whew.

So you'll see the same events happening, and you can measure a time interval between the two events by taking out your watch and writing down the time of the first event happening, and then writing the time of the second event happening, and then subtracting the two times to get a time interval. And as I've just told you before, that time interval isn't the same as the time interval that I get, and that's crazy. It's crazy that the time intervals should be different. I think it's crazy. You'd expect for time to be the same.

This effect is called time dilation, and it's called time dilation because actually, as you can derive from special relativity, it turns out that clocks that are moving relative to you actually tick at a slower pace than the clock that you're holding, that's stationary. So moving clocks runs slow. That's a phrase that's commonly used to encapsulate the meaning of time dilation. Moving clocks run slow.

So that means that if I-- if I measure a certain-- well, I could be a little bit-- I was about to be more quantitative in my explanation, but I think I'll just write down a formula for you, because this'll a very simple formula and you can appreciate it. And you'll have so much fun with it, believe me. And this is the-- I just want to make sure I don't forget it, because it's so important.

Let's say I measure-- let's say I measure a time interval of-- sorry. Let's say-- sorry, sorry. Let's say that you measure-- see, we have to be careful when we say who measures things, because so many things are relative. So many things are relative, so you have to be careful in how you phrase things. And that's-- it gets kind of confusing. It gets kind of confusing.

So let's say now-- let's say that you measure a certain time interval between two events. Let's say that you're stationary. You're holding a clock, and you measure some time interval between two events. And let's let that time interval be T0. T0 just because you're moving at zero speed. We just want to have some reference number. T0 is time interval that a stationary observer-- time interval that a stationary observer measures. A stationary-- OK. Now let's say that T is the time that an observer in motion measures. So I'm on the train, and I take out my watch, and I measure the time interval between two events. Let's call it T. Time interval that a moving observer measures. It turns out that you can relate the two in a very easy way, and in a very fascinating way. It turns out that T equals T0 divided by the square root of 1 minus V squared over c squared. I forgot to tell you what V is. V is the relative speed-- the relative speed of the two observers. Observers. And, of course, c is the speed of light, as usual. Approximately 3 times time to the eighth meters per second. Can you read this?

AUDIENCE: Yes.

PROFESSOR: OK. I'll make it a little bit nicer. Sorry. OK. That's supposed to be T equals T0 over the square root of 1 minus V squared over c squared. Yes?

AUDIENCE: Does c always stand for the speed of light?

PROFESSOR: c will always stand for the speed of light when I write it down.

AUDIENCE: OK.

PROFESSOR: Even when I write my name, N-I-C-K. I want for c to be the speed of light, but people don't understand that. Yes?

AUDIENCE: Can you read the [INAUDIBLE] again?

PROFESSOR: What's that?

AUDIENCE: Can you read the formula out again?

PROFESSOR: Oh, yeah. I'll read the formula again. Oh, I'm going to write down a couple of more formulas today, and I'll send them all to you with instructions on how to use them, so that you can all play with them and so that you can get them all done correctly. If you're not writing them down now, it's OK, because I'll send them all to you. So don't worry about writing them down. I'll read it out again. It says that T-- which is the time interval that a moving observer measures-- T equals T0 over the square root of 1 minus V squared over c squared. Yes?

AUDIENCE: What is the speed at which these effects become noticeable?

PROFESSOR: OK. The speed at which all of these effects that I'm discussing becomes noticeable is a speed that's close to the speed of light. You won't notice any of these effects if you're moving much smaller than speed of light, even 1% the speed of light. Probably 1% of the speed of light is when they start to be noticeable. So you'll never have any dream of observing these effects when you're walking your dog. Unless it's a space dog or something.

AUDIENCE: Yes? Well, why does the moving clock go slower?

PROFESSOR: Why does it go slower? I could derive this for you. It's as simple-- what's that?

AUDIENCE: Please do.

PROFESSOR: Please do. Would you be OK if I get did simple geome-- I guess I could. The question was, can you derive this for me? Sure. I-- I won't go through all the details, but I'll go through the essential details. I'm going to derive this first, and then you can ask the question. And I'm kind of going against the promise I made to the class about not being mathematical, but if you guys want to see how you get this, then I'll show it to you.

OK. So let's say-- can you guys guess what I'm drawing?

AUDIENCE: Train.

PROFESSOR: No, it's a school bus. Let's say that's the-- what? Let's say that the school bus is traveling at a speed of V relative to you. Say it's traveling at a speed of V relative to you. And let's say it has a height of h. Crazy h. It has a height of h. And let's say I'm inside of it. So I'm stationary relative to the train. Let's say-- let's say I'm hanging at the top. I'm at the top of the train. I'm hanging at the top of it. And I shine a flashlight going downwards. I shine a flash light going downwards, going down. This is the path that the light makes. I'll draw it again. A little squiggly path. This is the path the light makes. It goes down like this.

Now, I can measure the time interval between the two events of the emission of light and then when the light hits the bottom floor. I could calculate how long that takes. It's simply the height of the-- I almost said train, but I said this is the school bus. The height of the school bus divided by c. Let's say that-- OK. So T-- T as I've defined it there. T equals h over c.

Now, I ask-- sorry. I don't have that good handwriting. I'm trying really hard. T equals h over c. And now I ask, how long-- what's the time interval between these two events for you guys, with the train traveling at some speed? Well, for you-- I should make another drawing. For you, it doesn't look like this. For you, the light doesn't travel straight down. It actually travels on a diagonal path. It travels on a diagonal path because the train is moving this way. Oh, I said train, didn't I?

AUDIENCE: Yeah.

PROFESSOR: OK it's a train. It's a train. I'll remove this label now, because this actually is only what the path of the light looks like to somebody inside the train. It's not what it looks like to you. OK. So this is for an observer in the train. Observer in the train. And this is for the observer watching the train. For the observers watching the train, for you guys, the light actually will travel in a diagonal path, and then it'll hit the floor. The height of the train is still h, although you should be a little bit skeptical because I told you that there was something fishy about space. And there actually is an effect of the relativity of length. But it turns out that it's still h. It's still h, OK? It's still h. The height of the train is still h.

Now, this time the light doesn't travel h over c. It doesn't-- sorry. This time the light doesn't travel a distance of h. It actually travels a slightly longer-- slightly longer path, because now it's got to go down and it's got to go across. It's got to go vertical and horizontal. So the path that actually has to travel is now-- well, let me make a little diagram, a little triangle.

OK. This is h. This is a right triangle. h is just the height of the train. It's always going to be the height of the train. I can ask now-- what's that?

AUDIENCE: The bus.

PROFESSOR: What's that?

AUDIENCE: It's a bus.

PROFESSOR: I changed it. I couldn't help myself. It's a train again. It's always going to be a train. You're never going to hear me mention the word bus again, probably, at least not-- I have to really try. OK. It's a train. The height here is h, and-- does anybody know what this distance is? Yes?

AUDIENCE: [INAUDIBLE]

PROFESSOR: No, this distance is just how far the train's moved. Forget about length. This is just how far the train's moved. And the train has actually-- you can get that distance just by saying, well, it's the speed of the train times how long it takes for the light to get down. [INAUDIBLE] to make that diagonal path, and that time is T0. Same way we've defined it before.

This distance is V times T0. Rate times time. You get a distance. And so the total length the light travels is therefore-- using our little theorem, using the theorem all of you know-- it's just h squared plus V times T0 squared. That's a T. So that's the total distance the light travels. The total time-- the total amount of time that it'll take for the light to travel is therefore this distance divided by the speed of light.

And that distance is-- OK. That time-- I told you from the very beginning that time is T0. So we already know how long it is, but now we'll get an equation in which we can solve for T0. So that equation is now going to be T0 equals the square root of h squared plus V T0 squared over c. And you can solve for T0 now. You can solve for T0 to get an expression for T0. And then you can relate T0 to T, and then you can get this equation. Voila.

This equation is kind of complicated to solve. It's just quadratic, I guess. Yeah, you can do that if you want. But-- you don't really gain anything by solving it. I mean, you know how to solve it. You don't really gain anything. So that's where it comes from. That's where it comes from.

And this whole effect-- this effect of relativity of time intervals is called time dilation. I'll put this right here. I also learned what the word dilate means by learning about time dilation. Time dilation-- they call it dilation because it's a time interval that dilates, like your eyes dilate-- no, your pupils dilate. I learned about that after I learned about time dilation. Yes?

AUDIENCE: Why do all of these equations-- I guess except for that one-- use the speed of light squared? Except for that one.

PROFESSOR: Oh, except for this one? Oh--

AUDIENCE: Why does it have-- does it have to be-- why is it a square?

PROFESSOR: [INAUDIBLE] square? OK. The question is, why does the speed of light appear as a square in these equations? Well, you can see that pretty simply when you try to solve this equation. To solve this equation, you first square both sides so that you can get rid of the square root, and then you have a c squared. And then it travels around the equations and it ends up right there. And something similar happens when you get that equation. Something similar happens. And then once again, this effect is only noticeable for very high speeds, because if V small, then this ratio, V squared over c squared is very small. Then 1 minus a very small number is approximately 1, and then you get T is approximately T0. So the two intervals are approximately the same for everyday phenomena. Yes?

AUDIENCE: But it's still there?

PROFESSOR: Yeah. The phenomenon is still there. Time dilation is always occurring, just to a very-- totally imperceptible extent for humans. But this effect is actually measurable. It's not just theory. We've actually measured this in lots of particle physics experiments.

One characteristic of particles that you can measure is the half-life of a particle. Well-- a single particle doesn't really have a half-life. Half-life is really a statistical property. What I mean by half-life is, if you have many of these identical particles, then after the half-life time, after the half-life, approximately half of the particles will have decayed.

And you can actually notice-- you can notice time dilation for these particles because you're able to accelerate these particles to very high fractions of c, very high fractions of the speed of light. For instance, it might be that usually-- it might be that if you have a particle at rest, then it has a certain half-life-- which might be a microsecond, like one microsecond. And if I make that particle travel at a very high speed, then if I go into that-- I were traveling right next to that particle at that very high speed, then the half-life would still be the same. But if I'm at rest, sitting in the lab at rest, than that half-life will actually be dilated. That half-life will actually be much longer. And that's a direct confirmation of time dilation, which is pretty cool. Let's see. It's-- whoa. An hour has already passed. Does everybody else have 2:30?

AUDIENCE: Yeah.

PROFESSOR: Oh, OK. Wow. Let's take a short break. We'll take a short break for right now. When we come back-- when we come back, we'll talk about length contraction. It turns out the length is actually relative as well.

OK. We saw there's something fishy about time. We saw there something fishy about simultaneity. You saw there's something fishy about speed. Everything that we held so dear, things that we used to rely on-- time, speed, everything we used to rely on so dearly turned out to be just a misconception. And it turns out that there's something fishy about length, too. And by something fishy, I mean that length is relative. I mean that something-- yeah. Length is relative. Now, what exactly do I mean by length? Well, suppose you took an object-- suppose you took an object, you looked at the two ends of it at the same exact time. You wrote down the coordinates of this end, your coordinates of this end, and then you simply found the difference of the two coordinates an then took the absolute value. That's the length. That's the length of an object. You have to measure the coordinates of the endpoints at the same time, because if you don't measure them at the same time, then-- of course that's not length. if I have a water bottle traveling-- well, if I have a water bottle here, I measure the coordinate of this, then a minute later it's here and I measure the coordinate of this, well, of course this coordinate minus this coordinate's not the length. So you have to do it at the same time. It turns out that length is relative. And there's a very simple formula for the relation of lengths that you observe and the lengths that the train observer observes.

Again, let's say I'm on the train. I hold out a pole, and I measure the length of it and you measure the length of it. And we say, how do these two lengths-- we ask, how do these two lengths compare? Let's say that the length of-- let's say that the length of the pole that I'm holding is L0. Well, the length of the pole at rest. Say the length of the pole at rest is L0. And we call this-- we call it the proper length, just as we call T0 the proper time. Those are just words. L0 is-- I'll be more generic, and I'll say L0 is the length of object at rest.

Now I can ask, is that length going to be the same when the object starts moving? And it turns out the answer is no. The answer is no. And I'll call the length of the object when it's moving L, without the 0. L is the length of an object in motion. In motion moving at speed V. And it turns out that these two lengths are related by a simple formula-- namely that L equals L0 times the square root of 1 minus V squared over c squared. There's that quantity again, the square root of 1 minus V square over c squared.

I could derive this one again, similarly to the way I derived that one. But I think this is kind of a longer derivation than the previous one, and we only have a little bit of time left in the class, so I'll just give this to you. However, I will-- I'll send you some notes where I do derive it. I'll send you some notes where I derive the addition of velocities formula, too. I'll fix the square root. OK.

Let's say that you measure-- well, let's say that we measure the length of some stick to be-- some pole to be 10 feet, just to plug in some numbers. Let's say that we let the pole be-- I'll say 10 meters, or-- 10 meters is pretty big, but let's just say 10 meters. It's safe to use meters. It's safe to use meters because I know the speed of light in meters, and we should be consistent with our units. So we'll say L0 is 10 meters. And let's say that V-- well, let's actually say V over c. We'll say that's the fraction of the speed of light that V is. Let's say it's 0.6. So V is actually 60% the speed of light, which is very fast. That's how fast this pole is moving, 60% of the speed of light. And I ask, how fast does it-- I ask, how long is this pole? How long is this pole while it's in motion? And I will have this formula, and I just plug in these numbers. The pole is then 10 meters times the square root of 1 minus 0.6 square, which is 0.36. Square roots. 1 minus 0.36 is 0.64. The square root of 0.64 is 0.8. So the length-- let me fix this. So the length of the pole while it in motion is actually 8 meters. So it contracted. This is called length contraction. Question?

AUDIENCE: Yes. The original equation, L equals L0 times the square root of 1 minus V squared over c squared-- can't you simplify the square root of 1 minus V squared over c squared to 1 minus V over c?

PROFESSOR: You can say-- sure. You can simplify this question by writing 1 minus V squared over c. This is a difference of two squares, so you could simplify it if you wanted to. There's no reason to, but if you wanted to, you could say that 1 minus V squared equals 1 plus V over c times 1 minus V over c. But we don't need to. We're just plugging in numbers. But you could if you want. Question?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Does it matter? OK. Good question, good question.

AUDIENCE: Can you repeat the question?

PROFESSOR: The question is, does it matter which direction that the pole is moving and which direction the pole is oriented? Yeah, it does matter, actually. It does matter. Actually-- if I have a three-dimensional object-- if I have a three-dimensional object, like a box, that's moving at some speed relative to me, then it turns out-- I won't prove it for you. You'll have to take my word. Well, I'll prove it in some notes that I'll send to you.

It turns out that the only direction-- the only direction that the length is contracted is the direction of the box that's the same direction as the motion of the box. So if the box is moving this way, then this dimension is going to be contracted. It'll get shorter this way, but the two perpendicular directions actually are unaffected. They're unaffected, it turns out. It's pretty interesting. And this whole effect is called length contraction. Was there another question? Yes?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Oh, OK. So the question is, what happens as you approach the speed of light? That's a good question. The question is, what happens as you approach the speed of light? And the answer is that the closer that this object in motion approaches-- the closer this object gets to the speed of light, the shorter it will get. So it actually approaches 0 length as it approaches the speed of light.

And similarly, with the time dilation formula, as you approach speed of light, you have some number over a very, very small number, because now V squared over c squared is a very large number that's close to 1, very close to 1, so 1 minus this number is approximately 0, and some number over a number that's approximately 0 is going to be a really large number. So the time interval actually gets very, very dilated. It's much, much longer. The half-lives of the particles I mentioned get much longer than the half-lives of those particles at rest.

So there's something about space and time-- yes?

AUDIENCE: Is there any way to test length contraction?

PROFESSOR: Length? OK.

AUDIENCE: [INAUDIBLE]

PROFESSOR: The question is, are there ways of testing length contraction? I haven't heard of any direct experimental confirmation for length contraction, just because it's so hard to get objects of large size going at really large speeds so they could actually notice it occurring. I don't know of any direct experimental detections. But time dilation, there's a lot of experimental support for. Did you have a question? Oh, no, you're just doing this. OK.

Now, as I was starting to say, time and space are completely different from what you would you'd expect intuitively. They're completely different from how Newton envisioned time and space being. Newton thought that time intervals and space intervals-- which are lengths-- are the same for everybody. And he didn't question it at all. In fact, he said-- he said, "I do not define time, space, place, and motion, because they are well known to all." That's what he said.

Other people have speculated about the nature of time. It's a hard question to ask. What is time? Philosophers have thought about it. Physics have thought about it. Probably lots of other people have thought about it. And I have some interesting quotes that you might like.

Let's see. Here's a good one. Let me see. OK. "Nothing puzzles me more than time and space, and yet nothing troubles me less, as I never think about them." "Either this man is dead, or my watch has stopped." "It's good to reach 100, because very few people die after 100." That doesn't really have anything to do with time, does it? OK. Here's one of my favorites. "Time is nature's way to keep everything from happening at once."

We actually don't really define time in physics. We simply don't define it, because-- well, first of all, it's hard, and second, it might not even be meaningful. So what we do is, we define time operationally. We define time in terms of ticks of clocks, as I was mentioning earlier. And it seems to be a very successful way of doing things, but it makes you wonder, what really is time? it's hard to-- how do you define it, though? You can wonder for hours. What does time mean? And you probably won't get-- what's that?

AUDIENCE: You'll lose time.

PROFESSOR: You'll lose time. Yeah, you'll lose time by pondering time. Yeah, exactly. Exactly. OK. There are couple of paradoxes I wanted to talk about. I can talk about one paradox, one brain-- brain what? What do paradoxes do to brains? They don't-- they twist. Yeah, Brain twisting. I don't know. Brain tweaser? Teaser? Brain teaser. Yeah. OK.

But it's an important paradox, and it has to do with time dilation. It has to do with time dilation. And I'm not going to tell you the answer. I want you to think about it afterwards. It's known as the twin paradox, and it goes as follows. Suppose there's this astronaut who on her 21st-- 21st birthday decides to go out into space in a rocket very fast, very close to the speed of light. I'll give you some numbers, because I have them. You can check these numbers-- let me find it. You can check these numbers in a bit. Where do I have it? Gosh. Oh, here it is.

Suppose that there's an astronaut that travels-- there's this astronaut that travels into space on her 21st birthday, going at 12/13 of the speed of light, 12/13 c. And she goes-- she leaves on Earth and she travels for five years at this speed. She travels for five years at this speed on her clock, and then she decides to head back. So it takes her five years to get back. It takes her five years both directions.

I'll make-- I won't write the things down because I'm running out of time, but I'll send it in an email all precisely so you can think about it. So she goes five years this direction, then she comes back for five-- takes her five years to come back. Now, she'll arrive back on her 31st, birthday because 5 plus 5 is 10, and then we just add 10 to 21. Now, she has a twin. Suppose she has a twin that lives on Earth. And so her twin was also 21 when she left.

But-- the question is, how old will he be-- or she be. I'll say he. How old will he be when she comes back? Well, you can calculate this using this formula. Just plug in 12/13 and then multiply by 10 years, and you get that while it takes 10 years for her to travel out and come back, 26 years have actually passed on his watch. 26 years have actually passed on his watch. So when she's 31-- when she comes back, she's 31, and he's not going be 31. He'll be 47, because 26 plus 21 is 47.

It looks like there's something wrong here, because-- the astronaut twin you can actually do the same calculation. She can say, well-- she can say, well, my twin is moving relative to me-- because, remember, all of these observers are equal. I mean, there's not one preferred speed. They're all as good as any other. They're all as good as any other. She can say, well, my twin is moving-- my twin is moving at 12/13 speed. I'm stationary. So 26-- sorry, so 10 years should pass for my twin, and 26 years should pass for me. That's what the twin should say. You just run the calculation backwards.

All of these equations are-- I didn't emphasize, but all these equations that I've given you-- well, these two right here-- they're symmetric. If I see that-- if I say that you're moving relative to me, then you can equally say, well, you're moving relative to me. Did I just say that, or did I actually reverse it? I meant to reverse it. I meant to reverse it.

So the twin will think that, actually-- sorry, the astronaut twin will think that her Earth counterpart will have aged more than her, but only one of them can actually be true. Only one of the predictions can actually be true. Either the astronaut twin will have aged more than the Earth twin, or the Earth one will have aged more than the astronaut twin, or they'll both have aged the same.

Those are three possibilities. Only one of them can be true, and I'll tell you right now which one is true. I'll tell you right now which one is true. It's not that they both age the same. They haven't both aged the same. Actually, the astronaut twin ages less than the Earth twin. And that seems very crazy, because it's a completely symmetric situation-- or is it? I don't know if that was effective.

I'll let you ponder that, that twin paradox. It has a nice resolution, and I'll let you think about it.

But it goes against-- well, it goes against this perceived symmetry. Maybe it's not actually a symmetry. Maybe it's not. If your hand's up to answer the paradox-- if you're trying to resolve the paradox, then I'll have to ask-- I'll listen to you after class. If you have a different question, then I'll listen to you. Do you have a question?

AUDIENCE: No.

PROFESSOR: Oh, you had an answer. OK. Yeah. I want you guys to think about that paradox. And I'll send you an email with it stated more precisely so that you can really enjoy it, so you can really confused yourselves, so that you can really never sleep. Because I don't want you guys to sleep at all. You shouldn't be sleeping. Great summer. You should be up all night.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah. Should be up all night.