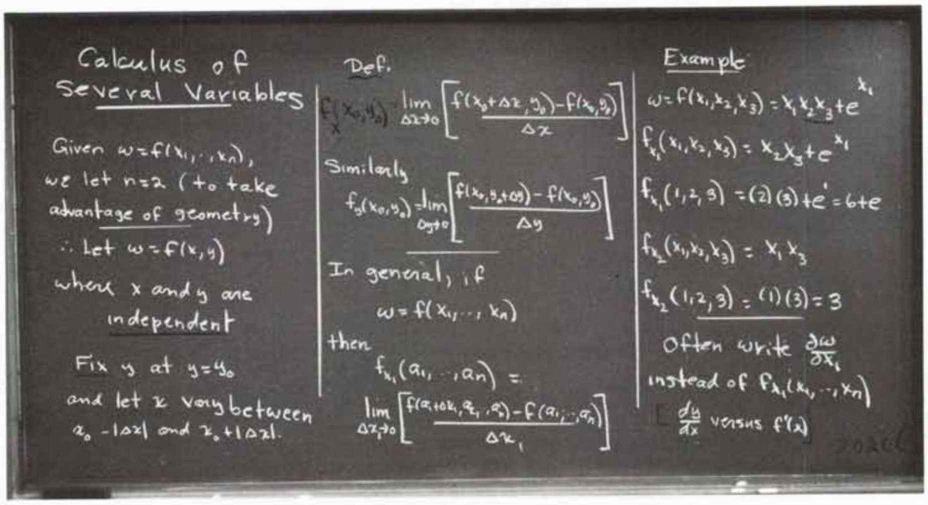
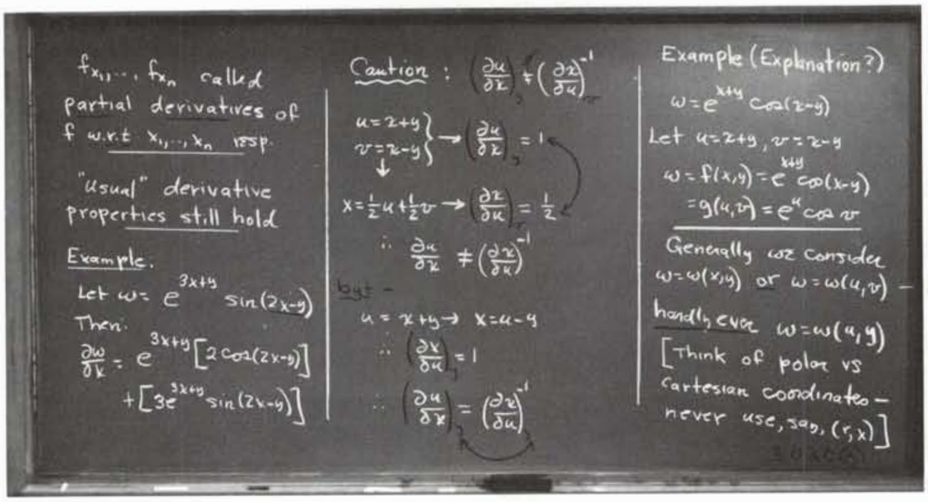
Unit 2: An Introduction to Partial Derivatives

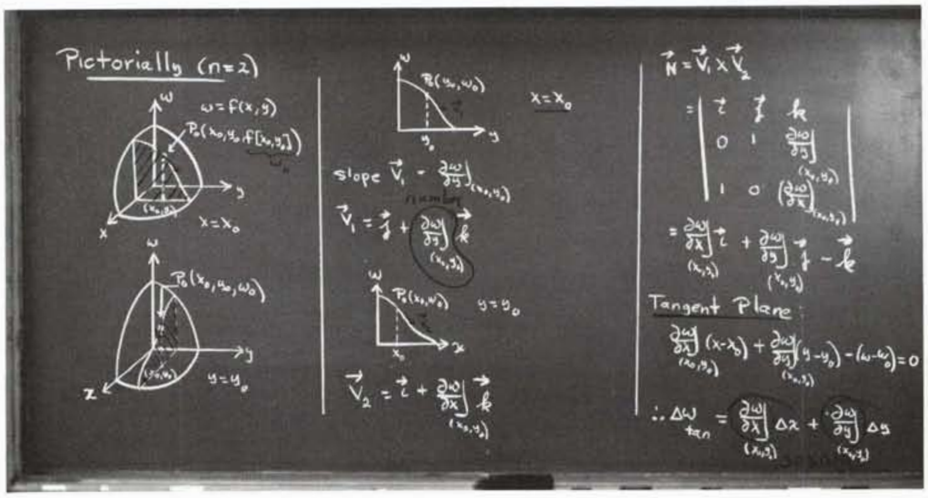
1. Lecture 3.020



a.



b.



c.

2. Read Thomas, Sections 15.2 and 15.3.

3. Exercises:

3.2.1(L)

a. If $f(x,y) = x^2 + y^3$, compute $f_x(1,2)$.

b. If $f(x,y) = x^3y + x^4 + y^5$, compute $f_{xx}(1,2)$ where $f_{xx}(1,2)$ means $\left.\frac{\partial}{\partial x}\left[\frac{\partial f}{\partial x}\right]\right|_{(1,2)}$.

c. Find $f_y(1,2,3,4)$ if $f(w,x,y,z) = w^2xy + z^3y^2 + x^3zw$.

3.2.2

a. Determine $f_w(w,x,y,z)$ and $f_{ww}(w,x,y,z)$ if $f(w,x,y,z) = w^3x^2y + x^3y^2z + wz^4$. In particular, determine $f_{ww}(1,2,3,4)$.

b. Compute $\frac{\partial z}{\partial x}$ if

$$z^3xy + z^5y + \cos z = 1.$$

3.2.3(L)

Let x and y be a pair of independent variables and define u and v by $u = 2x - 3y$ and $v = 3x - 4y$.

a. Show that u and v are then also a pair of independent variables.

b. Solve the above equations and express x and y in terms of u and v. From this compute $\frac{\partial x}{\partial u}$ and compare this with $\frac{\partial u}{\partial x}$.

c. Express x in terms of u and y, and then compute $\frac{\partial x}{\partial u}$. How does this answer compare with the result in (b)?

d. Express u in terms of x and v, and then compute $\frac{\partial u}{\partial x}$. In this case, does $\frac{\partial u}{\partial x} = \frac{1}{\left(\frac{\partial x}{\partial u}\right)}$ where $\frac{\partial x}{\partial u}$ is as in (b)?

3.2.4

Given that x and y are independent variables, define u and v by $u = x^2 - y^2$ and $v = 2xy$.

a. Explain why u and y are also independent variables.

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3.2.4 continued

b. Show that $\left(\frac{\partial x}{\partial u}\right)_y = \frac{1}{\left(\frac{\partial u}{\partial x}\right)_y}$.

c. Determine the value of $\left(\frac{\partial x}{\partial u}\right)_v$.

3.2.5(L)

From the polar coordinate relations, $y = r \sin \theta$ and $x = r \cos \theta$, compute $\frac{\partial \theta}{\partial y}$ where, from now on, $\frac{\partial \theta}{\partial y}$ will be interpreted to mean $\left(\frac{\partial \theta}{\partial y}\right)_x$ unless otherwise specified.

3.2.6

Given that x and y are independent variables, assume that u and v are functions of x and y [we usually denote this as either $u = u(x,y)$ and $v = v(x,y)$, or, if we feel that misinterpretation might arise, as $u = f(x,y)$ and $v = g(x,y)$] such that u and v are also independent variables. Assume further that we also know that u and v are related by $u^2 = y^2v$. Determine $\left(\frac{\partial u}{\partial y}\right)_x$.

3.2.7(L)

Let S be the surface defined by the Cartesian equation $z = x^2 + y^3$. Assume that there is a plane which is tangent to S at the point P(1,2,9). Find the equation of this plane.

3.2.8

Assuming that the surface defined by the Cartesian equation $z = x^3y^2 + x^5 + y^7$ has a tangent plane at the point (1,1,3), find the equation of this plane.

3.2.9(L)

Let the surface S have the Cartesian equation $x = g(y,z)$.

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3.2.9(L) continued

a. Assuming that S possesses a tangent plane at the point (x_0, y_0, z_0), find the equation of this plane.

b. The plane M is tangent to the surface $x = e^{3y-z}$ at the point (1,2,6). Find the equation of M.

c. Check the solution in (b) by expressing $x = e^{3y-z}$ in the form $z = f(x,y)$.

Resource: Calculus Revisited: Multivariable Calculus
Prof. Herbert Gross